EA3 Quiz 3
Section 21, Spring 2023

Name: ________________________________

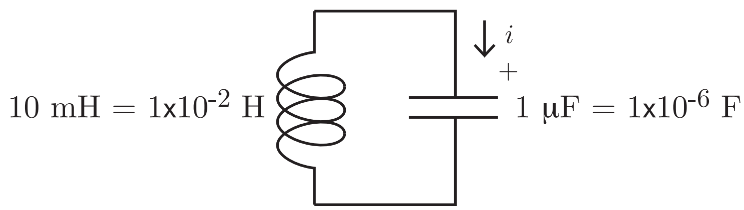
You are allowed to use only pens, pencils, and erasers. No electronics, other papers, etc.

Make sure to show all your work, and make sure your final answer is clear (for example, you can circle it). Full credit, or partial credit if your final answer is wrong, will only be given if your thought process is clear. If you think any question does not give you enough information to give an answer (i.e., there is a mistake on the test), then clearly write the extra assumptions you had to make to answer the question, and answer the question using those assumptions.

If you get a square root in your answer, like $\sqrt{12}$, it is preferred that you write it in its reduced form, $2\sqrt{3}$. Leaving it as $\sqrt{12}$ is also OK, but not preferred. DO NOT try to express it as an approximate decimal value, e.g., $\sqrt{12} = 3.46$ unless it has a simple exact decimal value.

Use the backs of the pages if you need more room for your work.

1. At time $t = 0$, the voltage held by the capacitor in the LC circuit below is 10 V (the positive side is labeled $+$), and no current is flowing. Current is defined to be positive according to the arrow shown.

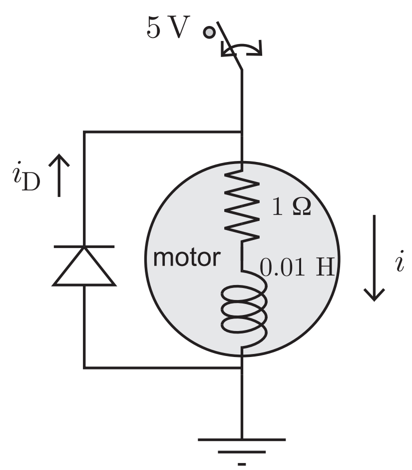


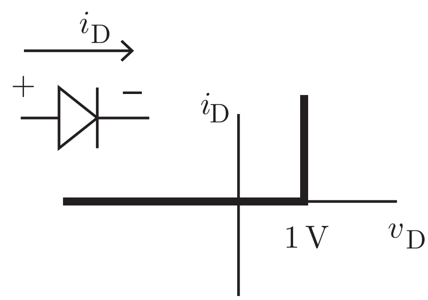
   (a) (2 pts) What is the natural frequency of this circuit? Provide your answer as both ω_n (radians/s) and f_n (Hz, i.e., cycles/s).

   (b) (4 pts) Plot the voltage across the capacitor as a function of time, starting from $t = 0$, correctly indicating the scale on the vertical axis (volts). Plot for time $T = 1/f_n$.

   (c) (2 pts) Plot the current i as a function of time, correctly indicating the scale on the vertical axis (current). Plot for time $T = 1/f_n$.

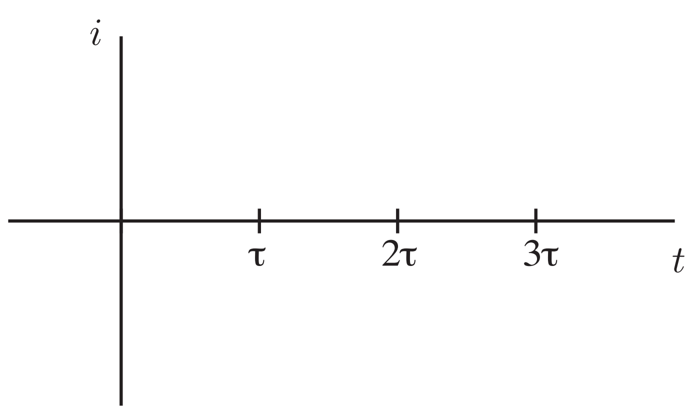
2. A simple electrical model of a motor is a resistor and inductor in series, and the torque produced by the motor is proportional to the current through it. (Assume the motor is prevented from spinning by an external restraint, allowing us to ignore a "back-emf" term in the motor model.) In the circuit below, the motor resistance is $1\ \Omega$ and the inductance is 0.01 H. The torque produced by the motor is controlled by switching one terminal of the motor between 5 V and disconnected (open circuit), while the other terminal is held at 0 V. There is also a diode in the circuit. The diode does not allow current to flow "down" as shown in the figure, and if current flows "up," the voltage at the bottom terminal of the diode is 1 V higher than the voltage at the top terminal. (This is summarized by the graph of the diode's idealized current-voltage constitutive law shown on the right.)





