ENGINEERING ANALYSIS 3

SYSTEM DYNAMICS

Quiz No. 3

June 2, 2023

Name : ____

You may use a calculator on this quiz. There are 2 problems with multiple sections. The quiz is out of 100 points.

Clearly mark your answers (circle your final answers, wherever applicable) and detail your problem-solving process. Credit will primarily be rewarded based on process (which demonstrates your conceptual understanding of the material) rather than results.

In this quiz, all resistors, capacitors, inductors, and voltage sources are ideal and linear. All wires are assumed to be perfect conductors.

Please specify units wherever necessary.

Potentially Useful Equations

Potential Energy = mgh

Elastic Potential Energy = ½ k x^2

Work = $\int F dx$

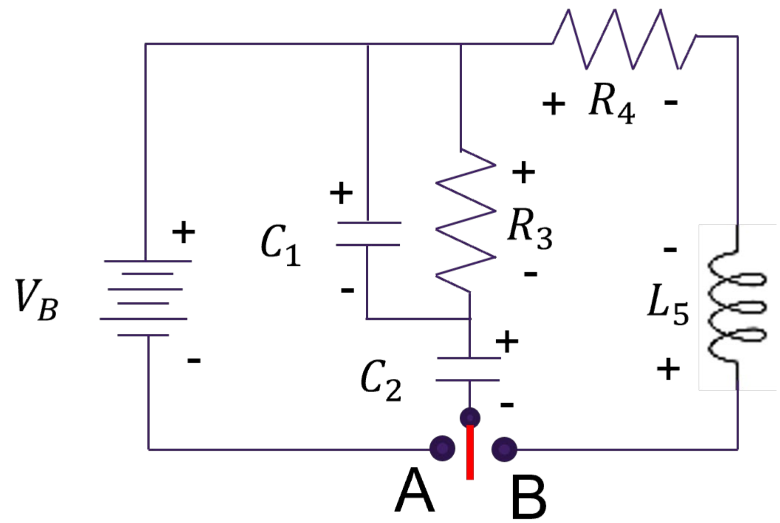
Kinetic Energy = ½ m v^2

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Do not start solving this quiz until you are told to do so.

Problem 1	_______	/ 30 points
Problem 2	_______	/ 70 points
Total	_______	/ 100 points

<u>Problem 1:</u> In the diagram below, the "switch" is flipped to location "A." After charging for a long time, the switch is flipped to location "B" at t=0. Solve the following prompts <u>using the defined voltage polarities</u>. (**<u>30 points</u>**)



<u>Step 1</u>: Determine the initial voltage and current through each element in the circuit the instant <u>after</u> the switch is flipped to location "B". (**<u>10 points</u>**)

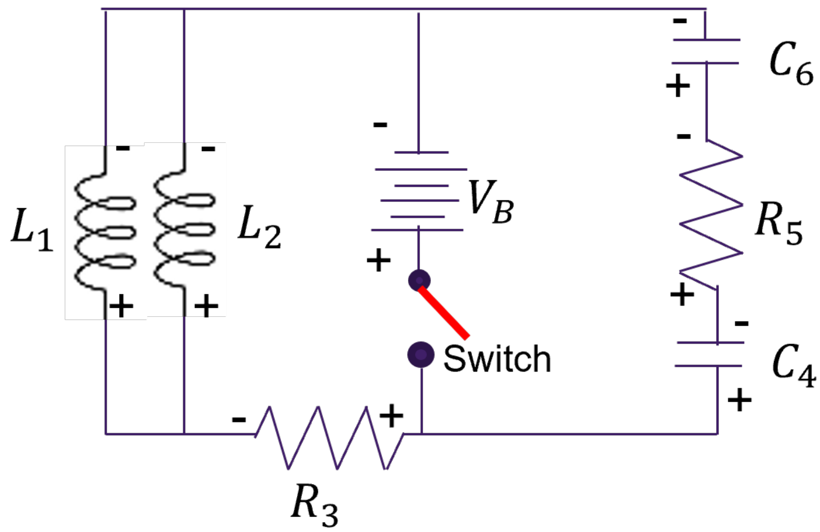
	Voltage	Current
C_1		
C_2		
R_3		
R_4		
L_5		

Step 2: Determine the voltage and current through each element in the circuit at steady state (i.e. a long time after the circuit is closed) (**10 points**)

