<u>24</u> 1. Determine if the following statements are True or False and circle **T or F**

A) **T / F :** Consider two identical springs hanging from a ceiling. The springs are connected to a bar at the bottom. When the bar is pushed up, these springs experience the same velocity and force.

B) **T / F :** The equivalent electrical circuit for the situation in A above is to have two identical capacitors that are connected in series and nothing else.

C) **T / F :** In a RC (Resistor-Capacitor) circuit, the march (with respect to the initial condition) that is completed towards the steady state in one time constant is (100 x e^{-1})%.

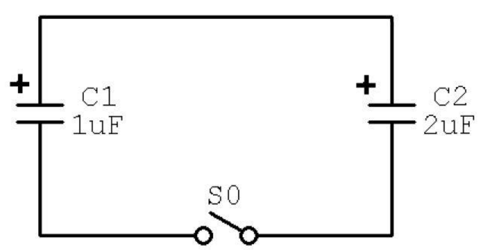
D) **T / F :** In one electrical circuit there are 5 identical capacitors of C μF each. There are three capacitors in parallel and then the three are connected in series to two capacitors with one on each side. The overall capacitance of this arrangement is (3/7)*C μF.

E) **T / F :** RLC circuit is energized with a 6V battery source and a switch. The switch is closed and a steady state is reached. At steady state the voltage across the capacitor is 2V.

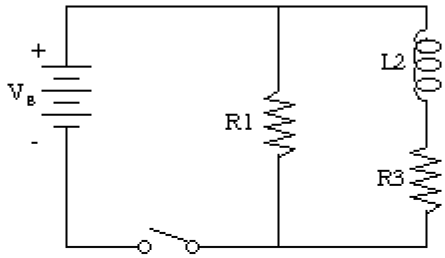
F) **T / F :** In a RC circuit, the capacitor discharge voltage has an exponential decay while in a stable RLC circuit the response may be oscillatory with decaying amplitude.

G) **T / F :** These units are dimensionally consistent: Amperes; Coulomb/Sec; Volt*Henry/Sec

H) **T / F :** In the diagram below, the initial voltages are V1=20V and V2=5V. Next the switch is closed till the steady state is reached. The V1 will now be 10V



52 2. Refer to the figure below.



6 a) Write two independent KVL loop equations
6 b) Write two KCL loop equations
6 c) Write three constitutive laws
10 d) Develop the one-variable differential equation for the current through the inductor, I_{L2}
4 e) What is the initial condition for the above equation?
10 f) What are the steady state values of V_{R1}; V_{R3}; V_{L2}; $I_{R1;}$ and I_{L2}
4 g) What are four descriptors of the differential equation above?
6 h) What is the characteristic equation for the differential equation above and the root(s)?

24 3. Analyze the following arrangements connected to 120V source.

8 a) Two strings of 20 lamps “1W 6V” each are daisy-chained (series connection). What is the current through each lamp and voltage across each lamp?

8 b) Two strings of 20 lamps “1W 6V” each are connected in parallel. What is the current through each lamp and voltage across each lamp?

8 c) One 20-lamps string consists of 17 lamps “1W 6V” each and 3 lamps of “1W 3V” each. What is the current through each lamp and voltages across each type of lamp?

Quiz 3

1\. A) T

Velocity for springs in parallel will be same. As the springs are identical forces will also be same.

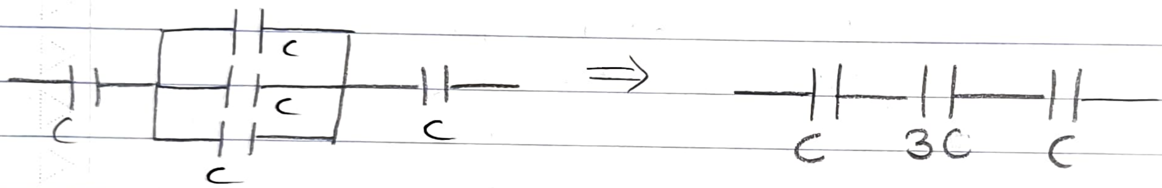
B) F

A battery to deliver EMF will also be required

C) T

$V = V_0 e^{-t/RC} \Rightarrow (100 \times e^{-1})\%$ in one time constant

D) T



$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{3C} + \frac{1}{C} = \frac{3+1+3}{3C} = \frac{7}{3C}.$$

