ENGINEERING ANALYSIS 3

SYSTEM DYNAMICS

Quiz No. 2

May 15, 2023

Name : ____

There are 2 problems with multiple sections.

Clearly mark your answers (circle your final answers, wherever applicable) and detail your problem-solving process. Credit will primarily be rewarded based on process (which demonstrates your conceptual understanding of the material) rather than results.

In this quiz, all masses, springs, and dampers are ideal and linear. All masses are non-deformable and can therefore be treated as "points" or "nodes." Springs, dampers, and levers are assumed to be massless. Walls are stationary if not otherwise specified in the problem statements.

Dimensional units will be provided for all necessary constants, but for ease of writing, your solutions do not need to include units. All numerical answers will be assumed to be in SI units (N, m, s), unless otherwise specified.

Potentially Useful Equations

Potential Energy = mgh

Elastic Potential Energy = ½ k x^2

Work = $\int F dx$

Power = $\frac{dU}{dt}$ = Fv

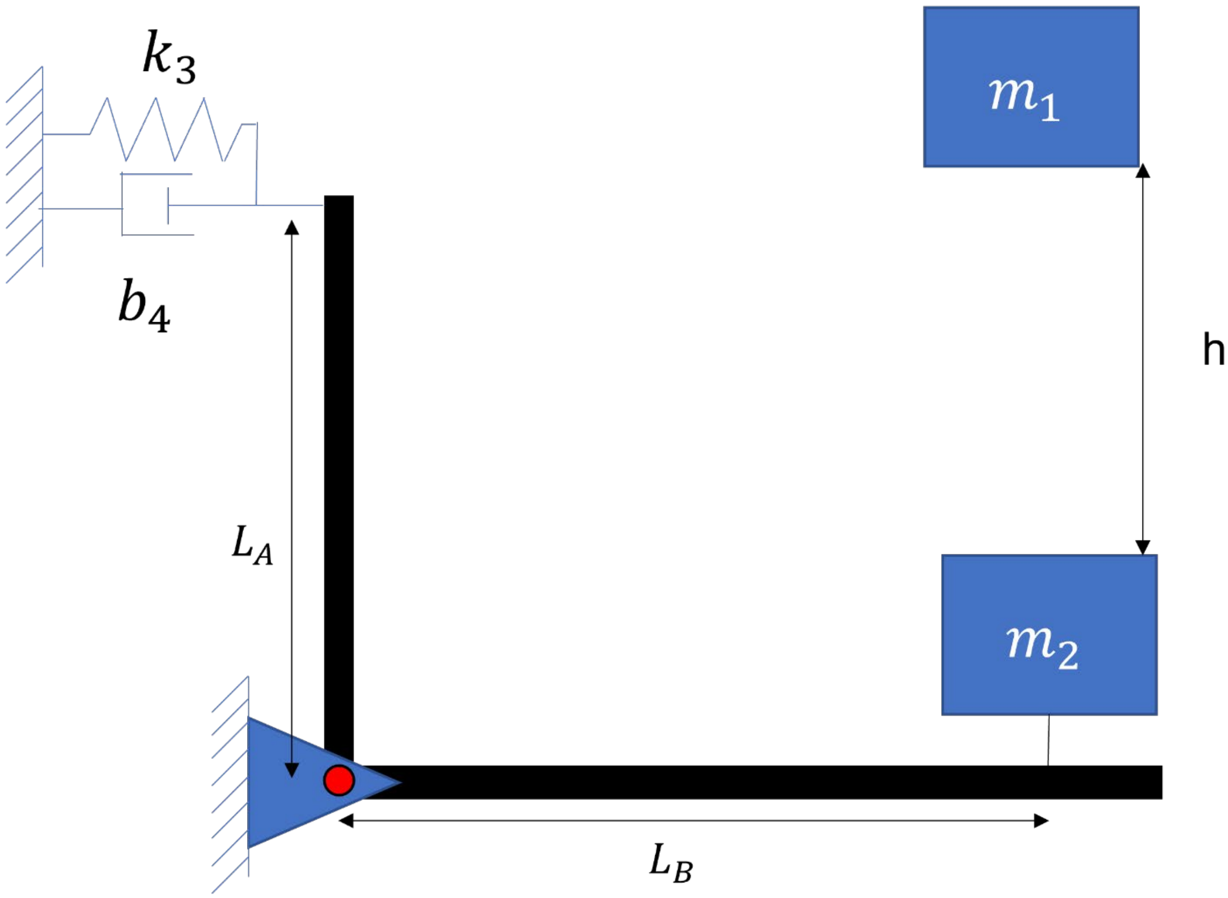
Kinetic Energy = ½ m v^2

Do not start solving this quiz until you are told to do so. *

Total ______ / 100 points

Problem 1

Observe the following spring-mass-damper system connected by an angled, massless lever. A mass, m1=0.5 kg, is dropped from a height of 1 meter and, upon collision, immediately sticks to a second mass, m2=0.5 kg. This collision results in movement of the lever. Solve the following prompts (**90 Pts)**



