EA3 Quiz 3
Prof. Lynch
Spring 2024

Name: ___________________________________

You are allowed to use only pens, pencils, and erasers. No electronics, other papers, etc.

Make sure to show all your work, and make sure your final answer is clear (for example, you can circle it). **Use the back of the previous page if you need more space, for your work or your final answer.** Full credit, or partial credit if your final answer is wrong, will only be given if your thought process is clear. If you think any question does not give you enough information to give an answer (i.e., there is a mistake on the test), then clearly write the extra assumptions you had to make to answer the question, and answer the question using those assumptions.

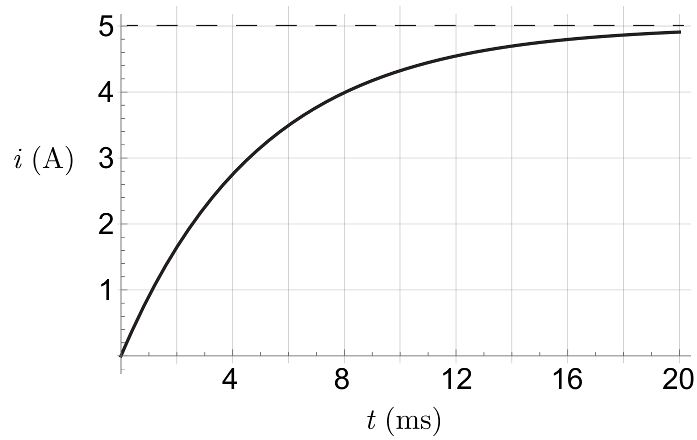
No significant calculations are needed, so you don't need a calculator. If you get an answer like $11/32$ or $\sqrt{5}$, just leave it like that. On the other hand, we would appreciate simple calculations, like reducing $6/2$ to 3 or $\sqrt{12}$ to $2\sqrt{3}$.

The "standard" second-order characteristic equation is $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$. The roots of a quadratic equation $as^2 + bs + c = 0$ are $s_{1,2} = (-b \pm \sqrt{b^2 - 4ac})/(2a)$. The following are approximate numerical values:

$$
\begin{aligned}
e^{-1} &= 0.37 \\
e^{-2} &= 0.14 \\
e^{-3} &= 0.05 \\
e^{-4} &= 0.02
\end{aligned}
$$

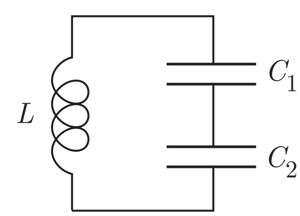
1. The electric motor you ordered just arrived, and you want to test it to make sure it matches its data sheet. You block the motor shaft so it cannot rotate, ensuring that the voltage v across the motor and the current i through the motor satisfy the equation $v = Ri + L(di/dt)$, where R is the resistance and L is the inductance of the motor. You use a power supply to control v to be zero for a long time, and then you switch to $v = 20$ V and measure the current i. The current $i(t)$ is shown in the plot below. (Note the units on the time axis are milliseconds.) The current reaches a steady state of 5 A.

   (a) (2 pts) What is the resistance R of the motor? Give units.



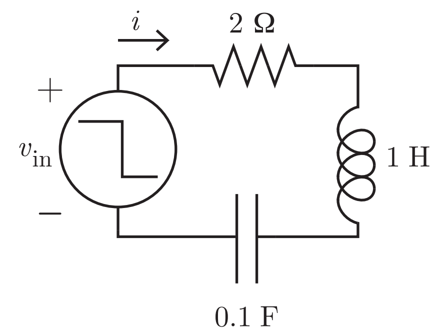
   (b) (2 pts) What is the approximate inductance L of the motor? Give a number with units and explain your answer.

2. (4 pts) At time $t = 0$ in the LC circuit below, the voltage across the inductor is 10 V and no current is flowing. What is the maximum current that flows through the inductor in the future?



