ENGINEERING ANALYSIS 3
SYSTEM DYNAMICS
Section 20

Quiz 1, October 23 2023

Name: (1pt)_________________________________ **Please also put your name on the back of the last page!**

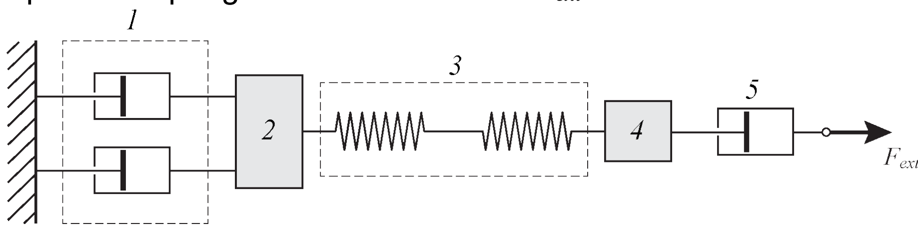
No electronic devices (phones, tablets, laptops, watches, etc.) are allowed during quizzes. No notes or scratch paper.

Do not ask for clarification of the questions; if you think that there is an ambiguity, clearly state your assumption and continue to answer the question.

There are 3 problems. Show all work, and CLEARLY MARK YOUR ANSWERS. Good luck!

Problem	Points
1	
2	
3	
Total	

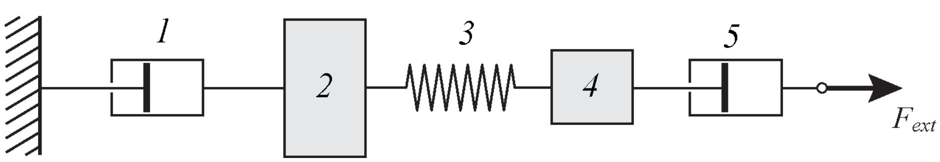
1. (27 pts) The system shown here consists of two dampers are lumped together to make one equivalent damper 1, and two springs are lumped together to make one equivalent spring 3. An external force F_{ext} acts on the distal end of damper 5.



1-1. (2 pts) If the two dampers making up equivalent damper 1 have the same damping constant b_0, what's the effective damping coefficient b_1 of the equivalent damper 1 in terms of b_0?

1-2. (2 pts) If the two springs making up equivalent spring 3 have the same spring constant k_0, what's the effective spring constant k_3 of the equivalent spring 3 in terms of k_0?

The system can then be simplified as the equivalent system showing below:



1-3. (6 pts) Write the force balance equations.

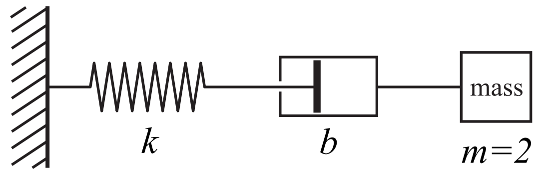
1-4. (4 pts) Write the geometric continuity equations.

1-5. (6 pts) Write the constitutive laws (use the lumped values b_1 and k_3).

1-6. (9 pts) Choosing the state variables v_2, x_3, and v_4, derive the three state equations. Show your work. Make sure your final answers are clearly noted.

2. (24 Points) The spring-mass-damper system shown here consists of a spring with spring constants $\boldsymbol{k}$, a damper with damping constant $\boldsymbol{b}$, and mass $\boldsymbol{m}=2$. With $\boldsymbol{x_s}$ the extension of the spring and $\boldsymbol{v_m}$ of the velocity of the mass, you've derived the state equations:

$$x_s' = v_m - 2x_s$$
$$v_m' = -2x_s$$

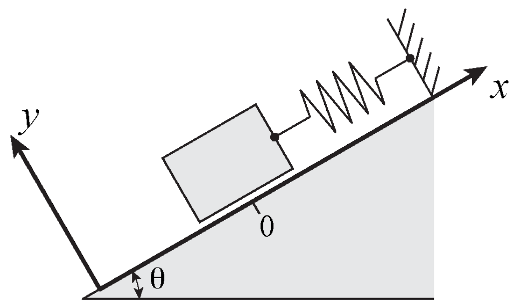


2-1. (8 pts) What are value of the spring constant $\boldsymbol{k}$ and damping coefficient $\boldsymbol{b}$?