1.A (5 points): Solve for the velocity of the combined mass after the collision (assume that the collision is instantaneous).

<u>1.B</u> (**<u>5 points</u>**): Draw Free Body Diagrams (FBDs) for each mass (the combined masses may be referred to as "m" moving forward), lever, and junction.

<u>1.C</u> (**<u>5 points</u>**): Set up a force (or moment) balance equation for each FBD and substitute in any relevant constitutive equations.

<u>1.D:</u> (**<u>5 points</u>**): Define all angular and linear kinematic constraints.

1.E (5 points): Solve for the state equations (equations of motion), in terms of your state variables and any additional constants.

1.F (5 points): Write a second-order, linear differential equation for the position of spring 3.

1.G (10 points): Using the given constants, determine the appropriate form of the general solution of the position of spring 3 as a function of time.

Parameter	Value	Parameter	Value
m_1	0.5 kg	L_A	1 m
m_2	0.5 kg	L_B	2 m
k_3	1.25 N/m	h	1 m
b_4	1 Ns/m	g	10 m/s^2

1.H (5 points): Specify the decay rate, damped frequency, and settling point of this solution. If there are multiple decay rates, specify both. If there is no oscillating frequency, simply write “Frequency N/A”.

1.I (10 points): Prior to the collision, the system was at rest (in static equilibrium). Determine the initial velocity and position of spring 3 following the instantaneous collision of mass 1 and mass 2.

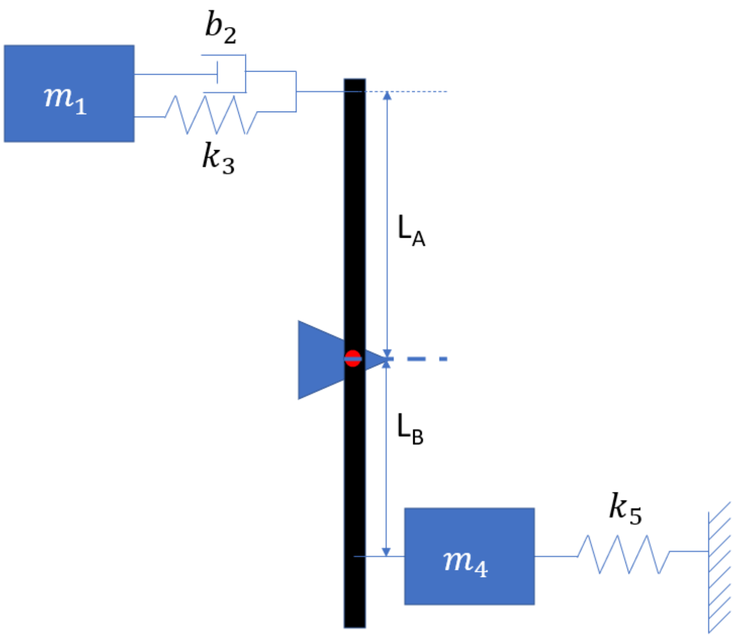
1.J (10 points): Solve for the particular form of the solution for spring 3’s position as a function of time.

1.K (10 points): Sketch a graph of the position of spring 3 over time. On the graph, label the initial position, the initial slope, the oscillating frequency (if relevant), and the maximum extension length of the spring.

1.L (10 points): How would manipulating each parameter (m, k_3, b_4, L_A, L_B, h) would alter the following (please write increase [or "up"], decrease [or "down"], or "N/A")?