3. For the circuit shown below, the voltage source provides a voltage $v_{\text{in}} = 12$ V for a long time, then it switches the voltage to $v_{\text{in}} = 0$ V at time $t = 0$.

   (a) (2 pts) What is the current i, as defined in the figure, just before the switch at $t = 0$? Give units.



   (b) (2 pts) What is the current immediately after the switch? Give units.

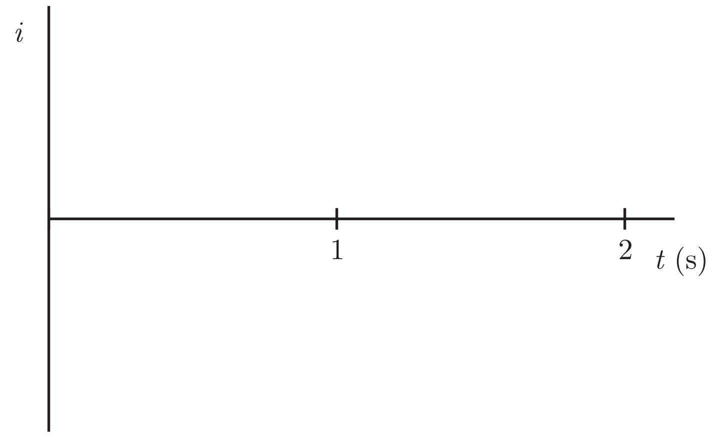
   (c) (2 pts) What is the rate of change of the current di/dt immediately after the switch? Give units.

   (d) (2 pts) Write the second-order differential equation governing the current i after $t = 0$.

   (e) (3 pts) Give the roots of the characteristic equation for your answer to the previous question. Indicate whether the system is overdamped, underdamped, critically damped, or none of these.

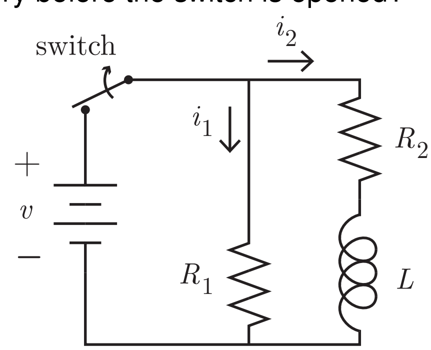
   (f) (4 pts) Write the particular solution $i(t)$ for the initial conditions when v_{in} switches to 0 V.

(g) (3 pts) Approximately hand sketch the response $i(t)$ from $t = 0$ to $t = 2$ s. Don't worry about the scale of the vertical (current) axis, but the response should have the correct basic shape. If the response decays, show the approximate decay. If it oscillates, the period of oscillation should be approximately correct.



4. In the circuit below, the switch is connected to the battery (with voltage v) for a long time, and then it is opened so the battery is disconnected. The positive directions of the currents i_1 and i_2 are indicated.

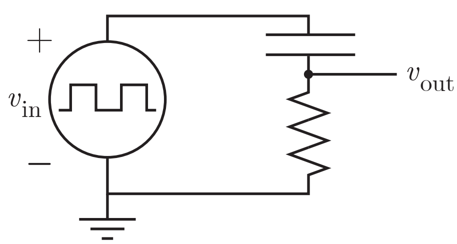
(a) (2 pts) What is the total current provided by the battery before the switch is opened?

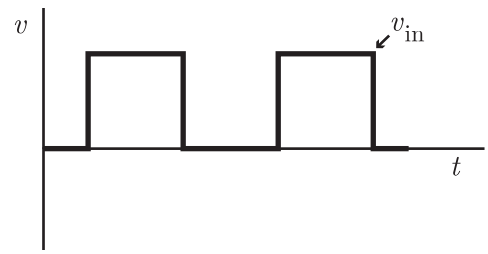


(b) (2 pts) What is the total power dissipated by the resistors immediately after the switch?

(c) (2 pts) What is di_2/dt immediately after the switch?

