Engineering Analysis 3
Fall 2024
Quiz #1

NAME: ________________________________

Date: Monday October 21, 2024; 50 minutes total

No Notes, Calculators, Smartphones, etc.

ATTEMPT ALL PROBLEMS (100 Points total). Show all of your work on these pages. Please read each problem thoroughly, circle any final answers, and clearly (and concisely) explain your findings, when prompted.

Neatness counts: if the graders can not understand your work, we can not promise that credit will be awarded.

Problem	Points	Score
#1	**25**	
#2	**25**	
2.1	10	
2.2	15	
#3	**50**	
3.1	10	
3.2	10	
3.3	20	
3.4	10	
Total	**100**	

Problem 1: The following represents a set of state equations, $\frac{dx}{dt}$ and $\frac{dy}{dt}$, in terms of a pair of state variables, x and y. These state equations represent the first derivative of the corresponding variable with respect to time.

$$\frac{dx}{dt} = 100x^2 + y$$

$$\frac{dy}{dt} = 3x$$

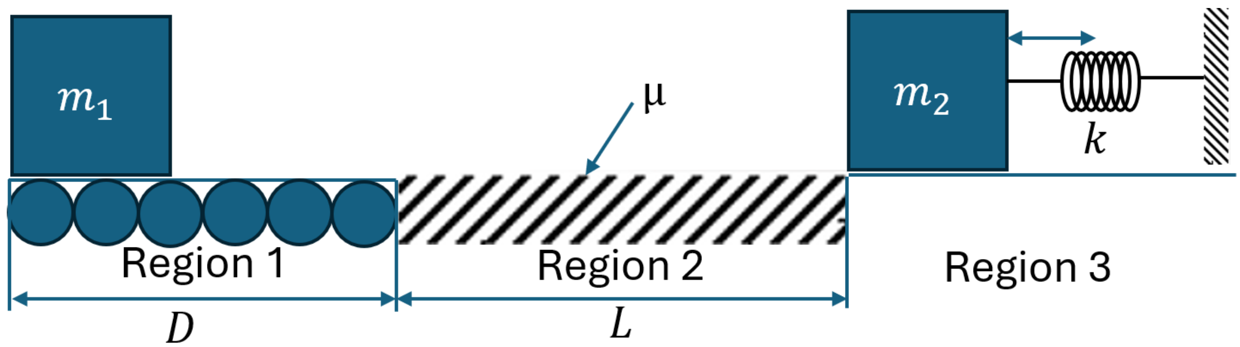
At a given time, $t = 0$ seconds, $x = 0$ and $y = 1$. Using Euler's method, find the approximate value of x and y at $t = 0.2$ seconds, using a time step $dt = 0.1$ seconds.

Problem 2: A box with mass, $\boldsymbol{m_1}$, is initially at rest ($v_{m1}(t=0) = 0\frac{m}{s}$). Gravity acts downward with a magnitude $\boldsymbol{g}$.

It is pushed to the right by "boosters" in Region 1 which exert a constant, rightward force, $\boldsymbol{F_1}$. This region has a length, $\boldsymbol{D}$.

At the end of Region 1, the box leaves the boosters and enters Region 2. Within this region, the box slides across a frictional surface with a coefficient of friction, μ. This region has a length, $\boldsymbol{L}$.

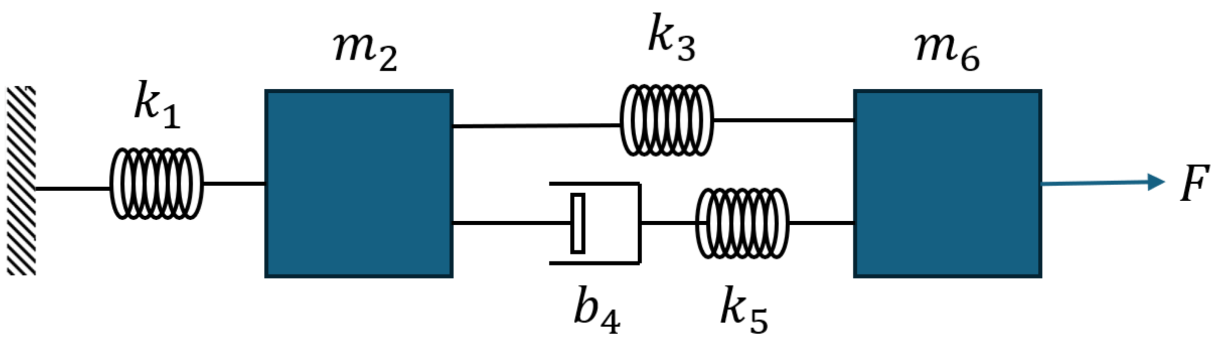
At the end of Region 2, the box collides with **and sticks to** a second box of mass, $\boldsymbol{m_2}$, attached to a spring and begins to compress the spring. Region 3 is frictionless. Answer the following prompts.