2-2. (4 pts) If the initial conditions of the state variables at time t=0 are $x_s(t=0)=1$ and $v_m(t=0)=4$, find their values at time $t=0.2$, i.e., $x_s(t=0.2)$ and $v_m(t=0.2)$, using a single step of forward Euler method with a time increment of $\Delta t = 0.2$.

2-3. (12 pts) You decide to refine your solution by reducing your time increment to $\Delta t = 0.1$. Using this new time increment, use two steps of forward Euler method to find $x_s(t = 0.2)$ and $v_m(t = 0.2)$.

3. (28 pts) Considering a mass $\boldsymbol{m}$ slides on a ramp with an angle $\boldsymbol{\theta}$. The coefficient of friction between mass and the ramp is μ_s. Take the acceleration of gravity of $\boldsymbol{g}$. The mass is attached to a spring to a wall. The Spring has a spring constant $\boldsymbol{k}$. The position of the mass on the ramp is $\boldsymbol{x}$ ($x=0$ when spring is relaxed). For what range of $\boldsymbol{x}$ can the mass be resting at equilibrium (not moving)?



Solve the following:

3-1. (4 pts) Draw a free body diagram (FBD) for the mass when it is resting up the ramp (***x>0***). To do this, redraw the mass, separated from the ramp and carefully label all forces acting on the mass. Clearly define each of the force.

3-2. (4 pts) Obtain the force balance equations along x and y direction, correspond to the scenario in 3-1. Considering the masses are stationary and $\mu_s > tan\theta$.

3-3. (6 pts) What's the maximum position x that mass can rest up the ramp?

3-4. (4 pts) Draw a free body diagram (FBD) for the mass when it is resting down the ramp (<u>$x<0$</u>). Clearly define each of the force.

3.5. (4 pts) Obtain the force balance equations along x and y direction, correspond to the scenario in 3-4. Considering the masses are stationary.

3-6. (6 pts) What's the maximum position $|x|$ that mass can rest down the ramp?

ENGINEERING ANALYSIS 3
SYSTEM DYNAMICS
Section 20

Quiz 1, October 23 2023

Name: (1pt)____________________________________ **Please also put your name on the back of the last page!**

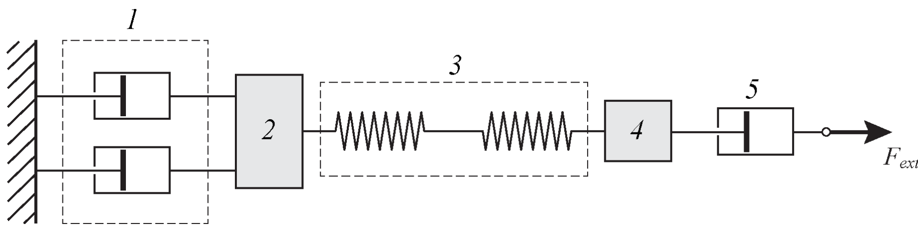
No electronic devices (phones, tablets, laptops, watches, etc.) are allowed during quizzes. No notes or scratch paper.

Do not ask for clarification of the questions; if you think that there is an ambiguity, clearly state your assumption and continue to answer the question.

There are 3 problems. Show all work, and CLEARLY MARK YOUR ANSWERS. Good luck!

Problem	
1	
2	
3	
Total	

1. (27 pts) The system shown here consists of two dampers are lumped together to make one equivalent damper 1, and two springs are lumped together to make one equivalent spring 3. An external force F_{ext} acts on the distal end of damper 5.



