ENGINEERING ANALYSIS 3

SYSTEM DYNAMICS

Quiz No. 3

December 5/6, 2024

Name : _____

You may NOT use notes or books or calculators or phones for this quiz. There are 2 problems with multiple sections.

Clearly mark your answers (circle your final answers, wherever applicable) and detail your problem-solving process. Credit will primarily be rewarded based on process (which demonstrates your conceptual understanding of the material) rather than results. **If your work is illegible, or cannot be followed**, we can not promise to award partial credit.

In this quiz, all resistors, capacitors, inductors, and voltage sources are ideal and linear. All wires are assumed to be perfect conductors.

Potentially Useful Relationships & Equations

R: Electrical Resistance (Ohms = V/A)

C: Capacitance (Farads = As/V)

L: Inductance (Henries = Vs/A)

General Exponential Decay Function: $x(t) = Ae^{rt} + C$

Overdamped System: $x(t) = A_1 e^{r_1 t} + A_2 e^{r_2 t} + C$

Underdamped System: $x(t) = e^{\alpha t}(A_1 \cos(\omega t) + A_2 \sin(\omega t)) + C$

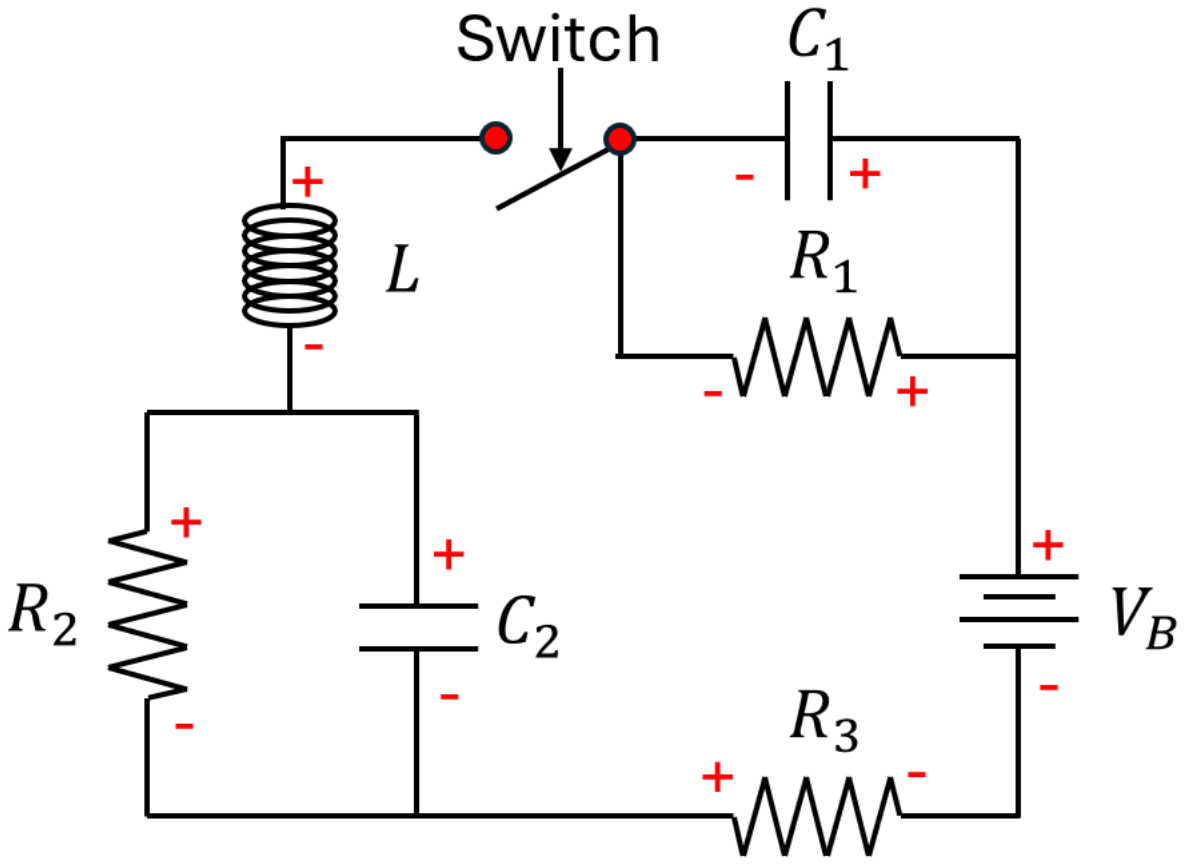
Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Do not start solving this quiz until you are told to do so. *