$$\boxed{C_{eq} = \frac{3}{7}C.}$$

E) F

At steady state the voltage should be 6V across capacitor as current through circuit is zero ⇒ so voltage drop across resistor is zero. Also at steady state voltage across inductor is zero. [Current is 0 as $i_{cap} = 0$ @ steady state]

F) T

$V_c = V_0 e^{-t/RC} \Rightarrow$ exponential decay

RLC can be oscillatory

G) F

$\text{Ampere} = \frac{\text{Volt} * s}{\text{Henry}}$ [from $V = L\frac{di}{dt}$]

H) T

Start $Q_1 = C_1 V_{C_1} = 1 \times 20 = 20\mu C$

$Q_2 = C_2 V_{C_2} = 2 \times 5 = 10\mu C$

$Q_{total} = Q_1 + Q_2 = 30\mu C$

Total charge will be conserved & at SS, both voltages woul
be equal for zero current

$Q_{total} = 30\mu C = Q_{1s} + Q_{2s}$

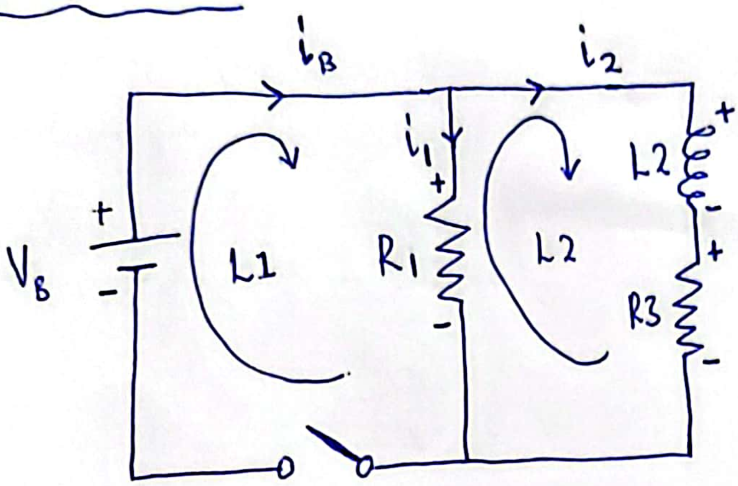
$\frac{Q_{1s}}{C_1} = \frac{Q_{2s}}{C_2} \implies Q_{2s} = 2Q_{1s}$

$Q_{1s} + 2Q_{1s} = 30\mu C$

$Q_{1s} = 10\mu C \implies V_{1s} = \frac{Q_{1s}}{C_1} = \frac{10\mu C}{1\mu F} = 10V$

also, $V_{2s} = \frac{Q_{2s}}{C_2} = \frac{20\mu C}{2\mu F} = 10V$

QUESTION 2



(a) Write two independent KVL loop equations

KVL

$V_B - V_{R1} = 0$ or $V_B = V_{R1}$ (3 points)

$V_{R1} - V_{L2} - V_{R3} = 0$ (3 points)

(6 total)

(b) Write two KCL loop equations

$i_B = i_1 + i_2$ (3 points)

$i_2 = i_{L2} = i_{R3}$ or $i_{L2} = i_{R3}$ (3 points)

(6 total)

(c) Write three constitutive laws:

$V_{R1} = i_{R_1} R_1$ (2 marks)

$V_{L2} = L_2 \dot{i}_{L2}$ (2 marks)

$V_{R3} = i_{R3} R_3$ (2 marks)

(6 total)

(d) Develop the one-variable differential equation for the current through the inductor, i_{L2}

From: $V_{L2} = L_2 i'_{L2}$

$$i'_{L2} = \frac{V_{L2}}{L_2}$$ (2 marks)