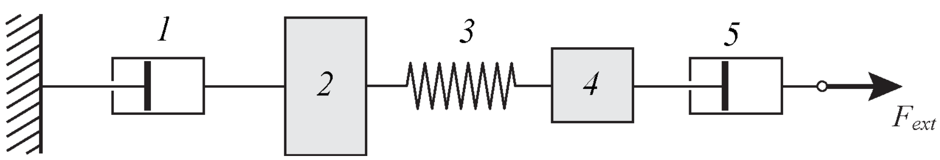
1-1. (2 pts) If the two dampers making up equivalent damper 1 have the same damping constant b_0, what's the effective damping coefficient b_1 of the equivalent damper 1 in terms of b_0?

$$b_1 = 2b_0$$

1-2. (2 pts) If the two springs making up equivalent spring 3 have the same spring constant k_0, what's the effective spring constant k_3 of the equivalent spring 3 in terms of k_0?

$$k_3 = \frac{k_0 k_0}{k_0 + k_0} = \frac{1}{2} k_0$$

The system can then be simplified as the equivalent system showing below:



1-3. (6 pts) Write the force balance equations.

$$F_3 - F_1 = m_2 a_2$$
$$F_5 - F_3 = m_4 a_4$$
$$F_5 - F_{ext} = 0$$

1-4. (4 pts) Write the geometric continuity equations.

$v_1 = v_2;$ $\qquad$ $v_3 = v_4 - v_2;$

1-5. (6 pts) Write the constitutive laws (use the lumped values $\boldsymbol{b_1}$ and $\boldsymbol{k_3}$).

$f_1 = b_1 v_1;\quad f_3 = k_3 x_3;\quad f_5 = b_5 v_5;$

(Optionally: $f_2 = m_2 a_2$; $f_4 = m_4 a_4$)

1-6. (9 pts) Choosing the state variables v_2, x_3, and v_4, derive the three state equations. Show your work. Make sure your final answers are clearly noted.

$$x_3' = v_3 = v_4 - v_2$$

$$\boxed{x_3' = v_4 - v_2}$$

$$v_2' = a_2 = \frac{1}{m_2}(F_3 - F_1) = \frac{1}{m_2}(k_3 x_3 - b_1 v_1) = \frac{1}{m_2}(k_3 x_3 - b_1 v_2)$$

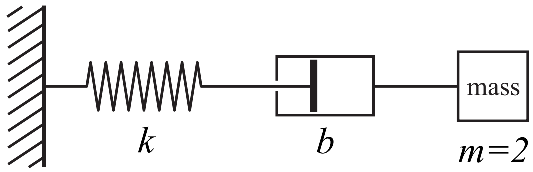
$$\boxed{v_2' = \frac{1}{m_2}(k_3 x_3 - b_1 v_2)}$$

$$v_4' = a_4 = \frac{1}{m_4}(F_{ext} - F_3) = \frac{1}{m_4}(F_{ext} - k_3 x_3)$$

$$\boxed{v_4' = \frac{1}{m_4}(F_{ext} - k_3 x_3)}$$

2. (24 Points) The spring-mass-damper system shown here consists of a spring with spring constants $\boldsymbol{k}$, a damper with damping constant $\boldsymbol{b}$, and mass $\boldsymbol{m}=2$. With $\boldsymbol{x_s}$ the extension of the spring and $\boldsymbol{v_m}$ of the velocity of the mass, you've derived the state equations:

$$x_s' = v_m - 2x_s$$
$$v_m' = -2x_s$$



$$x_s' = v_m - (k/b)x_s$$
$$v_m' = -(k/m)x_s$$

2-1. (8 pts) What are value of the spring constant $\boldsymbol{k}$ and damping coefficient $\boldsymbol{b}$?

$$k = 4; b = 2$$

2-2. (4 pts) If the initial conditions of the state variables at time t=0 are $x_s(t=0) = 1$ and $v_m(t=0) = 4$, find their values at time $t = 0.2$, i.e., $x_s(t=0.2)$ and $v_m(t=0.2)$, using a single step of forward Euler method with a time increment of $\Delta t = 0.2$.

$$x_s'(t=0) = v_m(t=0) - 2x_s(t=0) = 4 - 2 = 2$$
$$v_m'(t=0) = -2x_s(t=0) = -2$$

$$x_s(t=0.2) = x_s(t=0) + x_s'(t=0) \cdot \Delta t = 1 + 2 \times 0.2 = 1.4$$
$$v_m(t=0.2) = v_m(t=0) + v_m'(t=0) \cdot \Delta t = 4 + (-2) \times 0.2 = 3.6$$

