ENGINEERING ANALYSIS 3
SYSTEM DYNAMICS
Section 20

Quiz 2, Nov 13 2023

Name: (1pt)_____ **Please also put your name on the back of the last page!**

Only a calculator is allowed during quiz. No other electronic devices (phones, tablets, laptops, watches, etc.) are allowed. No notes or scratch paper.

There are 3 problems. Do not ask for clarification of the questions; if you think that there is an ambiguity, clearly state your assumption and continue to answer the question.

Please show all work, and clearly mark your answers in the PROVIDED BOXES. No points will be awarded for indicating your answer in any other way.

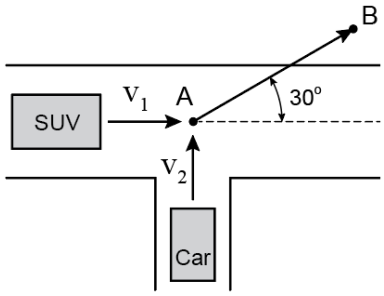
Good luck!

Problem	Points
1	
2	
3	
Total	

Quadratic formular for: $ax^2 + bx + c = 0$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. (30 points) Police are investigating a car crash involving an SUV and a car driving on two perpendicular streets, as shown in the figure. The car failed to yield to the cross traffic and collided with the SUV at the junction point A. It is a plastic impact (e=0), which results in the two vehicles becoming entangled and remaining together after the collision. The two vehicles slide over a distance of 40 meters before reaching a full stop at point B. Consider the coefficient of kinetic friction to be μ=0.5. The mass of the SUV is m_1=3,000 Kg, and the mass of the car is m_2=2,000 Kg. Use gravitational acceleration g=10 m/s^2.
Clearly write your answer in the box for each part below.



1-1. The driver of the car claims the SUV was speeding over the posted speed limit of 55 MPH. Can you calculate the speed of the SUV right before the collision? If so, what is the SUV's speed right before the collision?

[QUIZ CONTINUES ON NEXT PAGE]

1-2. Calculate the energy loss during the collision:

1-3. It's a snowy day, and the coefficient of kinetic friction reduces to $\mu=0.1$ on the icy surface. After the same plastic collision, how far do the two vehicles slide before reaching a full stop?

[QUIZ CONTINUES ON NEXT PAGE]

2. (24 Points) You are given the state equations below, for a two-state mechanical system as $\overrightarrow{X'} = A\vec{X}$ where $\vec{X}$ is the state vector.

$$\vec{X} = \begin{Bmatrix} x_1 \\ v_2 \end{Bmatrix}$$

and A is the square matrix

$$A = \begin{bmatrix} 0 & -1 \\ 4 & -3 \end{bmatrix}$$

Clearly write your answer in the box for each part below.

2-1. Write the state equations as a single second order equation in the variable x_1.

2-2. Consider a trial solution of the form $x_1(t) = Ce^{rt}$ where r and C are both constants. Find the characteristic equation for *r*.

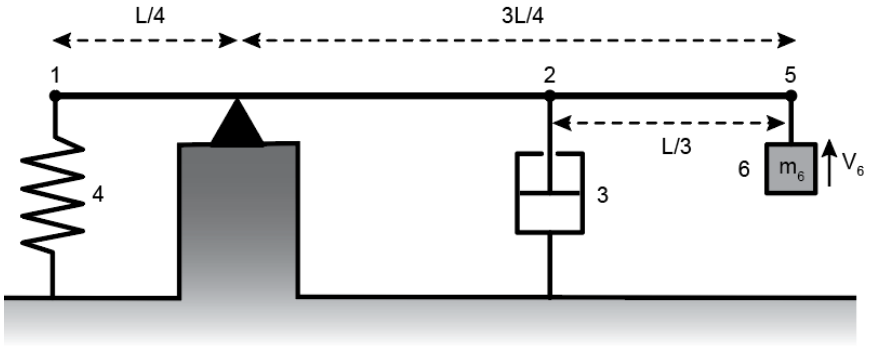
2-3. Write down the root(s) of the characteristic equation that you have found.

[QUIZ CONTINUES ON NEXT PAGE]

2-4. Write down the general solution $x_1(t)$ in real form. What kind of motion (i.e. oscillatory, damped oscillations, exponential decaying etc.) does this represent?

[QUIZ CONTINUES ON NEXT PAGE]

3. (45 pts) The rigid rod of length L in the diagram above pivots about a fulcrum ¼ of the way from its left end. A damper 3 with damping constant b_3, a spring 4 with spring constant k_4, and a mass m_6 with gravity acting upon it are attached at the points indicated. Use the upward direction as your positive sign convention for forces acting on the lever and velocity of the lever at points 1, 2, and 5. Label all parameters and dynamic variables with subscripts corresponding to the element numbers. Clearly write your answer in the box for each part below.



