Name _____

LAST First Name (Clearly written)

Quiz #2 **Engineering Analysis - 3** **Spring 2024**

Maximum time allowed is 50 minutes

Use only a pen and calculator.

No cellphones, computers, notes, books, or cheat sheets.

In this quiz, all the masses, springs, and dampers are ideal and linear.

All the referenced figures are on the last page, which you may tear off for your easy reference. Do not turn in the last page as it will not be graded.

Your Affirmation:

On my honor I have neither given nor received help on this quiz and I will not

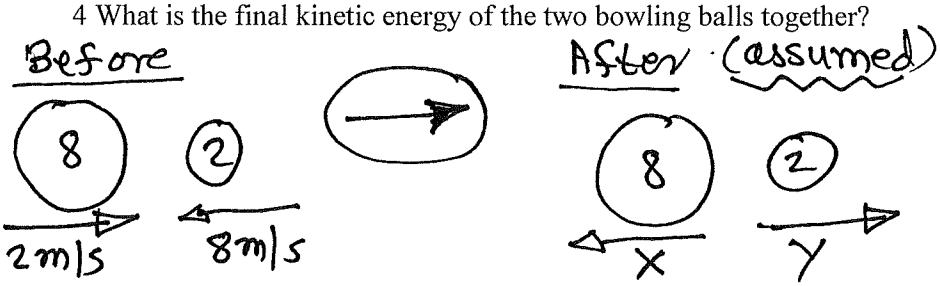
discuss content of the quiz with other students until 3 pm. (2 pt) _____

Sign your name above

40 1. Solve the following two short problems (20 points each):

A) Two bowling balls having unequal mass and traveling towards each other collide. The coefficient of restitution is 0.5. Ball A has a mass of 8 kg and initial velocity of 2 m/s while ball B has a mass of 2 kg and initial velocity of 8 m/s.

4 What is the initial momentum of the two bowling balls together?
4 What is the initial kinetic energy of the two bowling balls together?
8 What are the velocities (VA' and VB') and their directions after collision?
4 What is the final kinetic energy of the two bowling balls together?



B) Refer to Figure 1 on the last page. The hippo weighs 1 ton and the barge weighs 10 tons. Initially the hippo is in the middle of the barge (both are stationary and touching), which has length of 110 m. The hippo walks to the front of the barge and stops.

15 How far is the barge from the shore? Show your work
5 If the hippo had run instead, will the barge be closer/farther/about the same from the shore? Explain

50 2. Refer to Figure 2 on the last page. A bullet weighing mb kg traveling at Vb m/s is fired (through the hole in the left wall) upon the stationary mass m resulting in a **plastic** collision. The system dynamics begins as soon as the bullet is lodged inside the mass. Use the combined mass, M=m+mb for the system dynamics.

a) 10 Develop three equations of motion (EOM). Label the three equations.
b) 10 Reduce the EOM's into one differential equation for the spring displacement, x1
c) 10 Solve the above equation analytically.
d) 20 Assuming k1= k2 = 2N/m; mb = 1kg; Vb =100 m/s and m = 99 kg and that the system being initially in equilibrium with x1=0 and x2=0 just before the bullet is fired upon, what is the value of x1 and sign (compression or tension) at time t = 2 seconds?

08 3. If $(2 + 7i)*(1 - 2i) / 6i$ is equal to $a + i*b$, determine the constants a and b.

