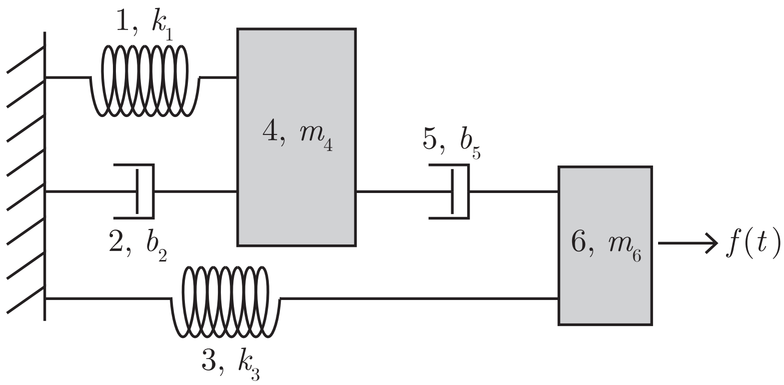
EA3 Quiz 1
Prof. Lynch
Spring 2024

Name: ___________________________________

You are allowed to use only pens, pencils, and erasers. No electronics, other papers, etc.

Make sure to show all your work, and make sure your final answer is clear (for example, you can circle it). **Use the back of the previous page if you need more space, for your work or your final answer.** Full credit, or partial credit if your final answer is wrong, will only be given if your work is clear. If you think any question does not give you enough information to give an answer or there is a mistake on the test, then clearly write the extra assumptions you had to make to answer the question, and answer the question using those assumptions.

No significant calculations are needed, so you don't need a calculator. If you get an answer like $11/32$ or $\sqrt{5}$, just leave it like that. On the other hand, we would appreciate simple calculations, like reducing $6/2$ to 3.



1. The six elements of the mass-spring-damper system above are numbered with their parameters. An external force $f(t)$ acts on m_6.
   (a) (4 pts) Write the force balance equations.

   (b) (4 pts) Write the geometric continuity equations (at least four independent equations).

   (c) (4 pts) Write the constitutive laws.

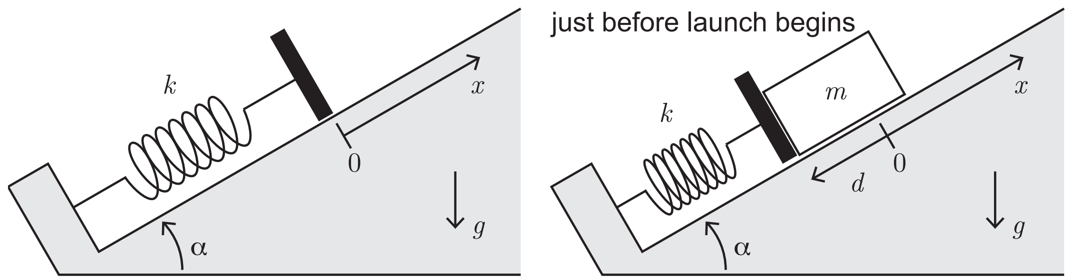
   (d) (4 pts) Write the state equations.

2. You've designed a new exercise bicycle that harnesses the energy expended by the user. When the user cycles at a speed v they feel a resistance force of magnitude bv, where b is like a damping constant. Instead of dissipating power, however, your exercise bike stores the energy the user puts into the bike (with 100% efficiency) by raising a mass m in gravity g.
   (a) (2 pts) If the user pedals at a speed v, what is the power they are putting into the mass?

   (b) (2 pts) Let h be the height of the mass. At the beginning of the exercise, $h = 0$. If the user cycles at a speed v, what is the total time T the user has to exercise to raise the mass to a height $h = 10$ m?

3. A system with one mass, one spring, and one damper has state variables x for the spring and v for the mass, and the state equations are $\dot{x} = -v$ and $\dot{v} = 3x - 2v + 10\cos t$.
   (a) (4 pts) At time $t = 0$, the initial conditions are $x(0) = 2$ and $v(0) = -1$. Approximately find the state variables at time $t = 0.5$ by using a single Euler step.

   (b) (6 pts) Draw the system corresponding to the state equations, including any walls, the spring, the damper, the mass, and the external force, using our usual conventions (positive displacement of a spring means it's stretched, positive velocity of a damper means it's lengthening, and positive velocity of a mass is to the right). Assume the mass is $m = 2$ and give the numerical values of the spring constant k, damping constant b, and the expression for the external force.



