Engineering Analysis 3
Fall 2024
Quiz #2

NAME: ______________________________

Date: Monday November 11, 2024; 50 minutes total

No Notes, Calculators, Smartphones, etc.

ATTEMPT ALL PROBLEMS (100 Points total). Show all of your work on these pages. Please read each problem thoroughly, circle any final answers, and clearly (and concisely) explain your findings, when prompted.

Neatness counts: if the graders can not understand your work, we can not promise that credit will be awarded.

All springs and dampers are assumed to be linear, ideal, infinite, and massless. Levers and connectors are assumed to be massless and non-deformable. Masses are treated as non-deformable point-masses.

Useful Formulae:

$$-\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Problem	Points	Score
#1	**40**	
1.1	15	
1.2	15	
1.3	10	
#2	**40**	
2.1	30	
2.2	10	
#3	**20**	
Total	**100**	

Problem 1: The following represents a set of state equations, $\frac{dx_s}{dt}$ and $\frac{dv_m}{dt}$, in terms of a pair of state variables, x_s and v_m, which represent the extension of a spring and the velocity of a mass, respectively. The additional terms in these equations, $q,\ p,\ n, and\ L$, are constants.

$$\frac{dx_s}{dt} = -qx_s + pnv_m$$

$$\frac{dv_m}{dt} = -nx_s - L$$

1. Derive a linear, homogeneous differential equation which describes the extension of spring (x_s).

2. Write an algebraic expression which will solve for the roots of this differential equation.

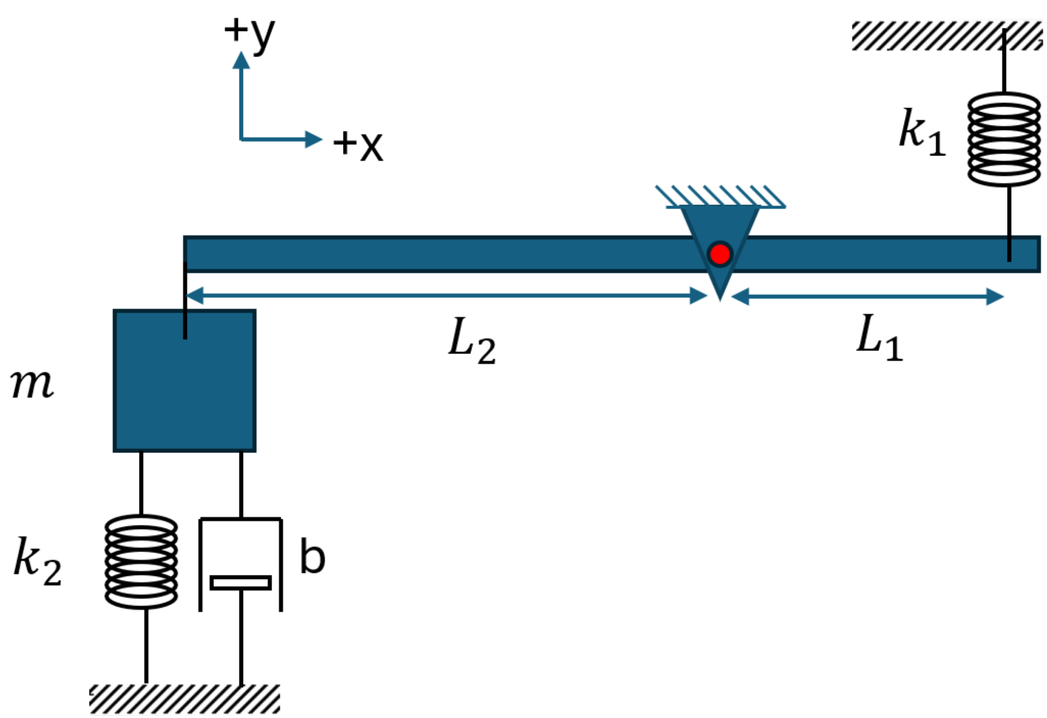
3. After solving for the above algebraic expression, you find that this system has two real roots, $r_{1,2} = -3 \pm 2$. It is also noted that the initial conditions are as follows:

$$x_s(0) = 0$$
$$v_m(0) = 0$$

Write an equation which represents the <u>particular</u> solution of the position of the spring (x_s) as a function of time (your equation <u>may</u> contain the constants $q,\ p,\ n, and/or\ L$).

Problem 2: The spring-mass-damper system shown below is **initially at rest in static equilibrium**, and the mass is acted upon by gravity in the downward (-y) direction.

1. Solve for the state equations of the following spring-mass-damper system attached to a lever in terms of only the state variables $x_{s1}, x_{s2}, and\ v_m$ (clearly circle your answers)



2. An impulse causes the mass to begin moving downward, and we observe that spring 2's position oscillates for a few seconds before coming to rest. If $L_2 > L_1$, which spring position graph (i.e., $x_{s1}(t)$ or $x_{s2}(t)$) do you expect to have a greater initial **amplitude** of oscillation?