1. Find the velocity at which box 1 collides with box 2. Express your answer algebraically in terms of the given constants ($\boldsymbol{m_1}$, $\boldsymbol{D}$, $\boldsymbol{L}$, μ, $\boldsymbol{F_1}$, $\boldsymbol{g}$). Label your answer as $\boldsymbol{V_1}$.

2. Find the maximum compression of the spring after the collision. Express your answer algebraically in terms of the given constants ($\boldsymbol{m_1}$, $\boldsymbol{m_2}$, $\boldsymbol{D}$, $\boldsymbol{L}$, μ, $\boldsymbol{k}$, $\boldsymbol{F_1}$, $\boldsymbol{g}$) and your answer from part 1, $\boldsymbol{V_1}$. Label your answer as $\boldsymbol{X_{max}}$.

Problem 3: At a given time, $t = 0\ seconds$, the combined spring-mass-damper system is subjected to a constant force, $\boldsymbol{F} = \mathbf{100}\ \boldsymbol{N}$. The values of all relevant constants are given below. Answer the following prompts:



$$\boldsymbol{k_1} = \mathbf{20}\,\frac{\boldsymbol{N}}{\boldsymbol{m}} \qquad \boldsymbol{k_3} = \mathbf{10}\,\frac{\boldsymbol{N}}{\boldsymbol{m}} \qquad \boldsymbol{k_5} = \mathbf{100}\,\frac{\boldsymbol{N}}{\boldsymbol{m}}$$

$$\boldsymbol{b_4} = \mathbf{100}\,\frac{\boldsymbol{Ns}}{\boldsymbol{m}} \qquad \boldsymbol{m_2} = \mathbf{10}\ \boldsymbol{kg} \qquad \boldsymbol{m_6} = \mathbf{100}\ \boldsymbol{kg}$$

1. Write three force balance equations which describe the above system.

2. Write three kinematic constraints to describe the velocities of elements within the system.

3. Derive a full set of **state equations** in terms of only **state variables**, any necessary constants, and the input force, F, which can be used to fully describe the behavior of this system.

4. After a long time ($t = +\infty$) after the initial application of force, F, the system comes to rest (all velocities are zero). At this time, find the final extension length of each spring in the system.

   a. $x_{s1}(t = +\infty)$=

   b. $x_{s3}(t = +\infty)$=

   c. $x_{s5}(t = +\infty)$=

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2.1	10	
2.2	15	
#3	**50**	
3.1	10	
3.2	10	
3.3	20	
3.4	10	
Total	**100**	

Problem 1: The following represents a set of state equations, $\frac{dx}{dt}$ and $\frac{dy}{dt}$, in terms of a pair of state variables, x and y. These state equations represent the first derivative of the corresponding variable with respect to time.

$$\frac{dx}{dt} = 100x^2 + y$$

$$\frac{dy}{dt} = 3x$$

At a given time, $t = 0$ seconds, $x = 0$ and $y = 1$. Using Euler's method, find the approximate value of x and y at $t = 0.2$ seconds, using a time step $dt = 0.1$ seconds.