idealized diode constitutive law

(a) (4 pts) The switch has been disconnected a long time, so everything is in steady state. Then the switch switches to 5 V at time $t = 0$. Plot by hand the current through the motor as a function of time on the axes given. Give the time constant τ (in seconds) for the horizontal axis and give the correct scale for the vertical (current) axis. Use the facts that $e^{-1} \approx 0.37$, $e^{-2} \approx 0.14$, and $e^{-3} \approx 0.05$.



After reaching steady state, the switch switches back to disconnected. The remaining questions refer to what happens just after this switch.

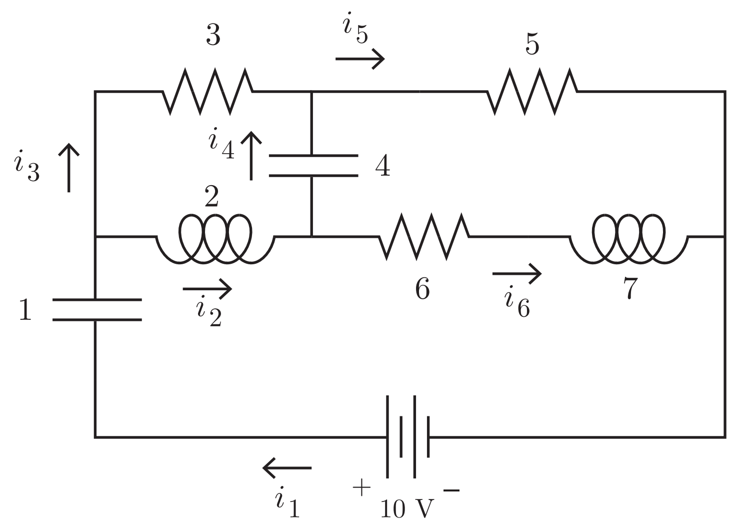
(b) (2 pts) What is the current i through the motor? Give units.

(c) (2 pts) What is the rate of change of current (di/dt) through the motor? Give units.

(d) (2 pts) The diode shown in the figure is often called a "flyback diode." What would have happened when switching to the disconnected state if we did not have a flyback diode? Explain in a clear sentence.

3. (2 pts) You are a member of NUSTARS, and you are experimenting with a new form of rocket propulsion, where the energy for propulsion is stored electrically in a capacitor, not in a liquid or solid propellant. Your rocket weighs 4 kg, and for your proof-of-concept test, you want to propel it to an altitude of 10 m. Assume the gravitational constant (at low altitudes) is 10 m/s^2. You choose a 2 F capacitor. Assuming the idealized case of 100% efficient conversion from electrical energy to mechanical energy and no air resistance, (a) to what minimum voltage would you have to charge the capacitor before launch, and (b) what is the corresponding minimum capacitor charge before launch? Both answers should be numerical with proper units.

4. Consider the circuit below. Six currents are labeled. In addition to the 10 V battery, there are seven passive elements labeled 1–7, with associated constants C_1, L_2, R_3, C_4, R_5, R_6, and L_7.

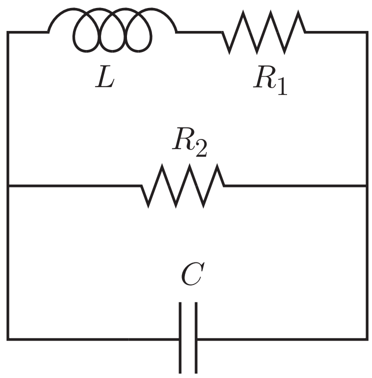


(a) (4 pts) Write a full set of KVL equations. You may give either a minimum complete set, or a set that includes additional linearly dependent equations. (Do not include constitutive laws in these equations; instead, just write v_1, v_2, etc.)

(b) (4 pts) Write a full set of KCL equations. You may give either a minimum complete set, or a set that also includes linearly dependent equations.

(c) (3 pts) Write the constitutive laws for elements 1, 2, and 3.

5. (8 pts) For the circuit below, derive two coupled first-order differential equations, where your state variables are the voltage of the capacitor and the current of the inductor.



EA3 Quiz 3
Section 21, Spring 2023

Name: Solutions

You are allowed to use only pens, pencils, and erasers. No electronics, other papers, etc.