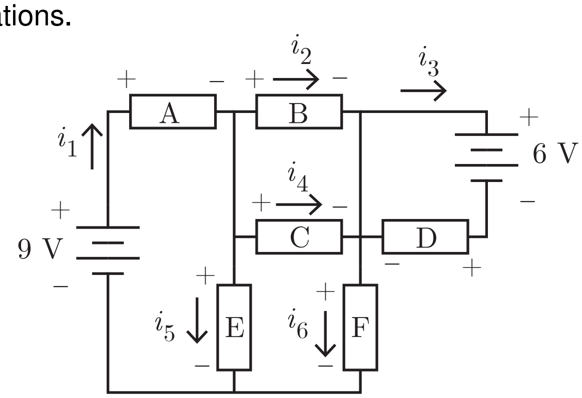
5. (2 pts) On the left is a circuit consisting of a capacitor, a resistor, and a square wave voltage generator: v_{in} cycles between zero and a positive voltage, and the time between switches is long enough for the current to approximately reach steady state between switches. The voltage v_{out} measures the voltage across the resistor. On the right is a plot of $v_{\text{in}}(t)$. On the same plot, qualitatively plot $v_{\text{out}}(t)$ and briefly explain your answer.





6. The circuit below contains resistors, capacitors, and inductors (all represented as rectangles) and batteries. The voltages across the elements are written v_A, ..., v_F using the polarity determined by the defined positive directions of the currents $i_1, \ldots, i_6$.

   (a) (4 pts) Write four independent KVL equations.

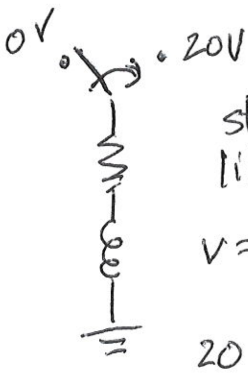


   (b) (3 pts) Write the KCL equations.

   (c) (2 pts) If A and B are resistors, C and D are capacitors, and E and F are inductors, what are the state variables of the coupled first-order differential equations?

1. The electric motor you ordered just arrived, and you want to test it to make sure it matches its data sheet. You block the motor shaft so it cannot rotate, ensuring that the voltage v across the motor and the current i through the motor satisfy the equation $v = Ri + L(di/dt)$, where R is the resistance and L is the inductance of the motor. You use a power supply to control v to be zero for a long time, and then you switch to $v = 20$ V and measure the current i. The current $i(t)$ is shown in the plot below. (Note the units on the time axis are milliseconds.) The current reaches a steady state of 5 A.

   (a) (2 pts) What is the resistance R of the motor? Give units.

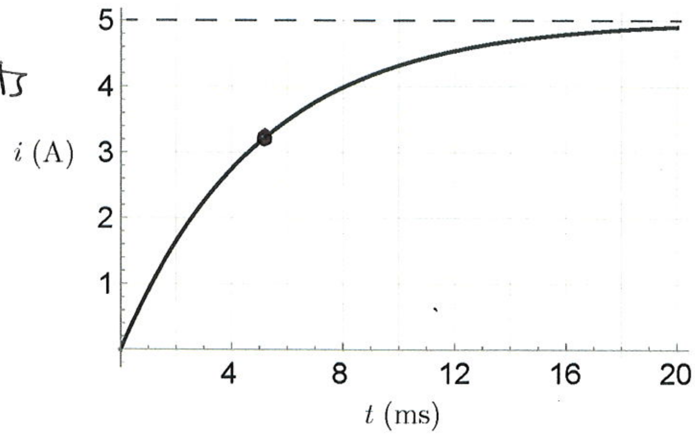


At steady state, inductor acts like a wire.

$$V = Ri + L\cancel{\frac{di}{dt}}^{\,0}$$

$$20 = R(5A)$$

$$R = 4\,\Omega$$



   (b) (2 pts) What is the approximate inductance L of the motor? Give a number with units and explain your answer.