Problem 3: A known system is represented by the following differential equation and particular solution (a, b, and c do not vary with time):

$$\frac{bc}{2}x_s'' + a\sqrt{b}x_s' + cx_s = F$$

$$x_s(t) = e^{-3t}(\cos(2t) - \sin(2t)) + 5$$

How will increasing the value of "**a**" change (i.e., increase, decrease, or not affect) the magnitude of the following parameters of the particular solution?

1. The settling point of $x_s(t)$ at $t = +\infty$

2. The oscillating frequency of $x_s(t)$

3. The decay rate of $x_s(t)$

How will increasing the value of "**b**" change (i.e., increase, decrease, or not affect) the magnitude of the following parameters of the particular solution?

4. The settling point of $x_s(t)$ at $t = +\infty$

5. The oscillating frequency of $x_s(t)$

6. The decay rate of $x_s(t)$

How will increasing the value of "**c**" change (i.e., increase, decrease, or not affect) the magnitude of the following parameters of the particular solution?

7. The settling point of $x_s(t)$ at $t = +\infty$

8. The decay rate of $x_s(t)$

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All springs and dampers are assumed to be linear, ideal, infinite, and massless. Levers and connectors are assumed to be massless and non-deformable. Masses are treated as non-deformable point-masses.

Useful Formulae:

$$-\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

General Exponential Decay Function: $x(t) = Ae^{rt} + C$

Overdamped System: $x(t) = A_1 e^{r_1 t} + A_2 e^{r_2 t} + C$

Underdamped System: $x(t) = e^{\alpha t}(A_1 \cos(\omega t) + A_2 \sin(\omega t)) + C$

Problem	Points	Score
#1	**40**	
1.1	15	
1.2	10	
1.3	15	
#2	**40**	
2.1	30	
2.2	10	
#3	**20**	
Total	**100**	

Problem 1: The following represents a set of state equations, $\frac{dx_s}{dt}$ and $\frac{dv_m}{dt}$, in terms of a pair of state variables, x_s and v_m, which represent the extension of a spring and the velocity of a mass, respectively. The additional terms in these equations, $q,\ p,\ n, and\ L$, are constants.

$$\frac{dx_s}{dt} = -qx_s + pnv_m$$

$$\frac{dv_m}{dt} = -nx_s - L$$

1. Derive a linear, homogeneous differential equation which describes the extension of spring (x_s).

   5 pts $\quad x_s'' = -qx_s' + pnv_m'$

   5 pts $\quad x_s'' = -qx_s' + pn(-nx_s - L)$

   5 pts $\quad x_s'' = -qx_s' - pn^2x_s - pnL$

   $$x_s'' + qx_s' + pn^2x_s = -pnL$$

2. Write an algebraic expression which will solve for the roots of this differential equation.

   10 pts $\quad r_{1,2} = -\frac{q}{2} \pm \frac{\sqrt{q^2-4pn^2}}{2}$

   (5 pts for setting up Ae^rt+C , 5 points for solving, if needed)

3. After solving for the above algebraic expression, you find that this system has two real roots, $r_{1,2} = -3 \pm 2$. It is also noted that the initial conditions are as follows:

   $$x_s(0) = 0$$
   $$v_m(0) = 0$$

   Write an equation which represents the particular solution of the position of the spring (x_s) as a function of time (your equation may contain the constants $q,\ p,\ n, and/or\ L$).

3 pts for form, 2 pts for set pt. $\quad x_s(t) = A_1e^{-5t} + A_2e^{-t} - L/n$

2 pts $\quad x_s'(t) = -5A_1e^{-5t} - A_2e^{-t}$

$$x_s(0) = 0 = A_1 + A_2 - L/n$$

3 pts $\quad x_s'(0) = 0 = -5A_1 - A_2$

5 pts (-2 for 1 wrong) $\quad A_1 = -\frac{L}{4n} \qquad A_2 = \frac{5L}{4n}$

$$x_s(t) = -\frac{L}{4n}(e^{-5t} - 5e^{-t} + 4)$$

$$x_s(t) = -\frac{L}{4n}e^{-5t} + \frac{5L}{4n}e^{-t} - \frac{L}{n}$$

Problem 2: The spring-mass-damper system shown below is **initially at rest in static equilibrium**, and **the mass is acted upon by gravity** in the downward (-y) direction.