	Voltage	Current
C_1		
C_2		
R_3		
R_4		
L_5		

Step 3: In the state where the switch is in location B, draw a mechanical translational system (of springs, masses, and dampers) which will display an identical dynamic behavior to this circuit. (**10 points**)

Problem 2: At t=0, the "switch" is closed to complete the circuit through the battery. Prior to closing the switch, all currents and voltages are zero. (**70 points**)



Step 1: Write at least 3 equations relating the currents through each element using Kirchoff's Current Law (**5 points**)

Step 2: Write at least 3 equations relating the voltages across each element using Kirchoff's Voltage Law (**5 points**)

Step 3: Write at least 3 equations describing the constitutive relationships which define the behavior of each electrical element in the system (you may exclude the battery from this step). (**3 points**)

Step 4: Define the state equations and state variables for this system. (**2 points**)

Step 5: Solve for each state equation in terms of the state variables. (**10 points**)

Step 7: Two Options: **Do not do both**

Option 1: Form two linear differential equations; one for $\boldsymbol{V_{C4}}$ and one for $\boldsymbol{i_{L1}}$.

Option 2: *Write the particular solution to the differential equations* for $\boldsymbol{V_{C4}}$ and $\boldsymbol{i_{L1}}$ *(hint: you may do this without first deriving the differential equation).*

Either solution is valid, and is worth **10 points**.

*For either option: you **may** choose to "lump" components in series and in parallel, as needed. If you do this, ensure that all algebraic relationships are clearly defined (e.g. for two springs in parallel: $k_{eq} = k_1 + k_2$).*

Step 8: Given:

$$V_B = \textbf{6 Volts}$$

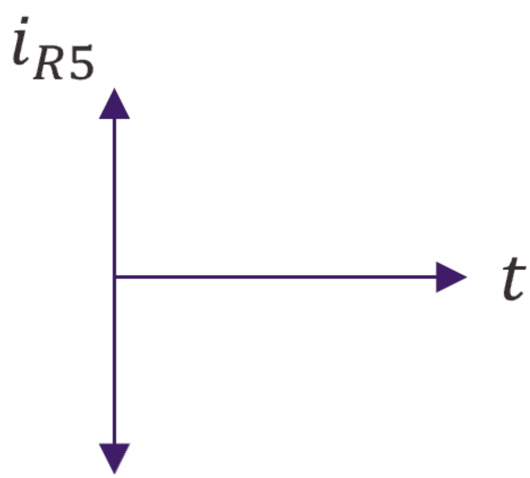
$$R_3 = R_5 = \textbf{2 Ohms}$$

$$L_1 = L_2 = \textbf{2 Henries}$$

$$C_4 = C_6 = \textbf{10 Farads}$$

Write a particular solution which describe the dynamics of $\boldsymbol{i_{R5}}$ as a function of time (**10 points**).

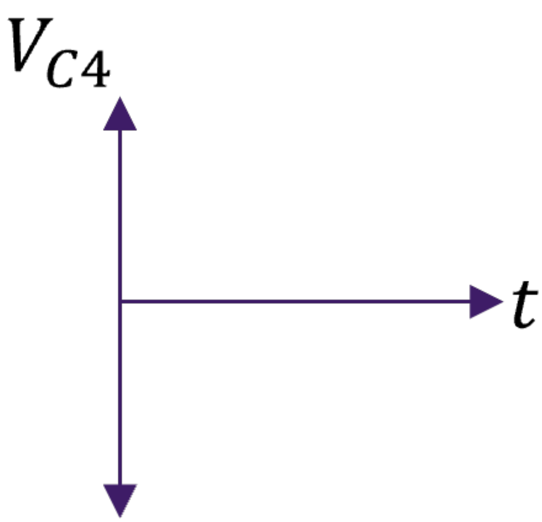
Step 10: Graph the behavior of $\boldsymbol{i_{R5}}$, indicating the initial value, the initial slope, settling time (i.e. 5*Time Constant), oscillating frequency (if necessary), and settling point. *Specify Units.* (**10 points**)



Parameter	Value (units)
Value at t=0+	
Slope at t=0+	
Oscillating Frequency (if irrelevant, write "N/A")	
Settling time (i.e. $5 \times Time\ Constant$)	
Settling Point	

<u>Step 11</u>: Find the total energy dissipated through resistor 5 during charging. *Specify Units* (**<u>10 points</u>**)

<u>Step 12:</u> After a long time of charging, the switch is flipped off. A differential equation is formed for this new circuit, which has roots of **-2 ± i.** Sketch the behavior of V_{C4}, indicating the initial value, rate of decay, oscillating frequency (if necessary), and the settling point of each graph. (**<u>5 points</u>**)