3 pts $\frac{dx}{dt}(0) = 100(0) + 1 = 1$

3 pts $\frac{dy}{dt}(0) = 3(0) = 0$

3 pts $x(0.1) = \frac{dx}{dt}(0)dt + x(0) = 0.1$

3 pts $y(0.1) = \frac{dy}{dt}(0)dt + y(0) = 1$

3 pts $\frac{dx}{dt}(0.1) = 100(0.1)^2 + 1 = 2$

3 pts $\frac{dy}{dt}(0.1) = 3(0.1) = 0.3$

3 pts $x(0.2) = \frac{dx}{dt}(0.1)dt + x(0.1) = 0.3$

3 pts $y(0.2) = \frac{dy}{dt}(0.1)dt + y(0.1) = 1.03$

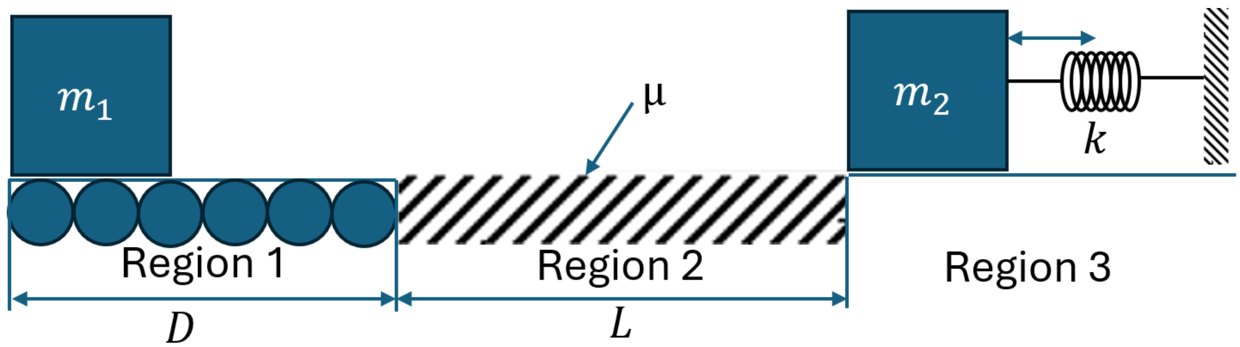
+1 for complete

Problem 2: A box with mass, $\boldsymbol{m_1}$, is initially at rest ($v_{m1}(t=0) = 0\,\frac{m}{s}$). Gravity acts downward with a magnitude $\boldsymbol{g}$.

It is pushed to the right by "boosters" in Region 1 which exert a constant, rightward force, $\boldsymbol{F_1}$. This region has a length, $\boldsymbol{D}$.

At the end of Region 1, the box leaves the boosters and enters Region 2. Within this region, the box slides across a frictional surface with a coefficient of friction, μ. This region has a length, $\boldsymbol{L}$.

At the end of Region 2, the box collides with **and sticks to** a second box of mass, $\boldsymbol{m_2}$, attached to a spring and begins to compress the spring. Region 3 is frictionless. Answer the following prompts.



1. Find the velocity at which box 1 collides with box 2. Express your answer algebraically in terms of the given constants ($\boldsymbol{m_1}$, $\boldsymbol{D}$, $\boldsymbol{L}$, μ, $\boldsymbol{F_1}$, $\boldsymbol{g}$). Label your answer as $\boldsymbol{V_1}$.

4 pts $$E_{before} + U_{added} = E_{after} + U_{lost}$$

4 pts $$0 + F_1 D = \frac{1}{2} m_1 V_1^2 + F_f L$$

$$0 + F_1 D = \frac{1}{2} m_1 V_1^2 + \mu m_1 g L$$

2 pts $$\boldsymbol{V_1 = \sqrt{\frac{2}{m_1}(F_1 D - \mu m_1 g L)}}$$

2. Find the maximum compression of the spring after the collision. Express your answer algebraically in terms of the given constants ($\boldsymbol{m_1}$, $\boldsymbol{m_2}$, $\boldsymbol{D}$, $\boldsymbol{L}$, μ, $\boldsymbol{k}$, $\boldsymbol{F_1}$, $\boldsymbol{g}$) and your answer from part 1, $\boldsymbol{V_1}$. Label your answer as $\boldsymbol{X_{max}}$.

5 pts $$m_1 V_1 = (m_1 + m_2) V_2$$

$$V_2 = \frac{m_1}{m_1 + m_2} V_1$$

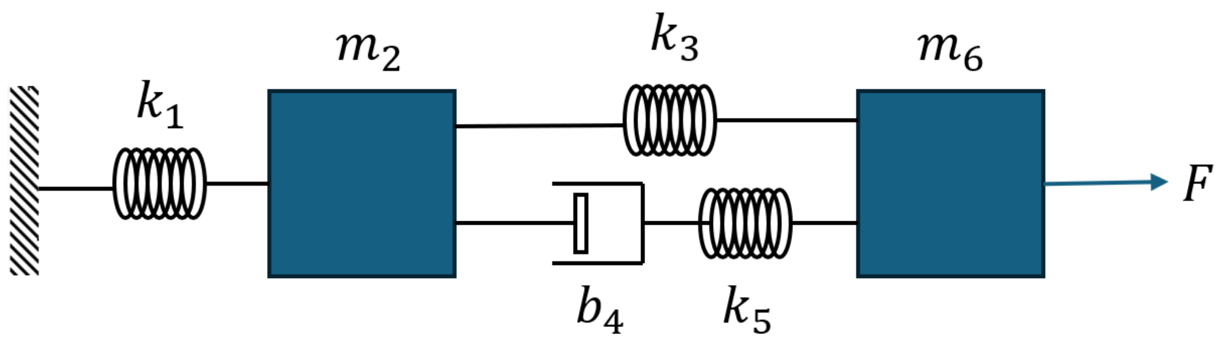
$$E_{before} + U_{added} = E_{after} + U_{lost}$$