	Input Parameters					
	m	k_3	b_4	L_A	L_B	h
Decay Rate(s)						
Frequency						
Settling Point						

2. Given the following diagram, determine whether the following are true or false. The velocity of each element will be defined by a numerical subscript ($e.g.\ v_1$ is velocity of spring 1), while the velocity and force on each point of the lever is assumed to be acting to the right **on the** lever, and will be defined with a letter subscript (e.g. v_A is velocity of the lever at position L_A above the fulcrum). For any false statements, a related, "corrected expression" must be written involving the relevant variables (and any other necessary variables or constants). (**10 points**)



	Statement	T/F	Corrected Expression
i.	$v_3 = -v_1$	___	____________________
ii.	$m_4 a_4 = k_5 x_5 + f_B$	___	____________________
iii.	$v_5 = \frac{L_B}{L_A}(v_1 - v_3)$	___	____________________
iv.	$\frac{L_A}{L_B} m_1 a_1 = k_5 x_5 - m_4 a_4$	___	____________________
v.	$\lvert f_A \rvert > \lvert f_B \rvert$	___	____________________

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In this quiz, all masses, springs, and dampers are ideal and linear. All masses are non-deformable and can therefore be treated as "points" or "nodes." Springs, dampers, and levers are assumed to be massless. Walls are stationary if not otherwise specified in the problem statements. The gravitational constant ***g*** will be assumed to be equal to 10 m/s^2, for ease of calculations.

Please specify the units of each answer, 1 point will be deducted for any missing units. All numerical answers will be assumed to be in SI units (N, m, s), unless otherwise specified.

Definitions of Constants

k: Spring Constant in Newtons/meter

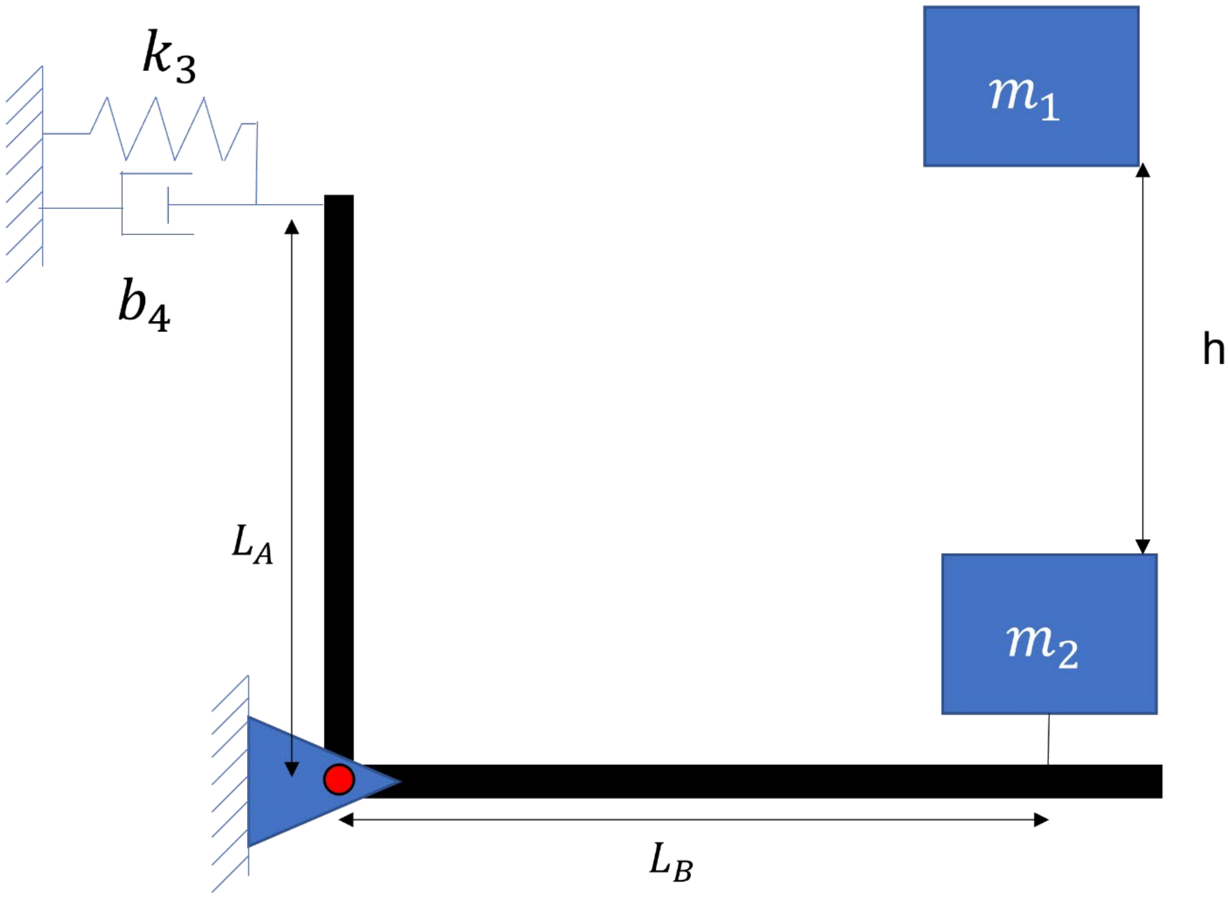
b: Damping Constant in Newton-Seconds/Meter