Current reaches 63% of final value at approx 5 ms.

$$i(t) = 5A(1 - e^{-t/\tau}) \quad \text{where } \tau = \frac{L}{R} = 5\,ms$$

$e^{-1} = 0.37$,
so $1 - e^{-1} = 0.63$

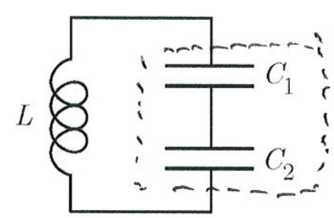
$$\frac{L}{4\Omega} = 5\,ms \rightarrow L = 20\,mH = 0.02\,H$$

2. (4 pts) At time $t = 0$ in the LC circuit below, the voltage across the inductor is 10 V and no current is flowing. What is the maximum current that flows through the inductor in the future?

Energy is conserved. Initially no current, so no energy in inductor ($\frac{1}{2}Li^2$).

All energy is in the capacitors,

$$\frac{1}{2} C_{eq} V^2 = \frac{1}{2} \frac{C_1 C_2}{C_1 + C_2} (10)^2$$



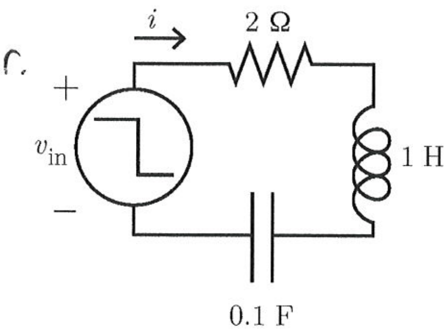
$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

When the equivalent capacitor is fully discharged and inductor fully charged,

$$\frac{1}{2} L i^2 = \frac{1}{2} 100 \frac{C_1 C_2}{C_1 + C_2} \rightarrow i = 10\sqrt{\frac{C_1 C_2}{L(C_1 + C_2)}}$$

3. For the circuit shown below, the voltage source provides a voltage $v_{\text{in}} = 12$ V for a long time, then it switches the voltage to $v_{\text{in}} = 0$ V at time $t = 0$.
   (a) (2 pts) What is the current i, as defined in the figure, just before the switch at $t = 0$? Give units.

Before the switch, $i = 0$ due to the fully charged (open circuit) capacitor.
$i = 0\ A$



   (b) (2 pts) What is the current immediately after the switch? Give units.

Inductor does not allow the current to change instantly.
$i = 0\ A$

   (c) (2 pts) What is the rate of change of the current di/dt immediately after the switch? Give units.

Cap is charged to 12V. KVL: $12V + Ri + L\frac{di}{dt} = 0$

$$12 + 2(0) + 1\left(\frac{di}{dt}\right) = 0$$

$$\frac{di}{dt} = -12\ A/s$$

   (d) (2 pts) Write the second-order differential equation governing the current i after $t = 0$.

KVL : $V_C + V_R + V_L = 0$

$$10\int i\,dt + 2i + \frac{di}{dt} = 0 \rightarrow \frac{d^2i}{dt^2} + 2\frac{di}{dt} + 10i = 0$$

   (e) (3 pts) Give the roots of the characteristic equation for your answer to the previous question. Indicate whether the system is overdamped, underdamped, critically damped, or none of these.

$$s^2 + 2s + 10 = 0 \rightarrow s_{1,2} = \frac{-2 \pm \sqrt{4-40}}{2}$$

$\boxed{\text{underdamped}}$ $\quad s_{1,2} = -1 \pm 3j$

   (f) (4 pts) Write the particular solution $i(t)$ for the initial conditions when v_{in} switches to 0 V.

$$i(t) = e^{-t}(A\cos 3t + B\sin 3t)$$

$i(0) = 0$ so $A = 0$

$$\frac{di}{dt}(0) = -12 = -e^{-t}(A\cos 3t + B\sin 3t) + e^{-t}(-3A\sin 3t + 3B\cos 3t)$$

