ENGINEERING ANALYSIS 3

SYSTEM DYNAMICS

Quiz No. 1

April 24, 2023

Name : ____

You may NOT use notes or books or calculators or phones for this quiz. There are 2 problems with multiple sections.

Clearly mark your answers (circle your final answers, wherever applicable) and detail your problem solving process. Credit will primarily be rewarded based on process (which demonstrates your conceptual understanding of the material) rather than results.

In this quiz, all masses, springs, and dampers are ideal and linear. All masses are non-deformable and can therefore be treated as “points” or “nodes.” Springs, dampers, and levers are assumed to be massless. Walls are stationary if not specified in the problem statements.

Dimensional units will be provided for all necessary constants, but for ease of writing, your solutions do not need to include units. In other words, velocity = 1 will be understood by the graders as equivalent to velocity = 1 m/s

<u>Potentially Useful Equations</u>

Potential Energy = mgh

Elastic Potential Energy = ½ k x^2

Work = $\int F dx$

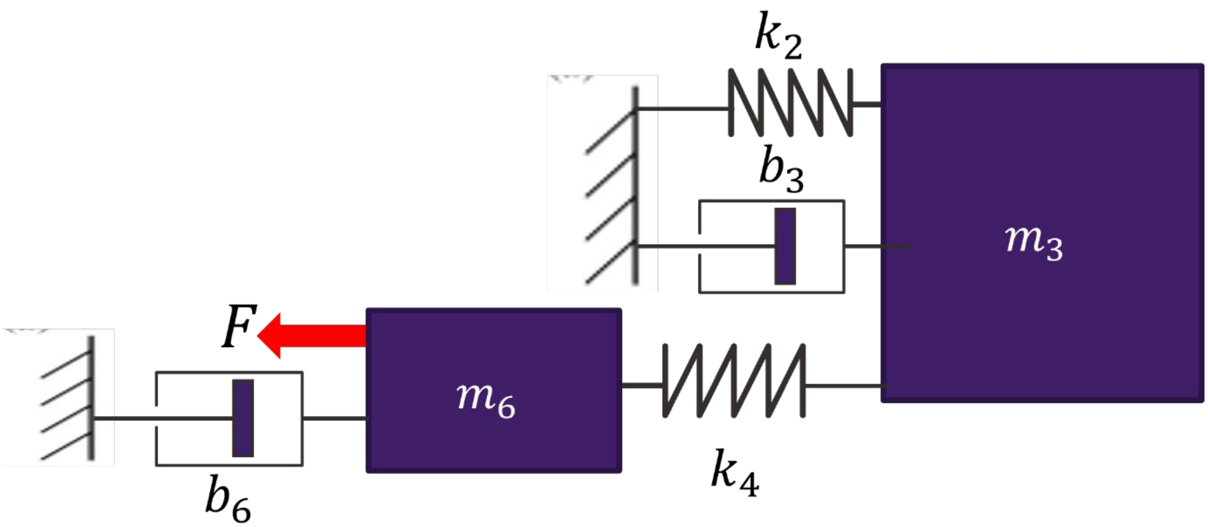
Work = $\int F dx$

Kinetic Energy = ½ m v^2

Do not start solving this quiz until you are told to do so. *

Problem 1

The following spring damper system is set up between two fixed walls. A constant, leftward external force is applied to mass 3. Develop a system of equations which describes the dynamics of this system in terms of its state variables (**55 Points Total**)



1.A (**10 points**): Draw Free Body Diagrams (FBDs) for each moving mass and junction in the system (you may draw in "dots" wherever necessary). Additional FBDs may be drawn as needed if they aid you in solving the problem.

1.B (**10 points**): Set up a force balance equation for each FBD and substitute in any relevant constitutive equations.

<u>1.C:</u> (**<u>10 points</u>**): Define all kinematic constraints/geometric continuity equations (4 are required to solve this problem, though other relationships may exist).

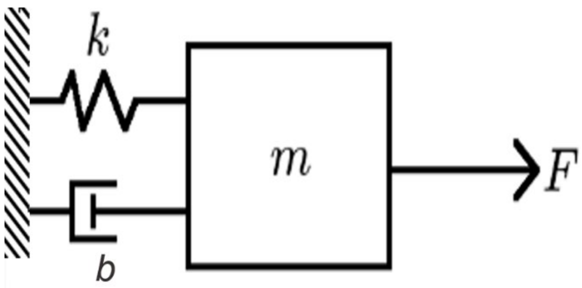
<u>1.D</u> (**<u>5 points</u>**): Define the state variables and state equations.