Do not start solving this quiz until you are told to do so. *

Problem 1	_______	/ 90 points
Problem 2	_______	/ 10 points
Total	_______	/ 100 points

Problem 1

Observe the following spring-mass-damper system connected by an angled, massless lever. A mass, m_1=0.5 kg, is dropped from a height, "h", of 1 meter and, upon collision, immediately sticks to a second mass, m_2 =0.5 kg. This collision results in movement of the lever. Solve the following prompts (**90 Pts)**



1.A (10 points): Solve for the velocity of the combined mass the instant **after** the collision (assume that the collision is instantaneous).

Conservation of Energy

$$m_1 gh = \frac{1}{2} m_1 v_1^2$$

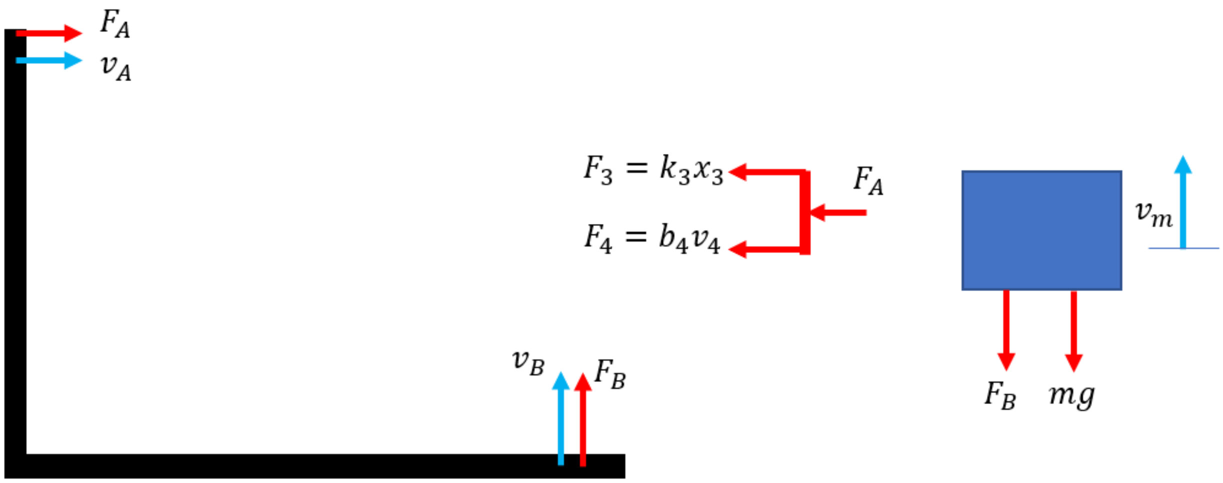
$$v_1 = \sqrt{2gh} = \sqrt{20}\frac{m}{s} = 2\sqrt{5}\frac{m}{s}$$

Conservation of Momentum

$$m_1 v_1 = (m_1 + m_2)v$$

$$v = \frac{m_1}{m_1+m_2} v_1 = \frac{\sqrt{20}}{2}\frac{m}{s} = \sqrt{5}\frac{m}{s} = 2.24\frac{m}{s}$$

<u>1.B</u> (**<u>5 points</u>**): Draw Free Body Diagrams (FBDs) for each mass (<u>the combined masses may be referred to as "m" moving forward</u>), lever, and junction.