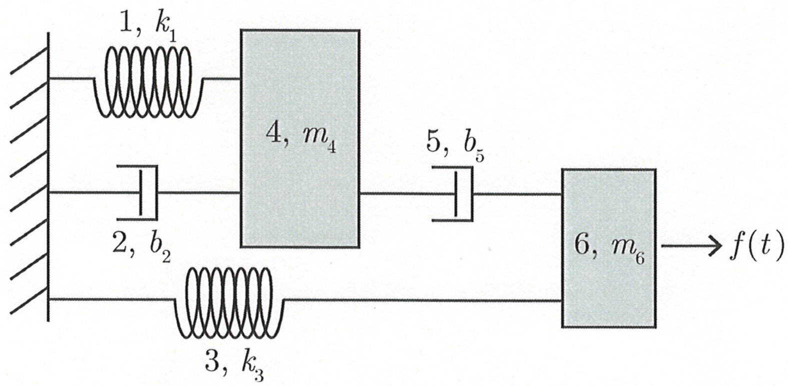
4. The figure on the left shows an infinitely long ramp with a launching spring (spring coefficient k). The spring's displacement is $x = 0$ and the force through the spring is zero. The angle of the ramp is α and gravity g acts downward. The figure on the right shows you have pushed a mass m against the spring to compress the spring by $d > 0$, so the displacement of the spring, and the position of the mass, is $x = -d$. (The position of the mass is measured at its left edge.) You then release the mass, and the spring launches the mass up the ramp. The mass slides on the ramp with friction coefficient μ. The spring loses contact with the mass when the spring and mass reach $x = 0$. After the mass breaks contact with the spring, the mass continues to slide up the ramp, and $x(t)$ denotes the position of the mass. The mass eventually stops ($\dot{x} = 0$) due to friction. Define the potential energy of the mass in gravity to be zero when its position is $x = 0$.

   (a) (2 pts) What is the total potential energy of the mass and spring on the right (the spring is compressed to $x = -d$) in terms of d and anything else needed?

   (b) (4 pts) After you release the mass, what is the kinetic energy of the mass when the spring breaks contact with the mass at $x = 0$?

   (c) (4 pts) Let K be the correct answer to the previous question, the kinetic energy when the mass leaves the spring at $x = 0$. Using K, at what position x_{max} does the mass come to rest on the ramp?

   (d) (2 pts) Describe in one sentence the motion of the spring after it breaks contact with the mass.



1. The six elements of the mass-spring-damper system above are numbered with their parameters. An external force $f(t)$ acts on m_6.
   (a) (4 pts) Write the force balance equations.

$$-f_1 - f_2 + f_5 = m_4 a_4 = m_4 \dot{v}_4$$
$$-f_3 - f_5 + f = m_6 a_6 = m_6 \dot{v}_6$$

   (b) (4 pts) Write the geometric continuity equations (at least four independent equations).

$v_1 = v_4$ $\quad\quad$ $v_4 + v_5 = v_6$

$v_2 = v_4$

$v_3 = v_6$

   (c) (4 pts) Write the constitutive laws.

$f_1 = k_1 x_1 \quad f_2 = b_2 v_2 \quad f_3 = k_3 x_3 \quad f_5 = b_5 v_5$

   (d) (4 pts) Write the state equations.

State variables: x_1, x_3, v_4, v_6

$$\dot{x}_1 = v_4$$
$$\dot{x}_3 = v_6$$
$$\dot{v}_4 = \frac{1}{m_4}(\underbrace{-k_1 x_1}_{-f_1} \underbrace{- b_2 v_2}_{-f_2} + \underbrace{b_5 v_5}_{f_5})$$
$$= \frac{1}{m_4}(-k_1 x_1 - b_2 v_4 + b_5 (v_6 - v_4))$$
$$\dot{v}_6 = \frac{1}{m_6}(-k_3 x_3 - b_5 v_5 + f)$$
$$= \frac{1}{m_6}(-k_3 x_3 - b_5 (v_6 - v_4) + f)$$

2. You've designed a new exercise bicycle that harnesses the energy expended by the user. When the user cycles at a speed v they feel a resistance force of magnitude bv, where b is like a damping constant. Instead of dissipating power, however, your exercise bike stores the energy the user puts into the bike (with 100% efficiency) by raising a mass m in gravity g.
   (a) (2 pts) If the user pedals at a speed v, what is the power they are putting into the mass?

$$\text{power} = fv = bv\,v = bv^2$$

   (b) (2 pts) Let h be the height of the mass. At the beginning of the exercise, $h = 0$. If the user cycles at a speed v, what is the total time T the user has to exercise to raise the mass to a height $h = 10$ m?