Parameter	Value (units)
Value at t=0+	
Oscillating Frequency (if irrelevant, write “N/A”)	
Decay Rate	
Settling Point	

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Name : ___________________Answer Key__________________________________

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Potentially Useful Equations

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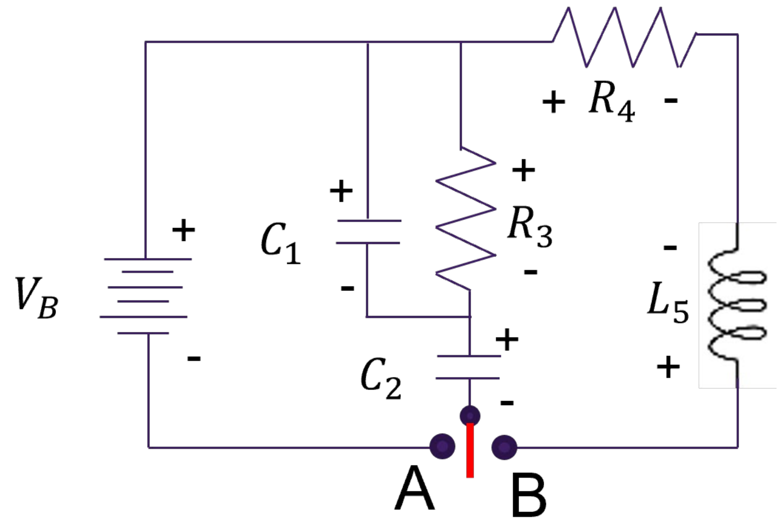
Do not start solving this quiz until you are told to do so.

Problem 1	_______	/ 30 points
Problem 2	_______	/ 70 points
Total	_______	/ 100 points

<u>Problem 1:</u> In the diagram below, the "switch" is flipped to location "A." After charging for a long time, the switch is flipped to location "B" at t=0. Solve the following prompts <u>using the defined voltage polarities</u>. (**<u>30 points</u>**)



<u>Step 1</u>: Determine the initial voltage and current through each element in the circuit the instant <u>after</u> the switch is flipped to location "B". (**<u>10 points</u>**)

	Voltage	Current
C_1	0	0
C_2	V_B	0
R_3	0	0
R_4	0	0
L_5	$-V_B$	0

<u>Non-zero values: +3 pts each ; -2 pts for sign errors</u>

<u>-2 pts for each other error; up to -10 pts total</u>

Step 2: Determine the voltage and current through each element in the circuit at steady state (i.e. a long time after the circuit is closed) (**10 points**)

	Voltage	Current
C_1	0	0
C_2	0	0
R_3	0	0
R_4	0	0
L_5	0	0

0 pts if any non-zero values are entered

Step 3: In the state where the switch is in location B, draw a mechanical translational system (of springs, masses, and dampers) which will display an identical dynamic behavior to this circuit. (**10 points**)

If drawing is accurate:+10 pts

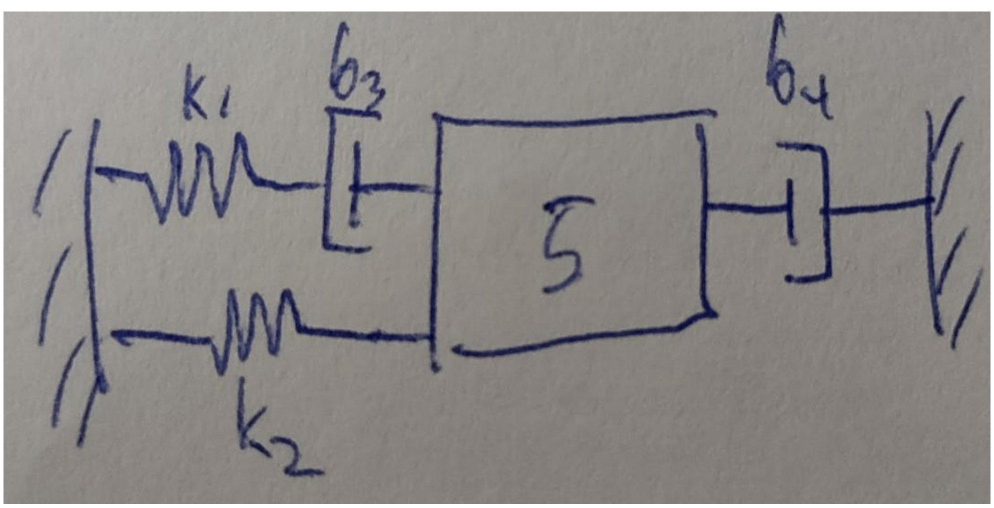
If drawing is inaccurate:

+3 pts for translating KCL

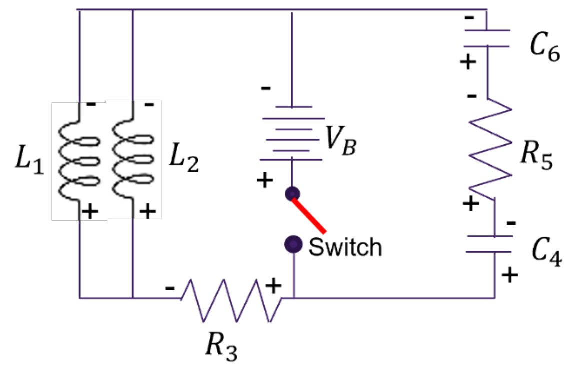
+3 pts for translating KVL

-1 pt for sign errors

-2 pts for very minor drawing errors



Problem 2: At t=0, the "switch" is closed to complete the circuit through the battery. Prior to closing the switch, all currents and voltages are zero. (**70 points**)



Step 1: Write at least 3 equations relating the currents through each element using Kirchoff's Current Law (**5 points**) (+1 pt for each; +2 pts for a set of 3; -2 for extra, inaccurate equations)

$$i_{c4} = i_{R5}$$

$$i_{c4} = i_{c6}$$

$$i_{L1} + i_{L2} = i_{R3}$$

Step 2: Write at least 3 equations relating the voltages across each element using Kirchoff's Voltage Law (**5 points**) (+1 pt for each; +2 pts for a set of 3 lin. indep. Eqs.)

$$V_{L1} = V_{L2}$$

$$V_B = V_{L1} + V_{R3}$$

$$V_B = V_{C4} + V_{R5} + V_{C6}$$

Step 3: Write at least 3 equations describing the constitutive relationships which define the behavior of each electrical element in the system (you may exclude the battery from this step). (**3 points**) (+1 pt for each)

$$V_C' = \frac{1}{C} i_c$$

$$V_R = i_R R$$

$$V_L = L i_L'$$

Step 4: Define the state equations and state variables for this system. (**2 points; 0.5 pt for each**): SVs: VC4, VC6, iL1, iL2; SE's: VC4', VC6', iL1', iL2'

Step 5: Solve for each state equation in terms of the state variables. (**10 points**)

+2 pts for each; bonus 2 for full set. Deduct 1 point per eq. for minor errors.

$$V'_{C4} = \frac{1}{R_5 C_4}(V_B - V_{C4} - V_{C6})$$

$$V'_{C6} = \frac{1}{R_5 C_6}(V_B - V_{C4} - V_{C6})$$

$$i'_{L1} = \frac{1}{L_1}(V_B - (i_{L1} + i_{L2})R_3)$$

$$i'_{L2} = \frac{1}{L_2}(V_B - (i_{L1} + i_{L2})R_3$$

Step 7: Two Options: **Do not do both**

Option 1: Form two linear differential equations; one for $\boldsymbol{V_{C4}}$ and one for $\boldsymbol{i_{L1}}$.

Option 2: *Write the particular solution to the differential equations* for $\boldsymbol{V_{C4}}$ and $\boldsymbol{i_{L1}}$ *(hint: you may do this without first deriving the differential equation).*

Either solution is valid, and is worth **10 points**.

*For either option: you **may** choose to "lump" components in series and in parallel, as needed. If you do this, ensure that all algebraic relationships are clearly defined (e.g. for two springs in parallel:* $k_{eq} = k_1 + k_2$*).*

Option 1 (+2 pts each; +2 bonus for set)

$$V''_{C4} + \frac{1}{R_5 C_{eq}} V'_{C4} = 0$$

$$C_{eq} = \frac{C_4 C_6}{C_4 + C_6}$$

$$i_{L1}'' + \frac{R_3}{L_{eq}} i'_{L1} = 0$$

$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

Option 2 (+2 pts each; +2 bonus for set)

$$V_{C4} = \frac{V_B}{1 + \frac{C_4}{C_6}}\left(1 - e^{-\frac{t}{R_5 C_{eq}}}\right)$$

$$C_{eq} = \frac{C_4 C_6}{C_4 + C_6}$$

$$i_{L1} = \frac{V_B}{R_3\left(1 + \frac{L_1}{L_2}\right)}\left(1 - e^{-\frac{R_3}{L_{eq}} t}\right)$$