<u>1.C</u> (**<u>5 points</u>**): Set up a force (or moment) balance equation for each FBD and substitute in any relevant constitutive equations.

$$ma_m = -F_B - mg$$

$$F_A = -k_3x_3 - b_4v_4$$

$$F_AL_A = F_BL_B$$

<u>1.D:</u> (**<u>5 points</u>**): Define all angular and linear kinematic constraints.

$$v_m = v_B$$

$$v_3 = v_4 = v_A$$

$$\frac{v_A}{L_A} = \frac{-v_B}{L_B} \; or \; \omega_A = \omega_B$$

1.E (**5 points**): Solve for the state equations (equations of motion), in terms of your state variables and any additional constants.

$$a_m = -g + \frac{L_A k_3}{L_B m} x_s - \frac{L_A^2 b_4}{L_B^2 m} v_m$$

$$x_s' = v_s = -\frac{L_A}{L_B} v_m$$

1.F (**5 points**): Write a second-order, linear differential equation for the position of spring 3.

$$m x_3'' + \left(\frac{L_A}{L_B}\right)^2 b_4 x_3' + \left(\frac{L_A}{L_B}\right)^2 k_3 x_3 = \frac{L_A}{L_B} mg$$

OR

$$\frac{L_B}{L_A} x_3'' + \frac{L_A b_4}{L_B m} x_3' + \frac{L_A k_3}{L_B m} x_3 = g$$

OR

$$x_3'' + \left(\frac{L_A}{L_B}\right)^2 \frac{b_4}{m} x_3' + \left(\frac{L_A}{L_B}\right)^2 \frac{k_3}{m} x_3 = \frac{L_A}{L_B} g$$

1.G (10 points): Using the given constants, determine the appropriate form of the **general solution** of the position of spring 3 as a function of time.

Parameter	Value	Parameter	Value
m_1	0.5 kg	L_A	1 m
m_2	0.5 kg	L_B	2 m
k_3	1.25 N/m	h	1 m
b_4	1 Ns/m	g	10 m/s^2

$$x_3(t) = \left(e^{-\frac{t}{8}}\left(A\cos\left(\frac{\sqrt{19}}{8}t\right) + B\sin\left(\frac{\sqrt{19}}{8}t\right)\right) + 16\right)\ meters$$

OR

$$x_3(t) = \left(e^{-\frac{t}{8}}\left(A\cos\left(\frac{\sqrt{19}}{8}t\right) + \varphi\right) + 16\right)\ meters$$

$$FYI: \frac{\sqrt{19}}{8} = 0.545$$

1.H (5 points): Specify the decay rate, damped frequency, and settling point of this solution. ***If there are multiple decay rates***, specify both. ***If there is no oscillating frequency***, simply write “Frequency N/A”.

Decay Rate: $0.125\frac{1}{s}\ OR\ \frac{1}{8}\frac{1}{s}$

Frequency: $\frac{\sqrt{19}}{8}\frac{rad}{s}\ OR\ 0.545\frac{rad}{s}$

Settling Point: 16 meters

If incorrect: check if values correspond to calculated values in previous step

1.I (10 points): Prior to the collision, the system was at rest (in static equilibrium). Determine the initial velocity and position of spring 3 following the instantaneous collision of mass 1 and mass 2.

$$x_3(0) = 8\ m$$

$$v_3(0) = x_3'(0) = \frac{\sqrt{5}}{2}\frac{m}{s}$$

1.J (10 points): Solve for the **particular** form of the solution for the position of spring 3 as a function of time.

$$x_3(0) = 8\ m = A + 16\ m$$

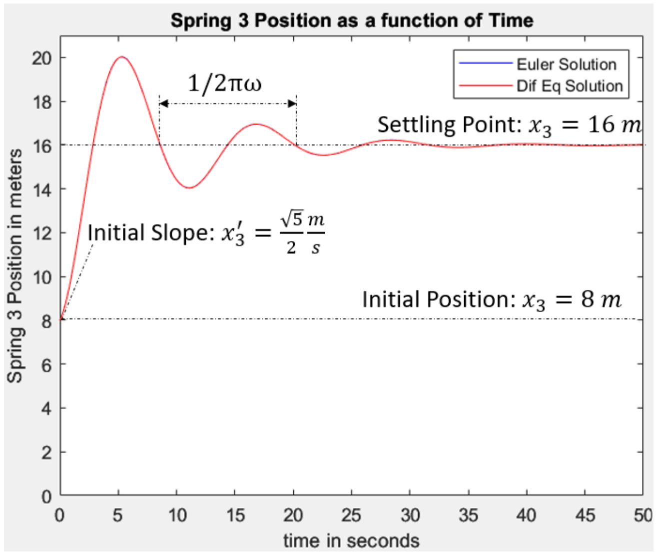
$$A = -8\ m$$

$$x_3'(0) = \frac{\sqrt{5}}{2}\frac{m}{s} = -\frac{1}{8}A + \frac{\sqrt{19}}{8}B$$

$$B = \frac{8}{\sqrt{19}}\left(\frac{\sqrt{5}}{2} - 1\right)\ m = 0.118\ m$$

$$x_3(t) = \left(e^{-\frac{t}{8}}\left(-8cos\left(\frac{\sqrt{19}}{8}t\right) + 0.118sin\left(\frac{\sqrt{19}}{8}t\right)\right) + 16\right)\ meters$$

1.K (10 points): Sketch a graph of the position of spring 3 over time. On the graph, label the initial position, the initial slope, the oscillating frequency (if relevant), and the settling point.