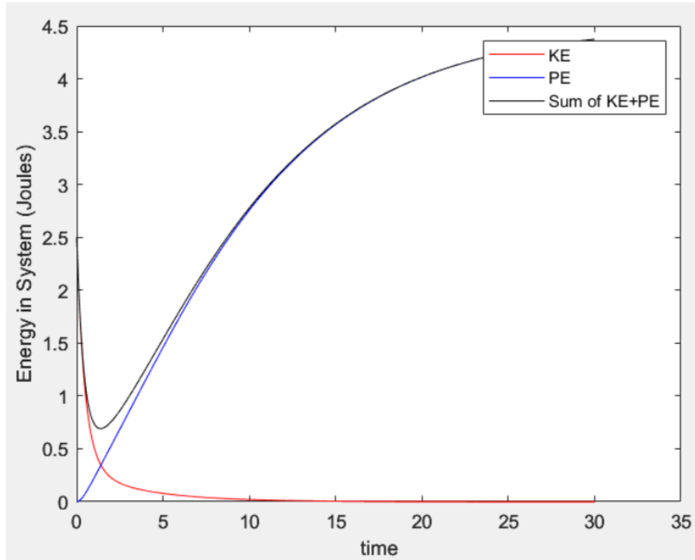
<u>1.E</u> (**<u>10 points</u>**): Solve for the state equations (equations of motion), in terms of your state variables and any additional constants.

1.F (**10 points**): Solve for the value of each state variable at time=0.1 second using the Forward Euler's Method with time steps of 1 second, given the following initial conditions.

$$k_2 = k_4 = 2\,\frac{N}{m} \qquad x_2(0) = 1\,m$$
$$b_3 = b_6 = 4\,\frac{Ns}{m} \qquad x_4(0) = 1\,m$$
$$m_1 = 1\,kg \qquad v_1(0) = 1\,\frac{m}{s}$$
$$m_5 = 2\,kg \qquad v_5(0) = 1\,\frac{m}{s}$$
$$F = 3N$$

<u>Problem 2</u>: The following graph presents the energy (kinetic and elastic potential) within a spring-mass-damper system. At the initial condition, the system has 2.5 Joules of energy, and at steady state (t=∞) the system has 4.5 Joules of energy. (**15 points**)



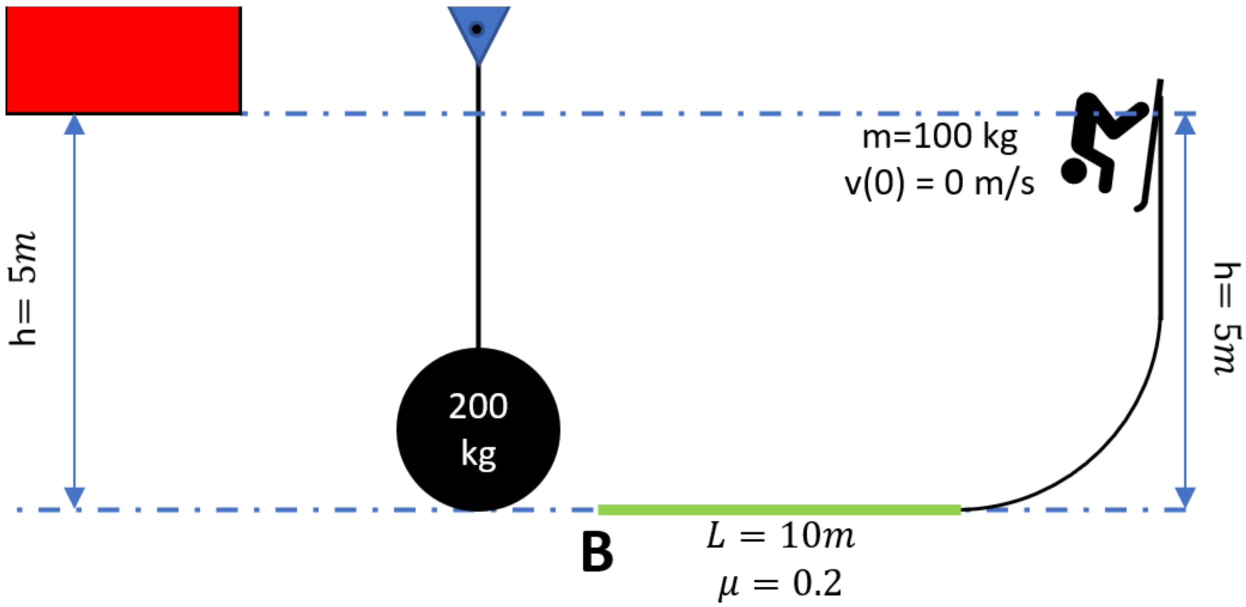


<u>2.A</u> (**<u>5 points</u>**): Note the trend of the total energy in the system (black trace). Why does the energy initially decrease? (Describe in terms of Power; please answer in 1 brief sentence)