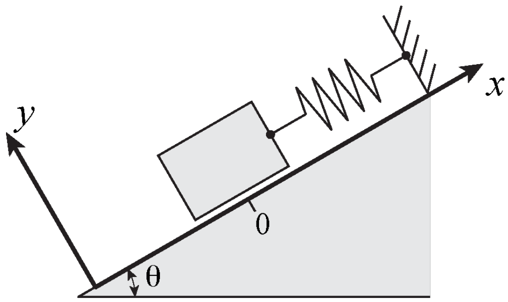
2-2. (12 pts) You decide to refine your solution by reducing your time increment to $\Delta t = 0.1$. Using this new time increment, use two steps of forward Euler method to find $x_s(t=0.2)$ and $v_m(t=0.2)$.

$$x_s(t=0.1) = x_s(t=0) + x_s'(t=0) \cdot \Delta t = 1 + 2 \times 0.1 = 1.2$$
$$v_m(t=0.1) = v_m(t=0) + v_m'(t=0) \cdot \Delta t = 4 + (-2) \times 0.1 = 3.8$$

$$x_s'(t=0.1) = v_m(t=0.1) - 2x_s(t=0.1) = 3.8 - 2 \times 1.2 = 1.4$$
$$v_m'(t=0.1) = -2x_s(t=0.1) = -2 \times 1.2 = -2.4$$

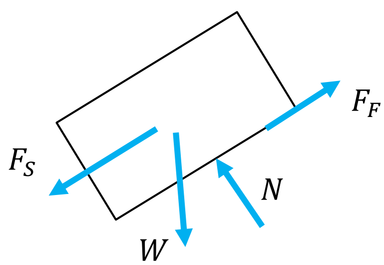
$$x_s(t=0.2) = x_s(t=0.1) + x_s'(t=0.1) \cdot \Delta t = 1.2 + 1.4 \times 0.1 = 1.34$$
$$v_m(t=0.2) = v_m(t=0.1) + v_m'(t=0.1) \cdot \Delta t = 3.8 + (-2.4) \times 0.1 = 3.56$$

3. (28 pts) Considering a mass ***m*** slides on a ramp with an angle θ. The coefficient of friction between mass and the ramp is μ_s. Take the acceleration of gravity of ***g***. The mass is attached to a spring to a wall. The Spring has a spring constant ***k***. The position of the mass on the ramp is ***x*** ($x=0$ when spring is relaxed). For what range of ***x*** can the mass be resting at equilibrium (not moving)?



Solve the following:

3-1. (4 pts) Draw a free body diagram (FBD) for the mass when it is resting up the ramp (***x>0***). To do this, redraw the mass, separated from the ramp and carefully label all forces acting on the mass. Clearly define each of the force.



3-2. (4 pts) Obtain the force balance equations along x and y direction, correspond to the scenario in 3-1. Considering the masses are stationary and $\mu_s > tan\theta$.

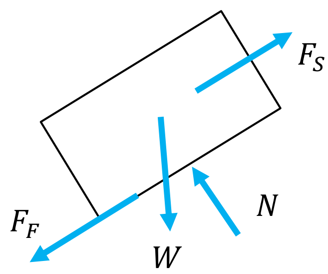
$$x\ direction: F_F - F_S - W \cdot sin\theta = 0$$

$$y\ direction: N - W \cdot cos\theta = 0$$

3-3. (6 pts) What's the maximum position x that mass can rest up the ramp?

$$x \leq \frac{\mu_s cos\theta - sin\theta}{k} mg$$

3-4. (4 pts) Draw a free body diagram (FBD) for the mass when it is resting down the ramp (***x<0***). Clearly define each of the force.



3.5. (4 pts) Obtain the force balance equations along x and y direction, correspond to the scenario in 3-4. Considering the masses are stationary.

$$x\ direction: F_s - F_F - W \cdot sin\theta = 0$$

$$y\ direction: N - W \cdot cos\theta = 0$$

3-6. (6 pts) What's the maximum position $|x|$ that mass can rest down the ramp?

$$|x| \leq \frac{\mu_s cos\theta + sin\theta}{k} mg$$