Solve the following:

3-1. At static equilibrium, express the extension of the spring (x_{equil}) in terms of m_6, g, *and* k_4:

3-2. The independent state variables for this system are:

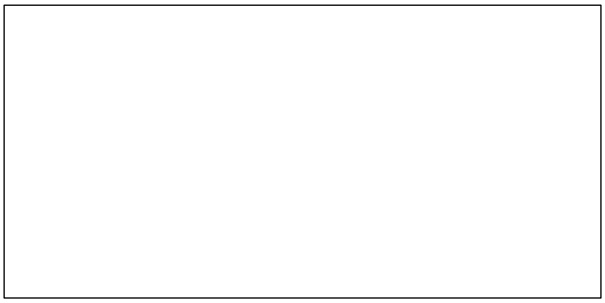
3-3. Write linear constitutive laws for the spring and damper.

3.4. Write the force balance equations for the connections 1,2,5 and mass 6.

3-5. Relate the forces acting on the lever to each other via moment equilibrium.

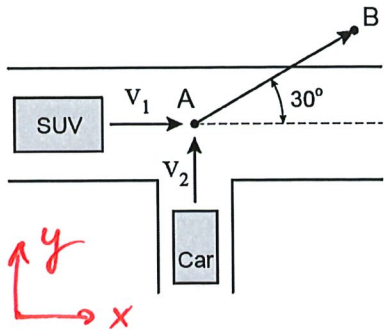
3-6. Write the geometric continuity equations for all elements (spring, damper, mass, lever).

3-7. Find the state equation(s) for the independent state variable(s).



[END OF QUIZ 2]

1. (30 points) Police are investigating a car crash involving an SUV and a car driving on two perpendicular streets, as shown in the figure. The car failed to yield to the cross traffic and collided with the SUV at the junction point A. It is a plastic impact (e=0), which results in the two vehicles becoming entangled and remaining together after the collision. The two vehicles slide over a distance of 40 meters before reaching a full stop at point B. Consider the coefficient of kinetic friction to be μ=0.5. The mass of the SUV is m_1=3,000 Kg, and the mass of the car is m_2=2,000 Kg. Use gravitational acceleration g=10 m/s^2.
Clearly write your answer in the box for each part below.



1-1. The driver of the car claims the SUV was speeding over the posted speed limit of 55 MPH. Can you calculate the speed of the SUV right before the collision? If so, what is the SUV's speed right before the collision? (14 points)

28.87 m/s

Step 1.. After Collision

(9pt.) method 1. Energy

$F \cdot S = \frac{1}{2}(m_1+m_2)V^2$ (3pt), $F = \mu \cdot (m_1+m_2)g$ (3pt)

$V = \sqrt{2\mu g s} = 20\ m/s$ (3pt)

method 2. Force and Acceleration.

$F = \mu(m_1+m_2)g$, (3pt) $a = \frac{F}{m} = \mu g$ (3pt)

$V^2 = 2as \Rightarrow V = \sqrt{2\mu g s} = 20 m/s$ (3pt)

(5pt) Step 2. for Collision

momentum in X. $m_1 \cdot V_1 = (m_1+m_2)V_2 \cdot \cos 30°$

$\Rightarrow V_1 = \frac{m_1+m_2}{m_1} \cdot V \cdot \cos 30° = 28.87\ m/s$

[QUIZ CONTINUES ON NEXT PAGE]

momentum in Y. $m_2 V_2 = (m_1+m_2) V \cdot \sin 30^\circ$ (3pt)

1-2. Calculate the energy loss during the collision: 8pt.

$V_2 = \frac{m_1+m_2}{m_2} \cdot V \cdot \sin 30^\circ = 25\ m/s$ (2pt)

875,215 J

$\Delta E = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 - \frac{1}{2}(m_1+m_2)V^2$

$= 875,215\ J$ (3pt)

1-3. It's a snowy day, and the coefficient of kinetic friction reduces to μ=0.1 on the icy surface. After the same plastic collision, how far do the two vehicles slide before reaching a full stop? (8 pt)

200 m

method 1. Energy.

$F \cdot S = \frac{1}{2}(m_1+m_2)V^2$ (4pt)

$S = \frac{(m_1+m_2)V^2}{2F} = \frac{(m_1+m_2)V^2}{2 \cdot \mu (m_1+m_2) g} = \frac{V^2}{2\mu g} = 200m$ (4 pt)

method 2 Force and Acceleration. (4pt)

$F = (m_1+m_2)a \Rightarrow a = \frac{F}{m_1+m_2} = \frac{\mu(m_1+m_2)g}{m_1+m_2} = \mu g$

$V^2 = 2as \Rightarrow S = \frac{V^2}{2a} = \frac{V^2}{2\mu g} = 200\ m$ (4pt)

[QUIZ CONTINUES ON NEXT PAGE]

2. (24 Points) You are given the state equations below, for a two-state mechanical system as $\overrightarrow{X'} = A\vec{X}$ where $\vec{X}$ is the state vector.

$$\vec{X} = \begin{Bmatrix} x_1 \\ v_2 \end{Bmatrix}$$

and A is the square matrix

$$A = \begin{bmatrix} 0 & -1 \\ 4 & -3 \end{bmatrix}$$

Clearly write your answer in the box for each part below.