Problem	Points	Score
#1	**48**	
1.1	**24**	
1.2	**24**	
#2	**65**	
2.1	**24**	
2.2	**6**	
2.3	**24**	
Total	**100**	

Free Page

Problem 1: At t=0, the "switch" is closed to complete the circuit through the battery. Prior to closing the switch, all elements (excluding the battery) are fully discharged. Battery voltage is $\boldsymbol{V_B}$.



The following constants are given:

$V_B = 12\ V$ $\quad$ $C_1 = 3F$ $\quad$ $C_2 = 5\ F$

$R_1 = 1\ \Omega$ $\quad$ $R_2 = 2\ \Omega$ $\quad$ $R_3 = 3\ \Omega$

$L = 6\ H$

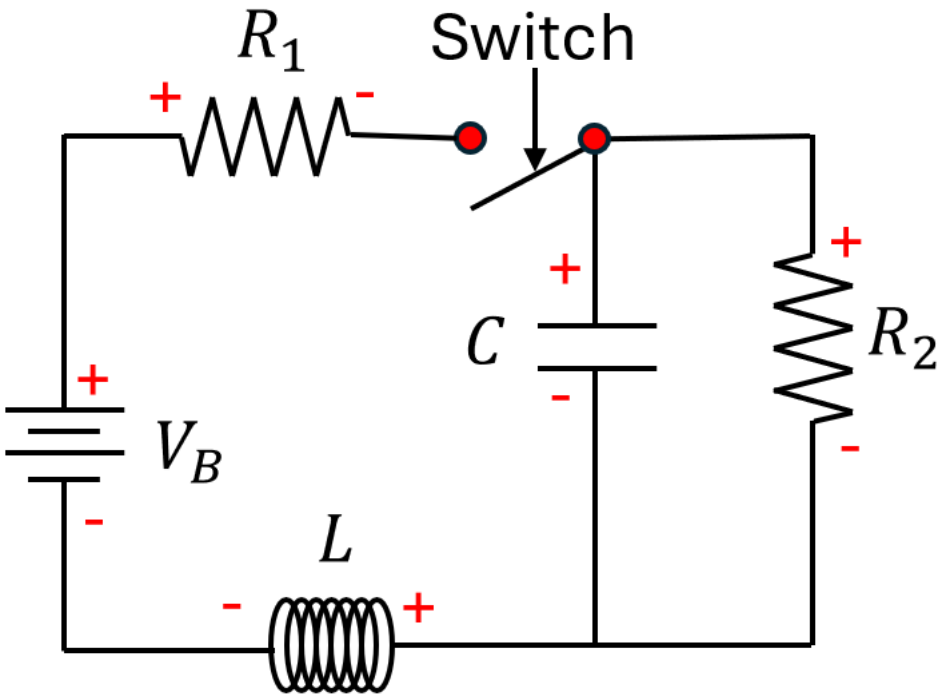
Question 1.1: Determine the initial voltage and current through each element in the circuit the instant after the switch is closed. (**24 pts)**

Component	Voltage	Current
L		
C_1		
R_1		
C_2		
R_2		
R_3		

Question 1.2: Determine the voltage and current through each element in the circuit at steady state, after the switch is closed. (**24 pts)**

Component	Voltage	Current
L		
C_1		
R_1		
C_2		
R_2		
R_3		

Problem 2: At t=0, the "switch" is closed to complete the circuit through the battery. Prior to closing the switch, all elements (excluding the battery) are fully discharged. Battery voltage is $\boldsymbol{V_B}$.