From: $V_{L2} = -V_{R3} + V_{R1}$

$$i'_{L2} = \frac{-V_{R3} + V_{R1}}{L_2}$$ (3 marks)

From: $V_{R3} = i_{R3} R_3$

$$i_{R3} = i_{L2}$$

$$\boxed{V_{R3} = i_{L2} R_3 \qquad V_{R1} = V_B}$$ (2 marks)

$$i'_{L2} = -\frac{i_{L2} R_3}{L_2} + \frac{V_B}{L_2}$$ (3 marks)

① $$\boxed{i'_{L2} + \frac{i_{L2} R_3}{L_2} - \frac{V_B}{L_2} = 0}$$

or ↓ differentiating further

② $$\boxed{i''_{L2} + i'_{L2}\frac{R_3}{L_2} = 0}$$

Total (10 marks).

(e) What is the initial condition for the above equation?

$i_L(t=0) = 0$ (4 marks)

(f) What are the steady state values of $V_{R1}, V_{R3}, V_{L2}, I_{R1}$, and I_{L2}

$V_{R1} = V_B$ (2 marks)

$V_{L2} = 0$

From constitutive law

$V_{L2} = L_2 i_2'$, $i_2' = 0$

(2 marks)

$V_{R3} = V_{R1} = V_B$ (2 marks)

$i_{R1} = \frac{V_{R1}}{R_1} = \frac{V_B}{R_1}$ (2 marks)

$i_{L2} = i_{R3} = \frac{V_B}{R_3}$ (2 marks)

10 marks Total.

(g) What are four descriptors of the differential equation above?

(second order for (2))

1. First order differential equation
2. Linear
3. Non homogenous (Homogenous for (2))
4. Constant coefficient.

(1 mark each)

(4 marks total)

(h) What is the characteristic equation for the differential equation above and the roots

For (d) (2)

$$i''_{L_2} + i'_{L_2}\frac{R_3}{L_2} = 0$$

$$\lambda^2 + \frac{R_3}{L_2} = 0$$

(3 marks)

$$\boxed{\lambda = \pm i\sqrt{\frac{R_3}{L_2}}}$$

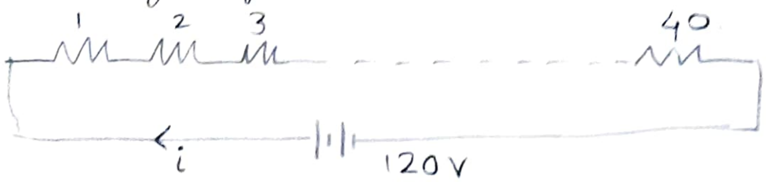
(1.5 marks each)

(3 marks)

(6 marks Total)

3) Source = 120 V

a) 2 Strings of 20 lamps (1W, 6V) in series.



(2 marks)

$P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} = \frac{6^2}{1} = 36\,\Omega$

$R_{lamp} = 36\,\Omega$ (2 marks)

$R_{eq} = 36 + 36 + 36 + \dots 36 = 36 \times 40 = 1440\,\Omega$

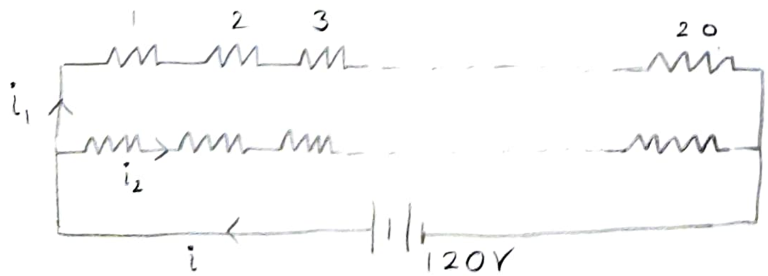
Current (i) $= \frac{V_{source}}{R_{eq}} = \frac{120}{1440} = \frac{1}{12} A = 0.0833 A$ (2 marks)