Make sure to show all your work, and make sure your final answer is clear (for example, you can circle it). Full credit, or partial credit if your final answer is wrong, will only be given if your thought process is clear. If you think any question does not give you enough information to give an answer (i.e., there is a mistake on the test), then clearly write the extra assumptions you had to make to answer the question, and answer the question using those assumptions.

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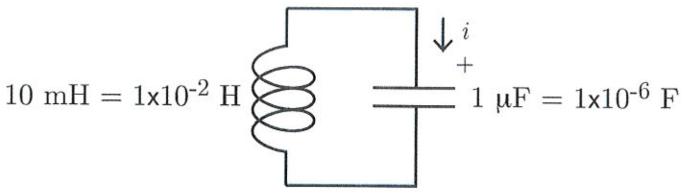
1. At time $t = 0$, the voltage held by the capacitor in the LC circuit below is 10 V (the positive side is labeled +), and no current is flowing. Current is defined to be positive according to the arrow shown.



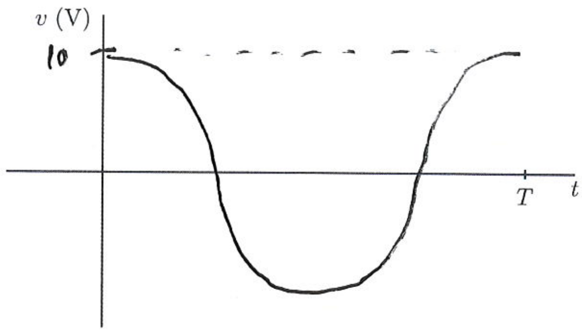
(a) (2 pts) What is the natural frequency of this circuit? Provide your answer as both ω_n (radians/s) and f_n (Hz, i.e., cycles/s).

KVL: $\frac{1}{C}\int i\,dt + L\frac{di}{dt} = 0 \rightarrow L\ddot{i}'' + \frac{1}{C} i = 0 \rightarrow \ddot{i}'' + \frac{1}{LC} i = 0 \rightarrow s^2 + \omega_n^2 = 0$

$\omega_n = \frac{1}{\sqrt{LC}} = 10^4$ rad/s $\quad f_n = (10^4/2\pi)$ Hz

(b) (4 pts) Plot the voltage across the capacitor as a function of time, starting from $t = 0$, correctly indicating the scale on the vertical axis (volts). Plot for time $T = 1/f_n$.

$i(t) = A\cos\omega_n t + B\sin\omega_n t$ so



$V_c(t) = C\cos\omega_n t + D\sin\omega_n t$

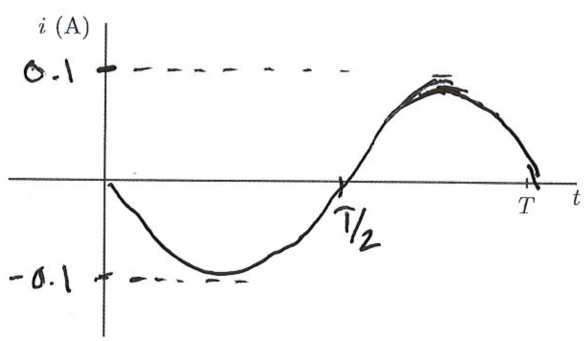
$V_c = 10$ V initially,

no current is flowing,

so $V_c'(0) = \frac{1}{C} i = 0$

$T = \frac{1}{f_n}$ so 1 cycle

(c) (2 pts) Plot the current i as a function of time, correctly indicating the scale on the vertical axis (current). Plot for time $T = 1/f_n$.