The following constants are given:

$V_B = 12\ V$ $\quad$ $R_1 = 2\ \Omega$ $\quad$ $R_2 = 2\ \Omega$

$L = 2\ H$ $\quad$ $C = 3F$

Question 2.1: Derive a **2nd order linear differential equation** describing the **voltage across the capacitor** (you do not need to solve the general or particular solution). You may substitute in constants whenever needed; the algebra may get messy. (**24 pts)**

Question 2.2: What will the voltage across the capacitor be at steady state (i.e., $V_C(t = +\infty)$)? (**6 pts)**

Question 2.3: Given the particular solution of the voltage across the capacitor, $V_C(t) = -24e^{-\frac{t}{2}} + 18e^{-\frac{2t}{3}} + 6$, find the particular solution for the current through R_2 as a function of time, $i_{R1}(t)$. (**24 pts)**

Free Page

Free Page

ENGINEERING ANALYSIS 3

SYSTEM DYNAMICS

Quiz No. 3

December 5/6, 2024

Name : _____

You may NOT use notes or books or calculators or phones for this quiz. There are 2 problems with multiple sections.

Clearly mark your answers (circle your final answers, wherever applicable) and detail your problem-solving process. Credit will primarily be rewarded based on process (which demonstrates your conceptual understanding of the material) rather than results. **If your work is illegible, or cannot be followed**, we can not promise to award partial credit.

In this quiz, all resistors, capacitors, inductors, and voltage sources are ideal and linear. All wires are assumed to be perfect conductors.

Potentially Useful Relationships & Equations

R: Electrical Resistance (Ohms = V/A)

C: Capacitance (Farads = As/V)

L: Inductance (Henries = Vs/A)

Elastic Potential Energy = ½ k x^2

Kinetic Energy = ½ m v^2

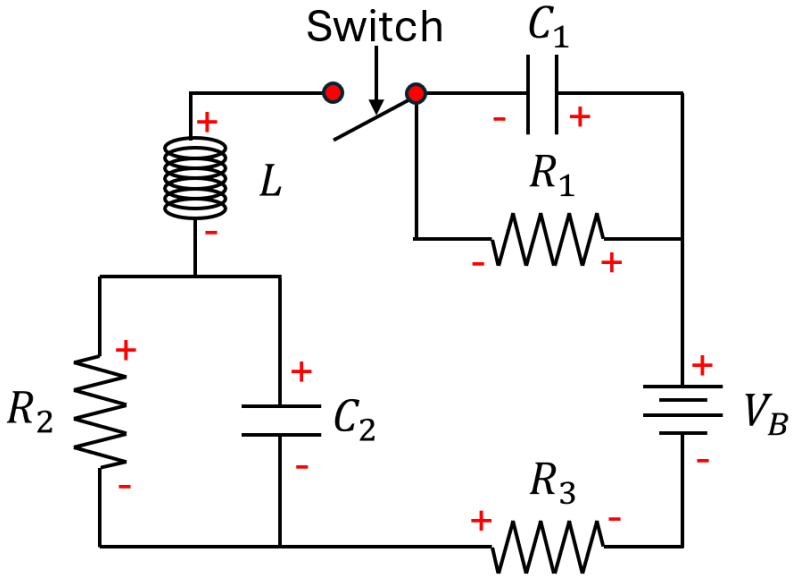
Work = $\int F dx$

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Do not start solving this quiz until you are told to do so. *

Problem	Points	Score
#1	**48**	
1.1	**24**	
1.2	**24**	
#2	**65**	
2.1	**24**	
2.2	**6**	
2.3	**24**	
Total	**100**	

Problem 1: At t=0, the "switch" is closed to complete the circuit through the battery. Prior to closing the switch, all elements (excluding the battery) are fully discharged. Battery voltage is $\boldsymbol{V_B}$.



The following constants are given:

$V_B = 12\ V$ $\quad$ $C_1 = 3F$ $\quad$ $C_2 = 5\ F$

$R_1 = 1\ \Omega$ $\quad$ $R_2 = 2\ \Omega$ $\quad$ $R_3 = 3\ \Omega$

$L = 6\ H$

$$V_{C1} = V_{R1}$$

$$V_{C2} = V_{R2}$$

$$V_B = V_{R1} + V_{R2} + V_{R3} + V_L$$

$$i_{R3} = i_L$$

$$i_L = i_{C2} + i_{R2}$$

$$i_L = i_{C1} + i_{R1}$$

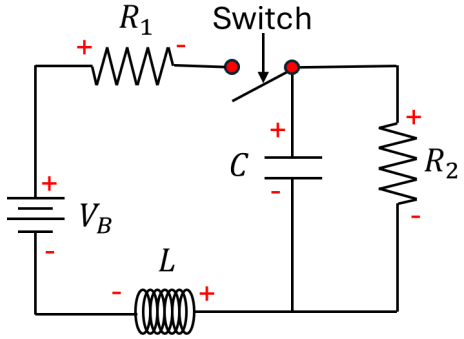
Question 1.1: Determine the initial voltage and current through each element in the circuit the instant after the switch is closed. (**24 pts)**