$$\text{power} \times \text{time} = \text{work (energy)}$$

$$bv^2 T = mgh = 10mg$$

$$T = \frac{10mg}{bv^2}$$

3. A system with one mass, one spring, and one damper has state variables x for the spring and v for the mass, and the state equations are $\dot{x} = -v$ and $\dot{v} = 3x - 2v + 10\cos t$.
   (a) (4 pts) At time $t = 0$, the initial conditions are $x(0) = 2$ and $v(0) = -1$. Approximately find the state variables at time $t = 0.5$ by using a single Euler step.

$$x(0.5) = x(0) + 0.5\,(-v(0)) = 2 + 0.5(1) = 2.5$$

$$v(0.5) = v(0) + 0.5(3x(0) - 2v(0) + 10\cos 0)$$

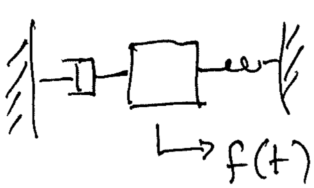
$$= -1 + 0.5(3\cdot 2 - 2(-1) + 10)$$

$$= -1 + 0.5(18)$$

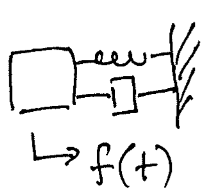
$$= 8$$

   (b) (6 pts) Draw the system corresponding to the state equations, including any walls, the spring, the damper, the mass, and the external force, using our usual conventions (positive displacement of a spring means it's stretched, positive velocity of a damper means it's lengthening, and positive velocity of a mass is to the right). Assume the mass is $m = 2$ and give the numerical values of the spring constant k, damping constant b, and the expression for the external force.

v for the mass is opposite $\dot{x}$ for the spring, from $\dot{x} = -v$, so a spring connects m to a wall on the right. Damper connects mass to a wall:



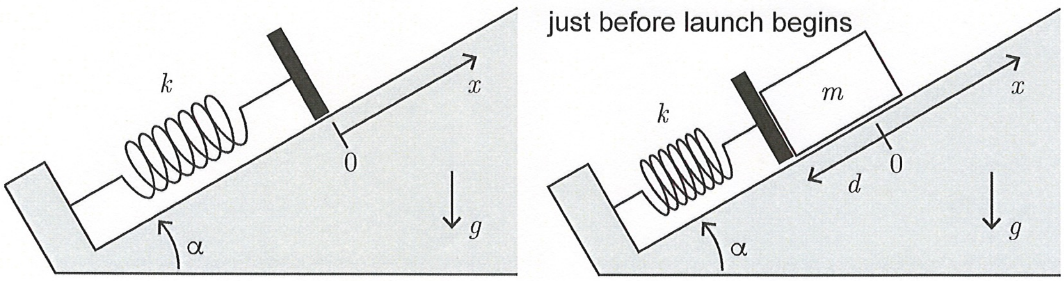
or



$$\dot{v} = \frac{1}{m}(-bv + kx) + \frac{1}{m}f$$

$$m = 2, \text{ so } \frac{-b}{2} = -2, \quad \frac{k}{2} = 3$$

$$b = 4, \; k = 6, \; f = 20\cos t$$



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   (a) (2 pts) What is the total potential energy of the mass and spring on the right (the spring is compressed to $x = -d$) in terms of d and anything else needed?

   PE = spring energy + gravitational potential

   $= \frac{1}{2} k d^2 + mgh$

   $= \frac{1}{2} k d^2 - mgd \sin\alpha$

   (b) (4 pts) After you release the mass, what is the kinetic energy of the mass when the spring breaks contact with the mass at $x = 0$?

   $\cancel{KE(0)} + PE(0) = KE(\text{launch}) + \cancel{PE(\text{launch})}$ + energy lost to friction

   $0 + \frac{1}{2} k d^2 - mgd \sin\alpha = KE + 0 + (\mu mg \cos\alpha)\, d$

   $\frac{1}{2} k d^2 - mgd(\sin\alpha + \mu \cos\alpha) = KE$

   (c) (4 pts) Let K be the correct answer to the previous question, the kinetic energy when the mass leaves the spring at $x = 0$. Using K, at what position x_{max} does the mass come to rest on the ramp?

   K = grav. potential at x_{max} + energy lost to friction

   $= mg\, x_{max} \sin\alpha + x_{max}\, \mu mg \cos\alpha$

   $= mg(\sin\alpha + \mu\cos\alpha)\, x_{max}$

   $x_{max} = K / (mg(\sin\alpha + \mu \cos\alpha))$

   (d) (2 pts) Describe in one sentence the motion of the spring after it breaks contact with the mass.

   It stops instantaneously at its rest length since $f_{spring} = 0$.