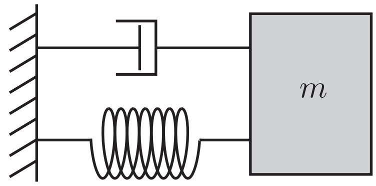
EA3 Quiz 1
Section 21, Spring 2023

Name: ________________________________

You are allowed to use only pens, pencils, and erasers. No electronics, other papers, etc.

Make sure to show all your work, and make sure your final answer is clear (for example, you can circle it). Full credit, or partial credit if your final answer is wrong, will only be given if your thought process is clear. If you think any question does not give you enough information to give an answer (i.e., there is a mistake on the test), then clearly write the extra assumptions you had to make to answer the question, and answer the question using those assumptions.

Use the backs of the pages if you need more room for your work.



1. For the mass-spring-damper system above, with x the extension of the spring and v the velocity of the mass, you've derived the state equations

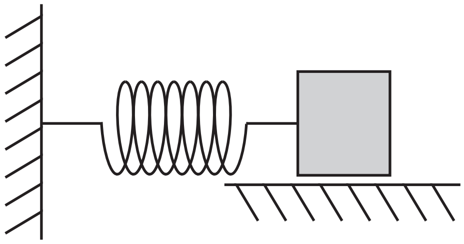
$$\begin{aligned}\dot{x} &= v \\ \dot{v} &= -2x - 3v.\end{aligned}$$

   (a) (2 pts) What is the mass m?

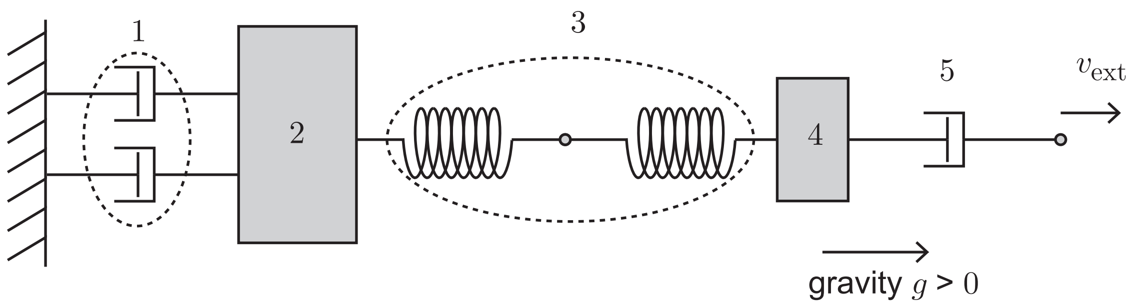
   (b) (4 pts) If the values of the state variables at time 0 are $x(0) = 0$ and $v(0) = 3$, find their values at time $t = 0.2$ s, i.e., $x(0.2)$ and $v(0.2)$, using a single step of forward Euler integration with a timestep of 0.2 s.

   (c) (4 pts) You decide to refine your solution by reducing your timestep to $\Delta t = 0.1$ s. Using this new timestep, use two steps of forward Euler integration to find $x(0.2)$ and $v(0.2)$. (Again, the initial conditions are $x(0) = 0$ and $v(0) = 3$.)

2. (2 pts) A mass m in gravity (g acts downward) slides on the ground with friction coefficient μ. The mass is attached to a spring to a wall. The spring has a spring constant k. The extension of the spring is x ($x = 0$ is relaxed). For what range of x can the mass be resting at equilibrium (not moving)?



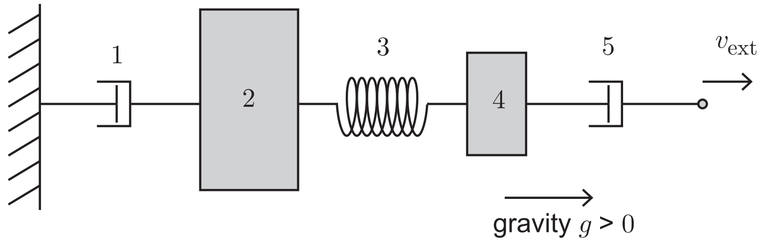
3. Consider the mechanism below. Two dampers are lumped together to make one equivalent damper 1, and two springs are lumped together to make one equivalent spring 3. An external velocity v_{ext} is imposed at one end of damper 5, and gravity $g > 0$ acts to the right.



(a) (2 pts) If the two dampers making up equivalent damper 1 have the same damping constant, what is each damper's damping constant (call it b_{each}) in terms of b_1?