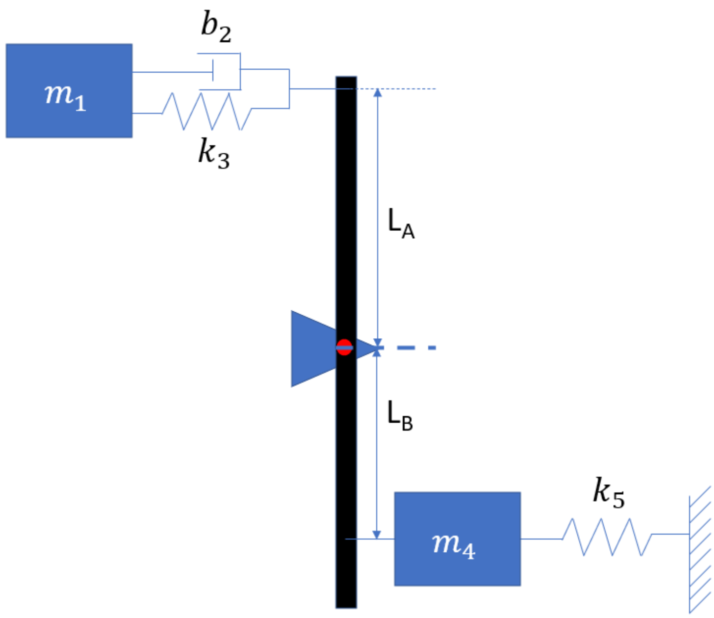


1.L (10 points): How would **increasing** each parameter (m, k_3, b_4, L_A, L_B, h) alter the following (please write increase [or "up"], decrease [or "down"], or "N/A")? If any parameters are irrelevant to the form of the particular solution you found previously, write "N/A" in each column.

	Input Parameters					
	m	k_3	b_4	L_A	L_B	h
Decay Rate(s)	Down	N/A	Up	Up	Down	N/A
Frequency	Down	Up	Down	Up	Down	N/A
Settling Point	Up	Down	N/A	Down	Up	N/A

$$\alpha = \frac{b_4 L_A^2}{2mL_B^2} \qquad \omega = \frac{L_A\sqrt{4mk_3L_B^2 - b_4^2L_A^2}}{2mL_B^2} \qquad C = \frac{L_B mg}{L_A k_3}$$

2. Given the following diagram, determine whether the following are true or false. The velocity of each element will be defined by a numerical subscript ($e.g.\, v_1$ is velocity of spring 1), while the velocity and force on each point of the lever is assumed to be acting to the right **on the** lever, and will be defined with a letter subscript (e.g. v_A is velocity of the lever at position L_A above the fulcrum). A statement will be treated as false, if it is false ***for any point in time.*** For any false statements, a related, "corrected expression" must be written involving the relevant variables (and any other necessary variables or constants). ***Assume*** $\boldsymbol{L_A > L_B}$ & $k_3 > k_5$ (**10 points**)



	Statement	True/False	Corrected Expression
i.	$v_3 = -v_1$	F	$v_3 = v_A - v_1$
ii.	$m_4 a_4 = k_5 x_5 + f_B$	F	$m_4 a_4 = k_5 x_5 - f_B$
iii.	$\lvert v_5\rvert \geq \lvert v_1 + v_3\rvert$	F	$\lvert v_5\rvert \leq \lvert v_1 + v_3\rvert$
iv.	$\frac{L_A}{L_B} m_1 a_1 = k_5 x_5 - m_4 a_4$	F	$\frac{L_A}{L_B} m_1 a_1 = m_4 a_4 - k_5 x_5$
v.	At settling point, $\lvert x_3\rvert > \lvert x_5\rvert$	F	$x_3(\infty) = x_5(\infty) = 0$