Component	Voltage	Current
L	12 V	**0**
C_1	**0**	0
R_1	0	0
C_2	**0**	0
R_2	0	0
R_3	0	0

Question 1.2: Determine the voltage and current through each element in the circuit at steady state, after the switch is closed. (**24 pts)**

Component	Voltage	Current
L	**0 V**	2 A
C_1	2 V	**0 A**
R_1	2 V	2 A
C_2	4 V	**0 A**
R_2	4 V	2 A
R_3	6 V	2 A

Problem 2: At t=0, the "switch" is closed to complete the circuit through the battery. Prior to closing the switch, all elements (excluding the battery) are fully discharged. Battery voltage is $\boldsymbol{V_B}$.



The following constants are given:

$V_B = 12\ V$ $\quad R_1 = 2\ \Omega$ $\quad R_2 = 2\ \Omega$

$L = 2\ H$ $\quad C = 3F$

Question 2.1: Derive a 2nd order linear differential equation describing the **voltage across the capacitor**. You may substitute in constants whenever needed. (**24 pts)**

KVL: 3 pts

$\boldsymbol{V_B = V_{R1} + V_L + V_C}$

$\boldsymbol{V_C = V_{R2}}$

KCL 3 pts

$\boldsymbol{i_L = i_{R1}}$

$\boldsymbol{i_L = i_C + i_{R2}}$

$\boldsymbol{V_C' = \frac{1}{C} i_C = \frac{1}{C}(i_L - i_{R2}) = \frac{1}{C}\left(i_L - \frac{V_{R2}}{R_2}\right) = \frac{1}{C}\left(i_L - \frac{V_C}{R_2}\right)}$ **5 pt**

$\boldsymbol{i_L' = \frac{1}{L} V_L = \frac{1}{L}(V_B - V_C - V_{R1}) = \frac{1}{L}(V_B - V_C - i_{R1}R_1) = \frac{1}{L}(V_B - V_C - i_L R_1)}$ **5 pt**

$\boldsymbol{V_C'' = \frac{1}{C}\left(i_L' - \frac{V_C'}{R_2}\right) = \frac{1}{C}\left(\frac{1}{L}(V_B - V_C - i_L R_1) - \frac{V_C'}{R_2}\right)}$ **5 pt**

$i_L = CV_C' + \frac{V_C}{R_2}$

$\boldsymbol{CLV_C'' + \left(CR_1 + \frac{L}{R_2}\right)V_C' + \left(1 + \frac{R_1}{R_2}\right)V_C = V_B}$ **3 pt**

$\boldsymbol{6V_C'' + 7V_C' + 2V_C = 12\ V}$

Question 2.2: What will the voltage across the capacitor be at steady state (i.e., $V_C(t = +\infty)$)? **(6 pts)**

$$\boldsymbol{V_C = \frac{V_B}{\left(1+\frac{R_1}{R_2}\right)} = 6\ V}$$

Question 2.3: Given the particular solution of the voltage across the capacitor, $V_C(t) = -24e^{-\frac{t}{2}} + 18e^{-\frac{2t}{3}} + 6\ V$, find the particular solution for the current through R_2 as a function of time, $i_{R1}(t)$. **(24 pts)**

$i_{R1}(t) = i_C(t) + i_{R2}(t) = CV_C'(t) + \frac{V_C(t)}{R_2}$ **7 pts**

$CV_C'(t) = 3\left(12e^{-\frac{t}{2}} - 12e^{-\frac{2t}{3}}\right) = 36e^{-\frac{t}{2}} - 36e^{-\frac{2t}{3}}$ **7 pts**

$\frac{V_C(t)}{R_2} = -12e^{-\frac{t}{2}} + 9e^{-\frac{2t}{3}} + 3$ **7 pts**

$\boldsymbol{i_{R1}(t) = 24e^{-\frac{t}{2}} - 27e^{-\frac{2t}{3}} + 3\ A}$ **3 pts**