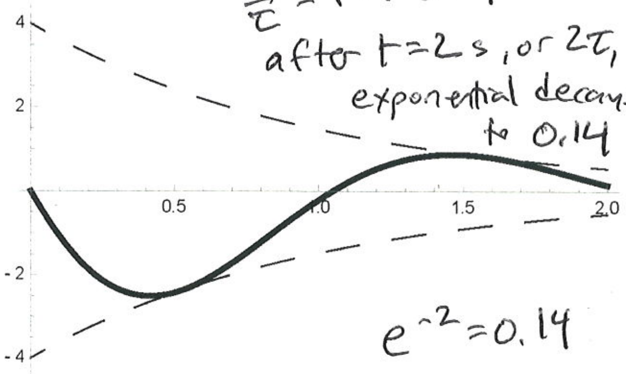
$$= 3B \rightarrow B = -4, \text{ so } i(t) = -4e^{-t}\sin 3t$$

(g) (3 pts) Approximately hand sketch the response $i(t)$ from $t = 0$ to $t = 2$ s. Don't worry about the scale of the vertical (current) axis, but the response should have the correct basic shape. If the response decays, show the approximate decay. If it oscillates, the period of oscillation should be approximately correct.

$\frac{1}{\tau} = 1 \rightarrow \tau = 1$

after $t = 2$ s, or 2τ, exponential decays to 0.14

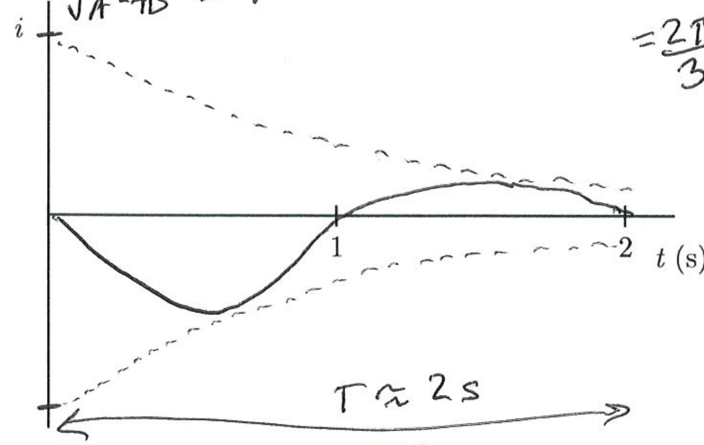
$e^{-2} = 0.14$



$\sqrt{A^2+B^2} = 4$

oscillation period $= T = \frac{2\pi}{\omega}$

$= \frac{2\pi}{3} \approx 2s$

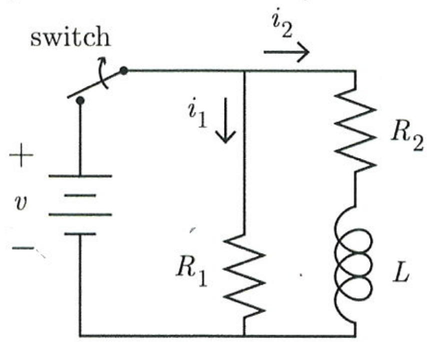


4. In the circuit below, the switch is connected to the battery (with voltage v) for a long time, and then it is opened so the battery is disconnected. The positive directions of the currents i_1 and i_2 are indicated.

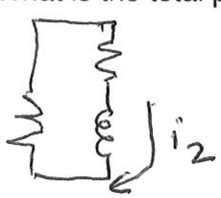
(a) (2 pts) What is the total current provided by the battery before the switch is opened?

$i_1 = \frac{V}{R_1}$ $\quad$ $i_2 = \frac{V}{R_2}$ (inductor acts like wire)

$i_{tot} = \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{R_1+R_2}{R_1 R_2}\right)$



(b) (2 pts) What is the total power dissipated by the resistors immediately after the switch?