$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

<u>Step 8</u>: Given:

$$V_B = \textbf{6 Volts}$$

$$R_3 = R_5 = \textbf{2 Ohms}$$

$$L_1 = L_2 = \textbf{2 Henries}$$

$$C_4 = C_6 = \textbf{10 Farads}$$

Write a particular solution which describe the dynamics of $\boldsymbol{i_{R5}}$ as a function of time (**<u>10 points</u>**).

+4 pts: $i_{R5} = C_4 V'_{C4}$

If option 1 from previous step:

+2 pts: Solve for roots of V'_{C4}: $r = -\frac{1}{R_5 C_{eq}} = -\frac{1}{10}$

+2 pts: Solve for coefficient A=3 using i_R5 (t=0)

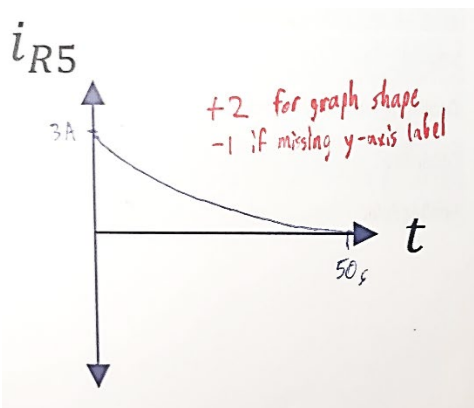
+2 pts: $i_{R5} = 3e^{-\frac{t}{10}}$

If option 2 from previous step:

+4 pts: $V'_{C4} = -\frac{3}{10}e^{-\frac{t}{10}}$

+2 pts: $i_{R5} = 3e^{-\frac{t}{10}}$

<u>Step 10</u>: Graph the behavior of $\boldsymbol{i_{R5}}$, indicating the initial value, the initial slope, settling time (i.e. 5*Time Constant), oscillating frequency (if necessary), and settling point. *Specify Units.* (**<u>10 points</u>**)



-1 for wrong/missing units

Parameter		Value (units)
Value at t=0+	+2	3 Amps
Slope at t=0+	+2	-0.3 Amps/sec
Oscillating Frequency (if irrelevant, write "N/A")		N/A
Settling time (i.e. 5 × *Time Constant*)	+2	50 sec
Settling Point	+2	0 Amps

Step 11: Find the total energy dissipated through resistor 5 during charging. *Specify Units* (**10 points**)

$+2\ pts: Pow = i_{R5}^2 R_5$

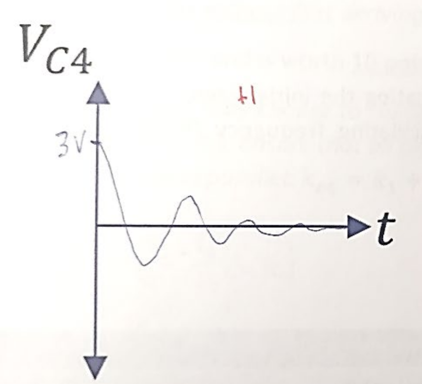
$$Pow = \left(3e^{-\frac{t}{10}}\,Amps\right)^2 (2\Omega) = 18e^{-t/5}\ Watts$$

+4 pts: $E = \int_0^\infty Pow\,dt = \int_0^\infty 18e^{-t/5}\,dt$

+4 pts: $E = 90\ Joules$

Deduct 1 pt for missing units; 1 additional point for each step with errors

Step 12: After a long time of charging, the switch is flipped off. A differential equation is formed for this new circuit, which has roots of **-2 ± i.** Sketch the behavior of V_{C4}, indicating the initial value, rate of decay, oscillating frequency (if necessary), and the settling point of each graph. (**5 points**)



$V_{c4}(0) = \frac{V_B}{1+\frac{C_4}{C_1}} = \frac{V_B}{2} = 3\ Volts$

-1 for wrong graph shape

-1 for wrong/missing units

Parameter	Value (units)
Value at t=0+ +1	3 Volts
Oscillating Frequency (if irrelevant, write "N/A") +1	1 $\frac{rad}{s}$
Decay Rate +1	2 $\frac{1}{s}$
Settling Point +1	0 Volts