Current through each of the lamps will be 0.0833 A as they are in series.
As the lamps are identical, voltage drop across each will also be identical (as current is also same)

Voltage across lamp 1 $(V_1) = i \times R_{lamp} = \frac{1}{12} \times 36 = \underline{3 V}$ (2 marks)

Hence, voltage across each lamp is 3V. Hence, the lamps would glow but considering the voltage is lower than 6V, they will be dim.

b)



$R_{lamp} = \frac{V^2}{P} = \frac{6^2}{1} = 36\,\Omega$

(2 marks)

Equivalent resistance of string 1 $= 20 \times R_{lamp} = 20 \times 36 = 720\,\Omega$

Similarly, Eq resistance of string 2 $= 720\,\Omega$ (2 marks)

$R_{eq\,(circuit)} = \frac{(R_{s_1 eq}) \times (R_{s_2 eq})}{(R_{s_1 eq} + R_{s_2 eq})} = \frac{720 \times 720}{2 \times 720} = 360\,\Omega$

Total Current (i) = $\frac{V_{source}}{R_{eq}} = \frac{120}{360} = \frac{1}{3} A = 0.3333 A$

As the equivalent resistance of both strings are identical, current passing through each string can be given as.

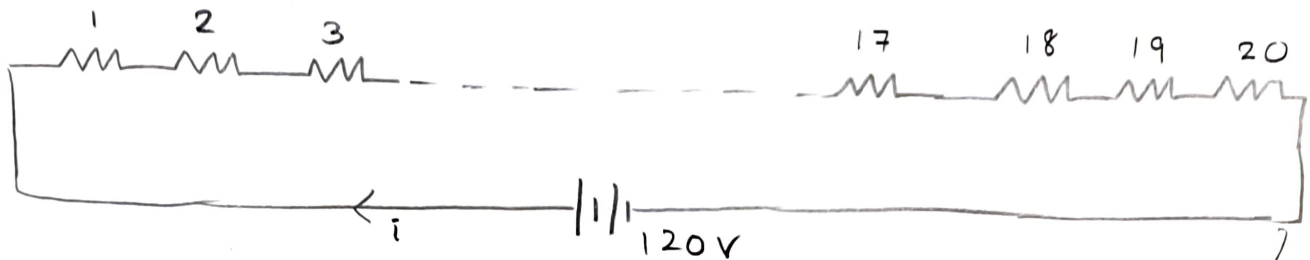
$i_1 = i_2 = \frac{i}{2} = \frac{1}{6} A = 0.1667 A.$ (2 marks)

Voltage across any lamp (V_{lamp}) = $i_1 \times R_{lamp} = \frac{1}{6} \times 36 = 6V$ (2 marks)

[As currents in each string are equal & current in series remains same through all variable, voltage across all lamps is 6V]

As the voltage across the lamp matches the rated voltage, the lamps would glow brightly

c)



Lamp 1-17 ⇒ "1W 6V" & lamp 18-20 ⇒ "1W 3V" (2 marks)

$R_{1-17} = \frac{V^2}{P} = \frac{36}{1} = 36\,\Omega$; $R_{18-20} = \frac{V^2}{P} = \frac{3^2}{1} = 9\,\Omega$

$R_{eq} = R_{1-17} \times 17 + 3 \times R_{18-20} = 17 \times 36 + 3 \times 9 = 639\,\Omega$ (for series)

Current (i) = $\frac{V_{source}}{R_{eq}} = \frac{120}{639} = 0.18779 A$ (2 marks)

Current across all elements will be 0.18779 A (as all in series)

Voltage across any element/lamp from 1-17 is given as

$V_{1-17} = i \times R_{1-17} = 0.18779 \times 36 = 6.76 V$ (2 marks)

Similarly, voltage across any lamp from 18-20 can be given as.

$V_{18-20} = i \times R_{18-20} = 0.18779 \times 9 = 1.69\,V$. (2 marks)

Herein, as the voltage across lamps 1 to 17 is slightly higher than 6V, they will flash brightly & soon burnout

Case (b) gives best working illumination.