Inductor keeps i_2 flowing. $i_2 = \frac{V}{R_2}$

power $= i^2 R = \left(\frac{V}{R_2}\right)^2 (R_1 + R_2)$

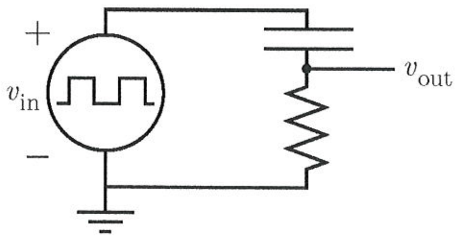
(c) (2 pts) What is di_2/dt immediately after the switch?

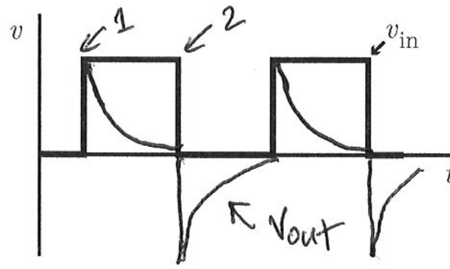
KVL: $V_{R1} + V_{R2} + V_L = 0$

$$R_1 \frac{V}{R_2} + R_2 \frac{V}{R_2} + L\frac{di_2}{dt} = 0$$

$$\frac{di_2}{dt} = \frac{-V}{L}\left(\frac{R_1+R_2}{R_2}\right)$$

5. (2 pts) On the left is a circuit consisting of a capacitor, a resistor, and a square wave voltage generator: v_{in} cycles between zero and a positive voltage, and the time between switches is long enough for the current to approximately reach steady state between switches. The voltage v_{out} measures the voltage across the resistor. On the right is a plot of $v_{in}(t)$. On the same plot, qualitatively plot $v_{out}(t)$ and briefly explain your answer.





This is a high-pass filter. At switch 1, the cap has no voltage, so Vout = Vin. But then the cap charges up, the current drops, and Vout goes to zero. At switch 2, the cap begins to discharge, high current flows opposite direction, and then drops to zero

6. The circuit below contains resistors, capacitors, and inductors (all represented as rectangles) and batteries. The voltages across the elements are written $v_A, \ldots, v_F$ using the polarity determined by the defined positive directions of the currents $i_1, \ldots, i_6$.

(a) (4 pts) Write four independent KVL equations.

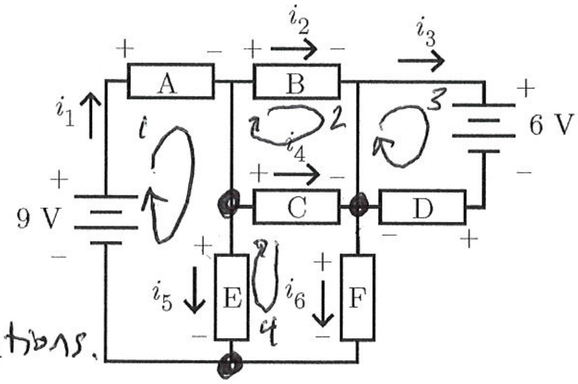
1: $-9 + V_A + V_E = 0$

2: $V_B - V_C = 0$

3: $6 + V_D = 0$

4: $-V_E + V_C + V_F = 0$

or you could choose other independent KVL set of equations.



(b) (3 pts) Write the KCL equations.

$i_1 = i_2 + i_4 + i_5$

$i_2 + i_4 + i_3 = i_3 + i_6 \rightarrow i_2 + i_4 = i_6$

$i_5 + i_6 = i_1 \rightarrow$ but this is not linearly independent of the first two

(c) (2 pts) If A and B are resistors, C and D are capacitors, and E and F are inductors, what are the state variables of the coupled first-order differential equations?

V_C, V_D, i_5, i_6

(But with these choices, not a well designed circuit...)