(b) (2 pts) If the two springs making up equivalent spring 3 have spring constants k_A and k_B, what is k_3 in terms of k_A and k_B?

Problem 3 continued. Figure copied from previous page.



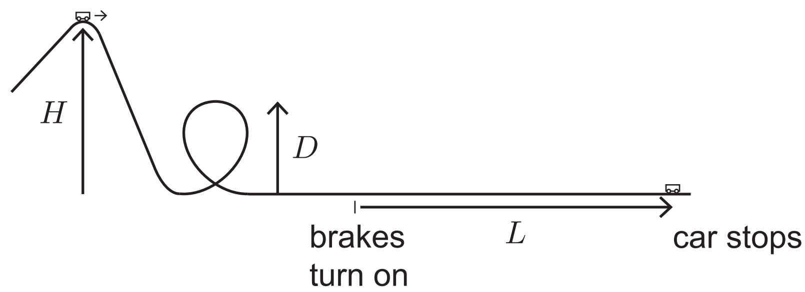
(c) (2 pts) Write the force balance equations at m_2 and m_4.

(d) (2 pts) Write the geometric continuity equations.

(e) (2 pts) Write the constitutive laws (use the lumped values b_1 and k_3).

(f) (3 pts) Choosing the state variables x_3, v_2, and v_4, derive the three state equations. Show your work. If you need to, use the back of the previous page for your work, but make sure your final answers are clearly noted.

4. Your roller coaster car, with a total mass m, starts down from the top of a hill of height H going at a speed v_0. It rolls without friction, through the loop of height D, and onto the flat section. Finally the brakes turn on, creating an effective friction coefficient μ with the track. The car comes to a stop after a distance L.

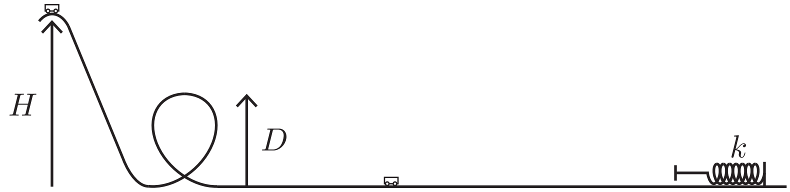


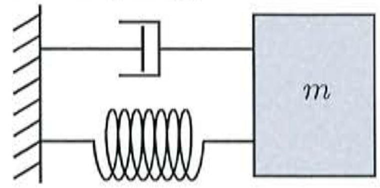
(a) (2 pts) What is the speed of the car v_1 at the top of the loop?

(b) (2 pts) What is the effective friction coefficient μ after the brakes turn on?

(c) (2 pts) What is the maximum negative power (energy dissipated per unit time) at any time by the brakes?

(d) (2 pts) The next time you ride, the brakes fail, so there is no friction on the flat section. The car runs into a big spring at the end of the track, which compresses by x to bring the car to a stop. What is the spring constant k of the spring? What happens after the car is brought to a stop?





1. For the mass-spring-damper system above, with x the extension of the spring and v the velocity of the mass, you've derived the state equations

$$\dot{x} = v$$
$$\dot{v} = -2x - 3v.$$

(a) (2 pts) What is the mass m?

FB: $m\dot{v} = -kx - bv$

$\dot{v} = -\frac{k}{m}x - \frac{b}{m}v$ $\quad$ $\frac{-8}{m} = -2$ and $\frac{-12}{m} = -3$, so $\boxed{m = 4}$

(b) (4 pts) If the values of the state variables at time 0 are $x(0) = 0$ and $v(0) = 3$, find their values at time $t = 0.2$ s, i.e., $x(0.2)$ and $v(0.2)$, using a single step of forward Euler integration with a timestep of 0.2 s.