$i = 0$ initially

$V_L(0) = L\frac{di(0)}{dt} = -10$ V

$\frac{di(0)}{dt} = -1000 \frac{A}{s}$ initially

max current calculated from

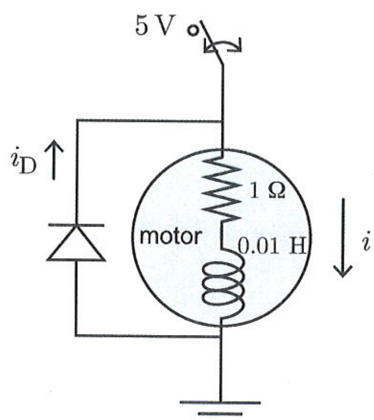
$\frac{1}{2}Cv^2 = \frac{1}{2}Li^2$

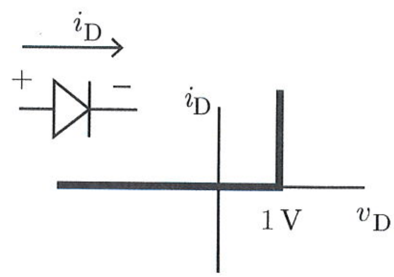
$(10^{-6})(100) = 10^{-2} i^2$

$10^{-2} = i^2$

$0.1\text{A} = i$

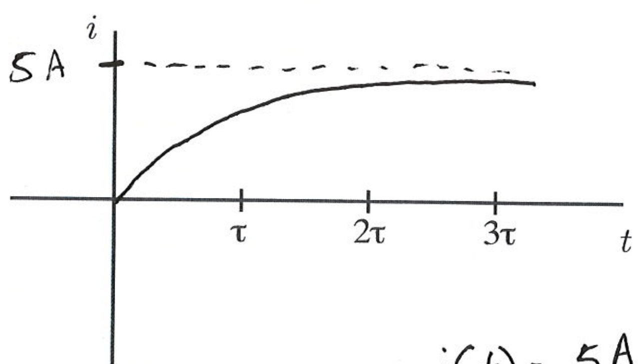
2. A simple electrical model of a motor is a resistor and inductor in series, and the torque produced by the motor is proportional to the current through it. (Assume the motor is prevented from spinning by an external restraint, allowing us to ignore a "back-emf" term in the motor model.) In the circuit below, the motor resistance is $1\ \Omega$ and the inductance is 0.01 H. The torque produced by the motor is controlled by switching one terminal of the motor between 5 V and disconnected (open circuit), while the other terminal is held at 0 V. There is also a diode in the circuit. The diode does not allow current to flow "down" as shown in the figure, and if current flows "up," the voltage at the bottom terminal of the diode is 1 V higher than the voltage at the top terminal. (This is summarized by the graph of the diode's idealized current-voltage constitutive law shown on the right.)





idealized diode constitutive law

(a) (4 pts) The switch has been disconnected a long time, so everything is in steady state. Then the switch switches to 5 V at time $t = 0$. Plot by hand the current through the motor as a function of time on the axes given. Give the time constant τ (in seconds) for the horizontal axis and give the correct scale for the vertical (current) axis. Use the facts that $e^{-1} \approx 0.37$, $e^{-2} \approx 0.14$, and $e^{-3} \approx 0.05$.



diode plays no role

$\tau = \frac{L}{R} = 0.01\,s$

inductor is open circuit initially, wire in steady state

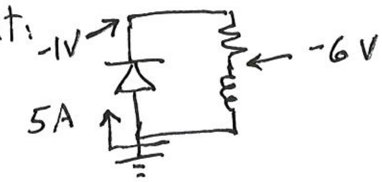
$i(t) = 5A\left(1-e^{-t/\tau}\right)$

After reaching steady state, the switch switches back to disconnected. The remaining questions refer to what happens just after this switch.

(b) (2 pts) What is the current i through the motor? Give units.

inductor wants to keep the current flowing. diode allows it.

new circuit:



$i = 5A$

(c) (2 pts) What is the rate of change of current (di/dt) through the motor? Give units.

-6 V across the inductor based on 1V diode voltage.

$$-6 = L\frac{di}{dt}$$

$$\frac{di}{dt} = -600 \text{ A/s}$$

(d) (2 pts) The diode shown in the figure is often called a "flyback diode." What would have happened when switching to the disconnected state if we did not have a flyback diode? Explain in a clear sentence.

Current through inductor would have to drop to zero instantly. $\frac{di}{dt} = -\infty$ so $V_L = L\frac{di}{dt} = -\infty$. A spark would occur at the switch.

3. (2 pts) You are a member of NUSTARS, and you are experimenting with a new form of rocket propulsion, where the energy for propulsion is stored electrically in a capacitor, not in a liquid or solid propellant. Your rocket weighs 4 kg, and for your proof-of-concept test, you want to propel it to an altitude of 10 m. Assume the gravitational constant (at low altitudes) is 10 m/s^2. You choose a 2 F capacitor. Assuming the idealized case of 100% efficient conversion from electrical energy to mechanical energy and no air resistance, (a) to what minimum voltage would you have to charge the capacitor before launch, and (b) what is the corresponding minimum capacitor charge before launch? Both answers should be numerical with proper units.