5 pts $$\frac{1}{2}(m_1 + m_2) V_2^2 = \frac{1}{2} k X_{max}^2$$

$$X_{max} = \sqrt{\frac{(m_1 + m_2)}{k} V_2^2} = V_2 \sqrt{\frac{(m_1 + m_2)}{k}}$$

5 pts $$X_{max} = \frac{\boldsymbol{m_1}}{\boldsymbol{m_1 + m_2}} \boldsymbol{V_1} \sqrt{\frac{\boldsymbol{(m_1 + m_2)}}{\boldsymbol{k}}} = \boldsymbol{m_1 V_1 \sqrt{\frac{1}{k(m_1 + m_2)}}}$$

<u>Problem 3:</u> At a given time, $t = 0\ seconds$, the combined spring-mass-damper system is subjected to a constant force, $\boldsymbol{F} = \mathbf{100}\ \boldsymbol{N}$. The values of all relevant constants are given below. The left boundary represents a fixed wall (v=0). Answer the following prompts:



$$\boldsymbol{k_1 = 20\frac{N}{m}} \qquad \boldsymbol{k_3 = 10\frac{N}{m}} \qquad \boldsymbol{k_5 = 100\frac{N}{m}}$$

$$\boldsymbol{b_4 = 100\frac{Ns}{m}} \qquad \boldsymbol{m_2 = 10\ kg} \qquad \boldsymbol{m_6 = 100\ kg}$$

1. Write three force balance equations which describe the above system.

3 pts $\boldsymbol{m_2 a_2 = F_3 + F_4 - F_1}$
3 pts $\boldsymbol{m_6 a_6 = F - F_3 - F_5}$
3 pts $\boldsymbol{F_4 = F_5}$
+1 for full set. – 1 per sign error, -3 pts per incorrect/missing term

2. Write three kinematic constraints to describe the velocities of elements within the system.

3 pts $\boldsymbol{v_{s1} = v_{m2}}$
3 pts $\boldsymbol{v_{s3} = v_{m6} - v_{m2}}$
3 pts $\boldsymbol{v_{s3} = v_{d4} + v_{s5}}$
+1 for full set. – 1 per sign error, -3 pts per incorrect term

3. Derive a full set of **state equations** in terms of only **state variables**, any necessary constants, and the input force, F, which can be used to fully describe the behavior of this system.

4 pts $\boldsymbol{v_{s1} = v_{m2}}$
4 pts $\boldsymbol{v_{s3} = v_{m6} - v_{m2}}$
4 pts $\boldsymbol{v_{s5} = v_{m6} - v_{m2} - \frac{k_5}{b_4} x_{s5}}$
4 pts $\boldsymbol{a_{m2} = \frac{k_3}{m_2} x_{s3} + \frac{k_5}{m_2} x_{s5} - \frac{k_1}{m_2} x_{s1}}$
4 pts $\boldsymbol{a_{m6} = \frac{F}{m_6} - \frac{k_3}{m_6} x_{s3} - \frac{k_5}{m_6} x_{s5}}$
– 1 per sign error, -3 pts per incorrect term

4. After a long time ($t = +\infty$) after the initial application of force, F, the system comes to rest (all velocities are zero). At this time, find the final extension length of <u>each</u> spring in the system.

a. $x_{s1}(t = +\infty)$=

$$\frac{F}{k_1} = \frac{100}{20} = \mathbf{5}\boldsymbol{m}$$

3 pts

b. $x_{s3}(t = +\infty)$=

$$\frac{F}{k_3} = \frac{100}{10} = \mathbf{10}\boldsymbol{m}$$

3 pts

c. $x_{s5}(t = +\infty)$=

$$\mathbf{0}\ \boldsymbol{m}$$

3 pts
+1 for all 3

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