1. Solve for the state equations of the following spring-mass-damper system attached to a lever in terms of only the state variables $x_{s1}, x_{s2}, and\ v_m$ (clearly circle your answers)

8 pts Force Balance $$ma_m = -mg - bv_d - k_2 x_{s2} - F_2$$

$$F_1 = k_1 x_{s1}$$

8 pts Moment Balance $$F_2 L_2 = F_1 L_1$$

8 pts Linear Kinematics $$v_1 = -v_{s1}$$

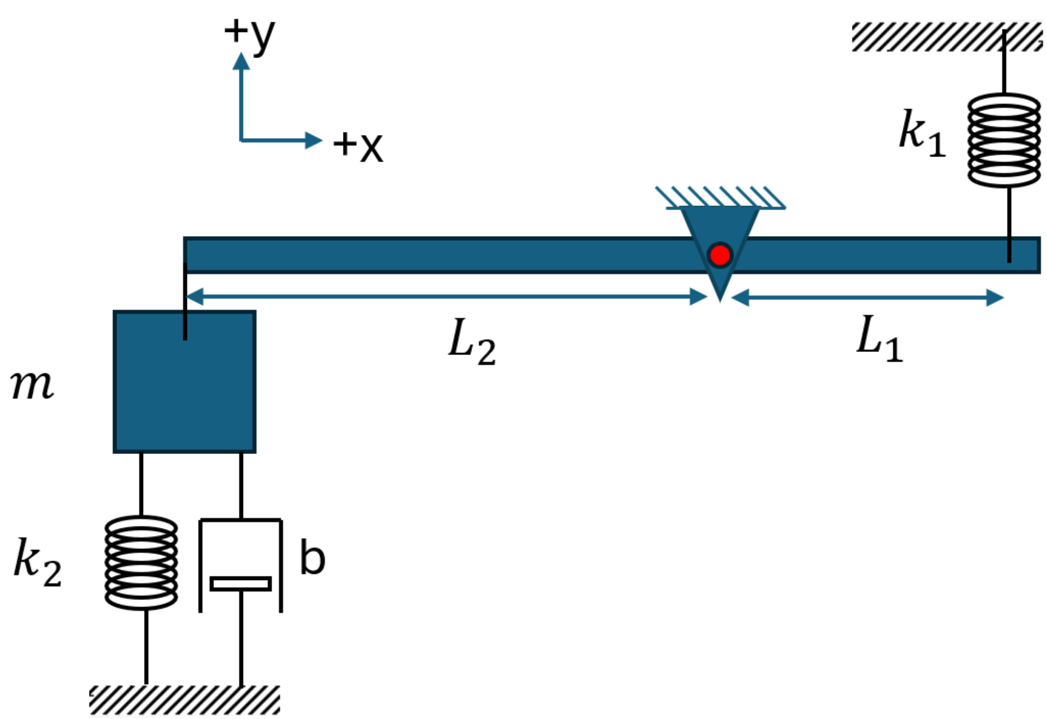
$$v_2 = v_m = v_{s2} = v_d$$

8 pts Angular Kinematics (-4 pts for sign error) $$\frac{v_1}{L_1} = -\frac{v_2}{L_2}$$

8 pts Solving State Equations $$a_m = -\frac{kL_1}{mL_2} x_{s1} - \frac{k_2}{m} x_{s2} - \frac{b}{m} v_m - g$$

$$v_{s1} = \frac{L_1}{L_2} v_m$$

$$v_{s2} = v_m$$



Problem 3: A known system is represented by the following differential equation and particular solution (a, b, and c do not vary with time):

$$\frac{bc}{2}x_s'' + a\sqrt{b}x_s' + \frac{c}{a}x_s = \frac{F}{a}$$

$$x_s(t) = e^{-3t}(\cos(2t) - \sin(2t)) + 5$$

$$\alpha = -\frac{a}{\sqrt{b}c}$$

$$\omega = \sqrt{\frac{1}{b}\left(\frac{2}{a} - \frac{a^2}{c^2}\right)}$$

2 pts each

How will <u>increasing</u> the value of "**a**" change (i.e., increase, decrease, or not affect) the magnitude of the following parameters of the particular solution?

1. The settling point of $x_s(t)$ at $t = +\infty$ No effect

2. The decay rate of $x_s(t)$ Increase

How will <u>increasing</u> the value of "**b**" change (i.e., increase, decrease, or not affect) the magnitude of the following parameters of the particular solution?

3. The settling point of $x_s(t)$ at $t = +\infty$ No effect

4. The oscillating frequency of $x_s(t)$ Decrease

5. The decay rate of $x_s(t)$ Decrease

How will <u>increasing</u> the value of "**c**" change (i.e., increase, decrease, or not affect) the magnitude of the following parameters of the particular solution?

6. The settling point of $x_s(t)$ at $t = +\infty$ Decrease

7. The oscillating frequency of $x_s(t)$ Increase

8. The decay rate of $x_s(t)$ Decrease

At what value of "**a**" (written in terms of the other variables "**b**" and "**c**") will the system switch from an oscillating (i.e., "underdamped") to a non-oscillating (i.e., "overdamped") system?

4 pts $\quad a = \sqrt[3]{2c^2}$

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