$x(0.2) = x(0) + 0.2(v(0)) = 0 + 0.2(3) = 0.6$

$v(0.2) = v(0) + 0.2(-2x(0) - 3v(0)) = 3 + 0.2(0-9) = 1.2$

(c) (4 pts) You decide to refine your solution by reducing your timestep to $\Delta t = 0.1$ s. Using this new timestep, use two steps of forward Euler integration to find $x(0.2)$ and $v(0.2)$. (Again, the initial conditions are $x(0) = 0$ and $v(0) = 3$.)

step 1: $x(0.1) = x(0) + 0.1(v(0)) = 0 + 0.1(3) = 0.3$

$v(0.1) = v(0) + 0.1(-2x(0) - 3v(0)) = 3 + 0.1(0-9) = 2.1$

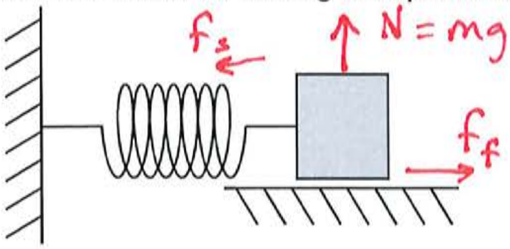
step 2: $x(0.2) = x(0.1) + 0.1(v(0.1)) = 0.3 + 0.1(2.1) = \boxed{0.51 = x(0.2)}$

$v(0.2) = v(0.1) + 0.1(-2x(0.1) - 3v(0.1))$

$= 2.1 + 0.1(-0.6 - 6.3)$

$= 2.1 + 0.1(-6.9) = \boxed{1.41 = v(0.2)}$

2. (2 pts) A mass m in gravity (g acts downward) slides on the ground with friction coefficient μ. The mass is attached to a spring to a wall. The spring has a spring constant k. The extension of the spring is x ($x = 0$ is relaxed). For what range of x can the mass be resting at equilibrium (not moving)?

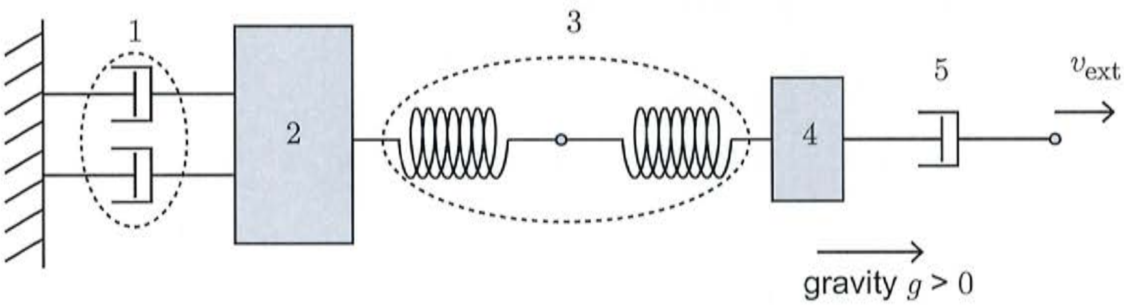


$$f_f = f_s$$

$$\mu mg = kx$$

$$\frac{\mu mg}{k} = x \qquad \boxed{|x| \leq \frac{\mu mg}{k}}$$

3. Consider the mechanism below. Two dampers are lumped together to make one equivalent damper 1, and two springs are lumped together to make one equivalent spring 3. An external velocity v_{ext} is imposed at one end of damper 5, and gravity $g > 0$ acts to the right.



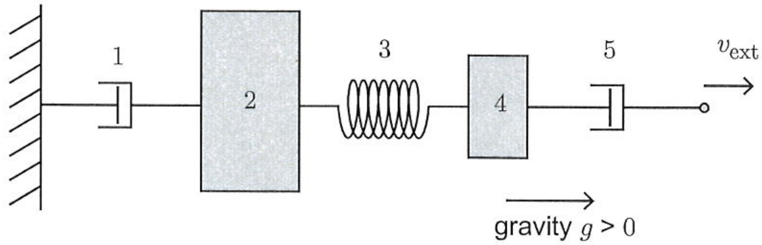
(a) (2 pts) If the two dampers making up equivalent damper 1 have the same damping constant, what is each damper's damping constant (call it b_{each}) in terms of b_1?

parallel: $b_1 = b_{\text{each}} + b_{\text{each}}$, so $b_{\text{each}} = \frac{b_1}{2}$

(b) (2 pts) If the two springs making up equivalent spring 3 have spring constants k_A and k_B, what is k_3 in terms of k_A and k_B?

series: $k_3 = \dfrac{k_A k_B}{k_A + k_B}$

Problem 3 continued. Figure copied from previous page.