Figure 1

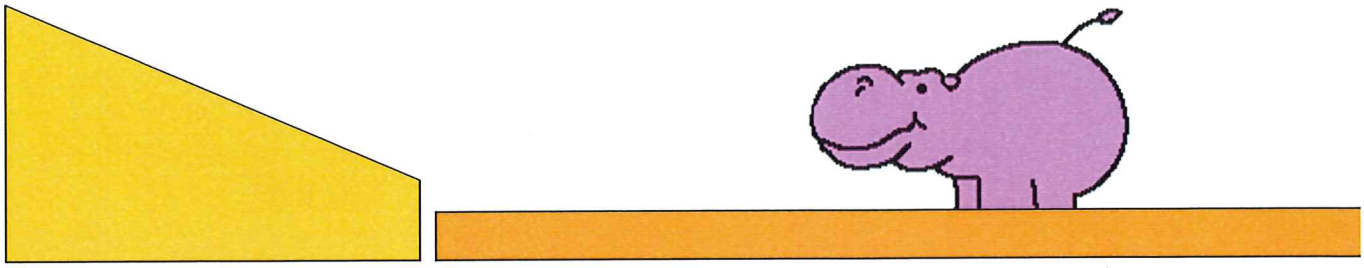
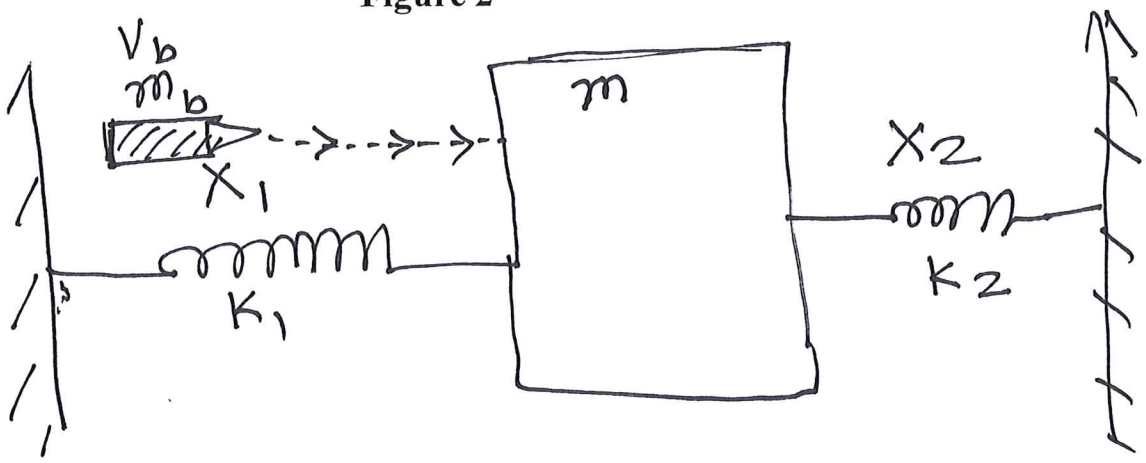


Figure 2



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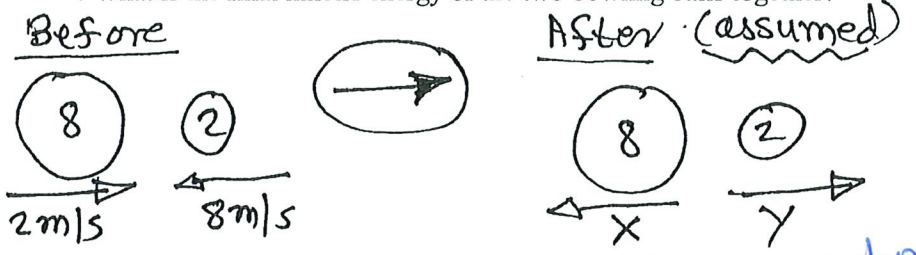

$V_A = 2 \quad V_B = -8$

$V_A' = -X \quad V_B' = Y$

(a) Initial momentum $= 8*2 - 2*8 =$ 0

(b) Initial KE $= \frac{1}{2} 8*4 + \frac{1}{2}*2*64$

$= 80J$

(c) $\therefore 0 = -8X + 2Y$

$0.5 = \frac{V_B - V_A}{V_A - V_B} = \frac{Y+X}{2+8}$

$Y = 4$ (To Right)

$\therefore X = 1$ (To Left)

(d) Final KE

$= \frac{1}{2}*8*1 + \frac{1}{2}*2*16$

$= 20$

B) Refer to Figure 1 on the last page. The hippo weighs 1 ton and the barge weighs 10 tons. Initially the hippo is in the middle of the barge (both are stationary and touching), which has length of 110 m. The hippo walks to the front of the barge and stops.

15 How far is the barge from the shore? Show your work
5 If the hippo had run instead, will the barge be closer/farther/about the same from the shore? Explain

∴ Hippo moves to left by 10 m & the barge moves to the right by 1 m.

∴ When hippo moves to the left by 50 m, the barge would move to the right by 5 m & Now hippo is at the front.