2-1. Write the state equations as a single second order equation in the variable x_1. (8 pt)

$x_1' = -v_2$ (2pt)

$v_2' = 4x_1 - 3v_2$ (2pt)

$x_1'' = -v_2' = -4x_1 - 3x_1' \Rightarrow x_1'' + 3x_1' + 4x_1 = 0$ (4pt)

or. $v_2'' = 4x_1' - 3v_2' = 4v_2 - 3v_2' \Rightarrow v_2'' + 3v_2' - 4v_2 = 0$

$x_1'' + 3x_1' + 4x_1 = 0$

2-2. Consider a trial solution of the form $x_1(t) = Ce^{rt}$ where r and C are both constants. Find the characteristic equation for r. (4 pt)

$r^2 + 3r + 4 = 0$

$r^2 + 3r + 4 = 0$

2-3. Write down the root(s) of the characteristic equation that you have found. (4 pt)

$$r_{1,2} = \frac{-3 \pm \sqrt{9-16}}{2} = -\frac{3}{2} \pm \frac{\sqrt{7}}{2} \cdot j$$

$$-\frac{3}{2} \pm \frac{\sqrt{7}}{2} j$$

[QUIZ CONTINUES ON NEXT PAGE]

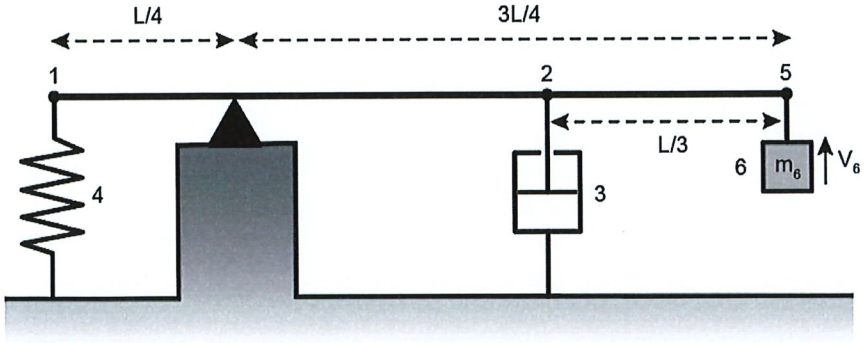
2-4. Write down the general solution $x_1(t)$ in real form. What kind of motion (i.e. oscillatory, damped oscillations, monotonic etc.) does this represent? (8 pt)

$x_1(t) = e^{-\frac{3}{2}t}\left[A\cos\left(\frac{\sqrt{7}}{2}t\right) + B\sin\left(\frac{\sqrt{7}}{2}t\right)\right]$ (6 pt)

Decaying (2 pt)

[QUIZ CONTINUES ON NEXT PAGE]

3. (45 pts) The rigid rod of length L in the diagram above pivots about a fulcrum ¼ of the way from its left end. A damper 3 with damping constant b_3, a spring 4 with spring constant k_4, and a mass m_6 with gravity acting upon it are attached at the points indicated. Use the upward direction as your positive sign convention for forces acting on the lever and velocity of the lever at points 1, 2, and 5. Label all parameters and dynamic variables with subscripts corresponding to the element numbers. Clearly write your answer in the box for each part below.



Solve the following:

3-1. At static equilibrium, express the extension of the spring (x_{equil}) in terms of m_6, g, *and* k_4: (9 pt)

+↻ ΣM : $F_4 \cdot \frac{L}{4} - F_3 \cdot \frac{1}{2}L - m_6 a_6 \frac{3}{4}L + m_6 g \frac{3}{4}L$ (4pt)

$\frac{3 m_6 g}{k_4}$

Static, $F_3 = 0$, $a_6 = 0$. $F_4 = k_4 \cdot x_4$ (2pt)

$\Rightarrow x_4 = \frac{3 m_6 g}{k_4}$ (3pt)

3-2. The independent state variables for this system are: (2pt)

V_6, m_4

3-3. Write linear constitutive laws for the spring and damper. (4pt)

$F_4 = k_4 x_4$

$F_3 = b_3 V_3$

3.4. Write the force balance equations for the connections and mass. (6 pt), 2 each

$F_1 = -F_4$

$F_2 = -F_3$

$m_6 a_6 = -F_5 - m_6 g$

3-5. Relate the forces acting on the lever to each other via moment equilibrium. (6 pt)

$3F_1 = 5F_2 + 9F_5$

3-6. Write the geometric continuity equations for all elements (spring, damper, mass, lever). (6 pt)

$V_1 = V_4$ $\quad 5V_1 = -3V_2$

$V_2 = V_3$ $\quad 9V_2 = 5V_5$

$V_5 = V_6$ $\quad 3V_1 = -V_5$

3-7. Find the state equation(s) for the independent state variable(s). (12 pt)

$X_4' = -\frac{1}{3} V_6$ (6 pt)

$V_6' = \frac{k_4}{3m_6} x_4 - \left(\frac{25}{81}\right)\left(\frac{b_3}{m_6}\right) V_6 - g$ (6 pt)

[END OF QUIZ 2]