(c) (2 pts) Write the force balance equations at m_2 and m_4.

$$-f_1 + f_3 + m_2 g = m_2 a_2, \quad -f_3 + f_5 + m_4 g = m_4 a_4$$

(d) (2 pts) Write the geometric continuity equations.

$$v_2 = v_1 \qquad v_4 + v_5 = v_{ext}$$
$$v_4 = v_2 + v_3$$

(e) (2 pts) Write the constitutive laws (use the lumped values b_1 and k_3).

$$f_1 = b_1 v_1 \qquad f_3 = k_3 x_3 \qquad f_5 = b_5 v_5$$

(optionally $f_2 = m_2 a_2$, $f_4 = m_4 a_4$)

(f) (3 pts) Choosing the state variables x_3, v_2, and v_4, derive the three state equations. Show your work. If you need to, use the back of the previous page for your work, but make sure your final answers are clearly noted.

$$\boxed{\dot{x}_3 = v_3 = v_4 - v_2}$$

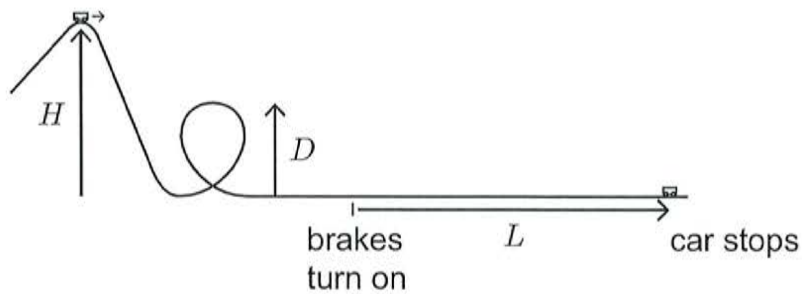
$$a_2 = \dot{v}_2 = \frac{-f_1}{m_2} + \frac{f_3}{m_2} + g = \frac{-b_1 \overset{v_2}{v_1}}{m_2} + \frac{k_3 x_3}{m_2} + g$$

$$\boxed{\dot{v}_2 = \frac{-b_1 v_2}{m_2} + \frac{k_3 x_3}{m_2} + g}$$

$$a_4 = \dot{v}_4 = \frac{-f_3}{m_4} + \frac{f_5}{m_4} + g = \frac{-k_3 x_3}{m_4} + \frac{b_5 \overset{v_{ext} - v_4}{v_5}}{m_4} + g$$

$$\boxed{\dot{v}_4 = \frac{-k_3 x_3}{m_4} + \frac{b_5 (v_{ext} - v_4)}{m_4} + g}$$

4. Your roller coaster car, with a total mass m, starts down from the top of a hill of height H going at a speed v_0. It rolls without friction, through the loop of height D, and onto the flat section. Finally the brakes turn on, creating an effective friction coefficient μ with the track. The car comes to a stop after a distance L.



gravity = g

(a) (2 pts) What is the speed of the car v_1 at the top of the loop?

$KE(0) + PE(0) = KE + PE$

$\frac{1}{2}\cancel{m}v_0^2 + \cancel{m}gH = \frac{1}{2}\cancel{m}v_1^2 + \cancel{m}gD \Rightarrow \frac{1}{2}v_1^2 = \frac{1}{2}v_0^2 + g(H-D)$

$v_1 = \sqrt{v_0^2 + 2g(H-D)}$

(b) (2 pts) What is the effective friction coefficient μ after the brakes turn on?

friction dissipates energy μmgL (force × distance)

initial energy is $\frac{1}{2}mv_0^2 + mgH$. so $\mu mgL = \frac{1}{2}mv_0^2 + mgH$

$\mu = \frac{1}{gL}\left(\frac{1}{2}v_0^2 + gH\right)$

(c) (2 pts) What is the maximum negative power (energy dissipated per unit time) at any time by the brakes?

velocity when brakes turn on = v_2 $\quad \frac{1}{2}\cancel{m}v_2^2 = \frac{1}{2}\cancel{m}v_0^2 + \cancel{m}gH$

$v_2 = \sqrt{v_0^2 + 2gH}$

max neg power $= -fv_2 = -\mu mgv_2$ where μ given above, v_2 here

(d) (2 pts) The next time you ride, the brakes fail, so there is no friction on the flat section. The car runs into a big spring at the end of the track, which compresses by x to bring the car to a stop. What is the spring constant k of the spring? What happens after the car is brought to a stop?

energy in spring $= \frac{1}{2}kx^2$

$\frac{1}{2}kx^2 = \frac{1}{2}mv_0^2 + mgH$

$k = \frac{1}{x^2}\left(mv_0^2 + 2mgH\right)$

Since, by our simple model, there is no dissipation, the car will be launched in reverse, through the loop and back up the hill!