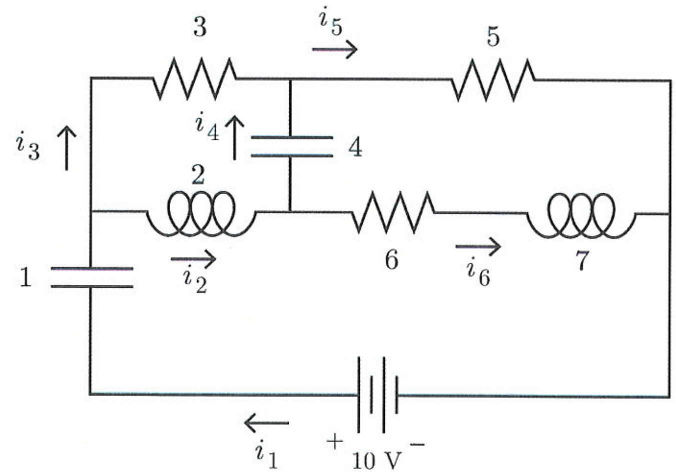
$$\frac{1}{2}Cv^2 = mgh \quad \rightarrow \text{equate electrical and mechanical PE}$$

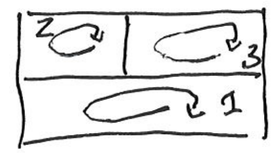
$$\frac{1}{2}(2)v^2 = (4)(10)(10)$$

$$v^2 = 400 \text{ J/F}$$

a) $\boxed{v = 20\text{ V}}$ b) $\boxed{q = Cv = 40\text{ C}}$

4. Consider the circuit below. Six currents are labeled. In addition to the 10 V battery, there are seven passive elements labeled 1–7, with associated constants C_1, L_2, R_3, C_4, R_5, R_6, and L_7.





for KVL

4: 

5: 

(a) (4 pts) Write a full set of KVL equations. You may give either a minimum complete set, or a set that includes additional linearly dependent equations. (Do not include constitutive laws in these equations; instead, just write v_1, v_2, etc.)

Any 3 of

1: $-10 + V_1 + V_2 + V_6 + V_7 = 0$

2: $V_3 - V_4 - V_2 = 0$

3: $V_4 + V_5 - V_7 - V_6 = 0$

4: $-10 + V_1 + V_3 - V_4 + V_6 + V_7 = 0$

5: $-10 + V_1 + V_2 + V_4 + V_5 = 0$

(b) (4 pts) Write a full set of KCL equations. You may give either a minimum complete set, or a set that also includes linearly dependent equations.

$i_1 = i_2 + i_3$ $\qquad$ $i_3 + i_4 = i_5$

$i_2 = i_4 + i_6$ $\qquad$ $i_5 + i_6 = i_1$

(c) (3 pts) Write the constitutive laws for elements 1, 2, and 3.

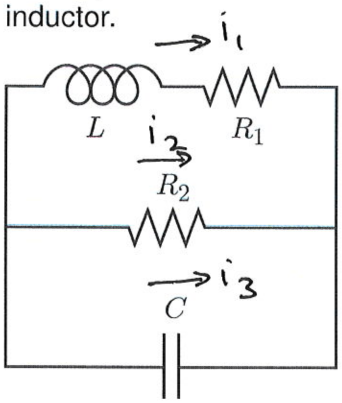
$$V_1 = \frac{1}{C_1} q_1 = \frac{1}{C_1} \int i_1 \, dt$$

$$V_2 = L_2 \frac{di_2}{dt}$$

$$V_3 = R_3 i_3$$

5. (8 pts) For the circuit below, derive two coupled first-order differential equations, where your state variables are the voltage of the capacitor and the current of the inductor.

v $\quad$ i



KVL: $V_L + V_{R_1} = V_{R2} = V_C$

KCL: $i_1 + i_2 + i_3 = 0$

CL: $V_c = \frac{1}{C} q_3$

$V_{R2} = R_2 i_2$

$V_L = L \frac{di_1}{dt}$

$V_{R1} = R_1 i_1$

Express state eqs. in terms of V_c & i_1 (or v & i)

$V_L + V_{R1} = V_c$

$L \frac{di_1}{dt} + R_1 i_1 = V_c$

$$\boxed{\frac{di_1}{dt} = \frac{1}{L}(V_c - R_1 i_1) \quad \text{or} \quad \frac{di}{dt} = \frac{1}{L}(v - R_1 i)}$$

$V_c = \frac{1}{c} q_3 \longrightarrow \frac{dV_c}{dt} = \frac{1}{C} i_3 = \frac{-1}{C}(i_1 + i_2)$

$i_2 = \frac{V_{R2}}{R_2} = \frac{V_c}{R_2}$

$$\boxed{\frac{dV_c}{dt} = -\frac{1}{C}\left(i_1 + \frac{V_c}{R_2}\right) \quad \text{or} \quad \frac{dv}{dt} = -\frac{1}{C}\left(i + \frac{v}{R_2}\right)}$$