(a) ∴ Barge is 5 m from the shore.

(b) In this ideal case, the barge will still be 5 m away from shore. Just in Less Time!

50 2. Refer to Figure 2 on the last page. A bullet weighing mb kg traveling at Vb m/s is fired (through the hole in the left wall) upon the stationary mass m resulting in a **plastic** collision. The system dynamics begins as soon as the bullet is lodged inside the mass. Use the combined mass, M=m+mb for the system dynamics.

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(a) $\Sigma F = m_3 V_3' = -K_1 X_1 + K_2 X_2$

$V_1 + V_2 = 0$

$V_3 = V_1$

$\therefore$ $\boxed{X_1' = V_3}$ EOM-1

$\boxed{X_2' = -V_3}$ EOM-2

$\boxed{V_3' = -\frac{K_1 X_1}{m_3} + \frac{K_2 X_2}{m_3}}$ EOM-3

(b) $\therefore X_1'' = V_3' = -\frac{K_1 X_1}{m_3} + \frac{K_2 X_2}{m_3}$

$X_1 + X_2 = \text{constant} = 0$ [From initial Values]

$\therefore X_2 = -X_1$

$\therefore \boxed{X_1'' + \left(\frac{K_1}{m_3} + \frac{K_2}{m_3}\right) X_1 = 0}$

$\therefore \lambda^2 + \left(\frac{K_1 + K_2}{m_3}\right) = 0$ $\therefore \lambda = \pm j\sqrt{\frac{K_1 + K_2}{m_3}}$

$$\therefore X_1 = C_1 e^{-j\theta t} + C_2 e^{+j\theta t}$$

Where $\theta = \sqrt{\frac{k_1+k_2}{m_3}}$

$X_1 = A\cos(\theta t) + B\sin(\theta t)$

① @ $t = 0$ $\quad X_1 = 0$

$X_1' = V_3$

From momentum conservation for a plastic collision we obtain,

initial $V_3 = \frac{1 * 100}{(1+99)} = 1 \text{ m/s}$

$\theta = \sqrt{\frac{2+2}{100}} = 0.2$

$\therefore$ @ $t = 0$, $0 = A$

$\therefore X_1 = B\sin(0.2t)$

$X_1' = 1 = 0.2B\cos(0.2t)$

$\therefore B = 5$

$\therefore X_1 = 5\sin(0.2t)$

@ 2 seconds, $X_1 = 5\sin(0.4) = +1.95$

Tension / Compression

08 3. If $(2 + 7i)*(1 - 2i) / 6i$ is equal to $a + i*b$, determine the constants a and b.

$$\frac{(2+7i)(1-2i)}{6i}$$

$$= \frac{2+3i+14}{6i}$$

$$= \frac{16+3i}{6i} * \frac{i}{i}$$

$$= \frac{16i-3}{-6}$$

$$= \frac{1}{2} - \frac{16}{6}\,i$$

$$= a + ib$$

$a = \frac{1}{2}$

$b = -16/6$ — either is okay!

$b = -8/3$

Figure 1

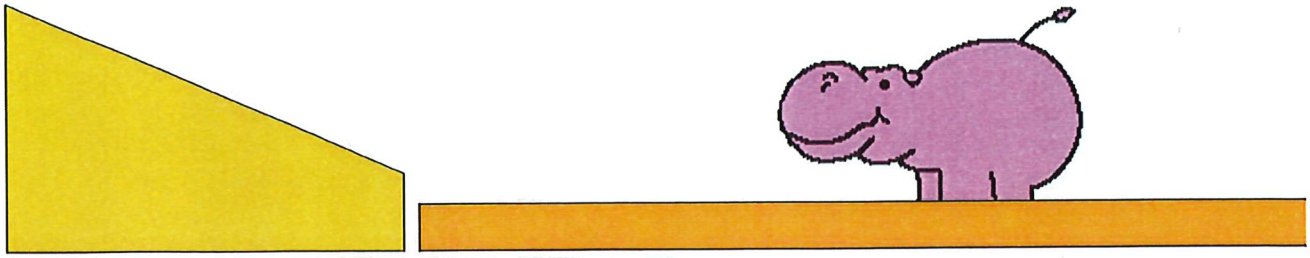

Figure 2