<u>2.B (5 points)</u>: Following the initial decrease, why does the energy begin to increase? (Describe in terms of Power; please answer in 1 brief sentence)

<u>2.C (5 points):</u> Why is this process sequential? That is, why doesn't the energy continuously change in one direction or the other? (1 brief sentence)

Problem 3: A skier (m=100 kg) slides 5 meters down a frictionless slope onto a horizontal, frictional surface ($\mu = 0.2$). After sliding across the surface, they collide with and stick to a wrecking ball (m=200kg) which is free to swing in either direction. A fixed, red "ceiling" exists to the left of the wrecking ball, 5 meters above the frictional surface. (**30 pts total**)



3.A (10 pts): What is the velocity of the skier at the left end of the frictional surface (Point "B")?

3.B (10 pts): What is the velocity of the wrecking ball immediately after colliding with the skier?

<u>3.C (5 pts):</u> In the swinging motion that follows the collision between the skier and the wrecking ball, does the wrecking ball collide with the red ceiling? If not, how high does the wrecking ball swing? (*treat the wrecking ball as a point mass)*

<u>3.D (5 pts)</u>: If the frictional surface were smoothed out (i.e. made completely frictionless), would the wrecking ball collide with the red ceiling? No calculations necessary; please justify your answer in one, brief sentence.

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Name : Answer Key

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Potentially Useful Equations

Potential Energy = mgh

Elastic Potential Energy = ½ k x^2

Work = $\int F dx$

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Kinetic Energy = ½ m v^2

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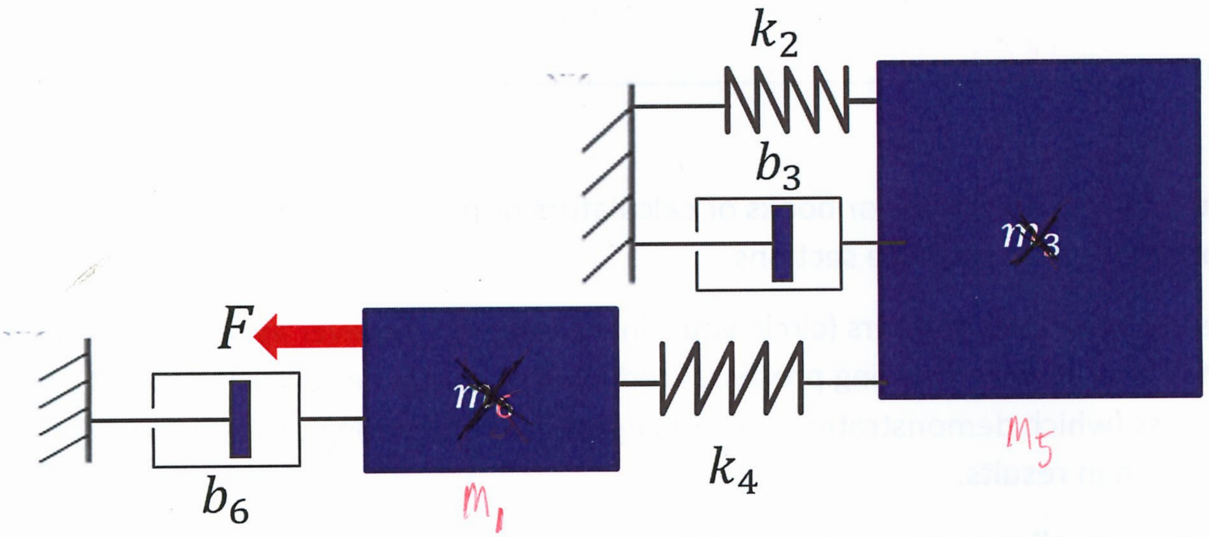
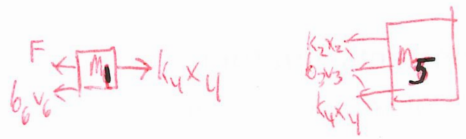
Problem 1

The following spring damper system is set up between two fixed walls. A constant, leftward external force is applied to mass 3. Develop a system of equations which describes the dynamics of this system in terms of its state variables (**60 Points Total**)



1.A (**10 points**): Draw Free Body Diagrams (FBDs) for each moving mass and junction in the system (you may draw in "dots" wherever necessary). Additional FBDs may be drawn as needed if they aid you in solving the problem.



- Deduct 2 points for each missing force.
- Deduct 1 point for each force in incorrect direction
- Maximum of 10 pts of deduction

1.B (**10 points**): Set up a force balance equation for each FBD and substitute in any relevant constitutive equations.

$$m_1 a_1 = k_4 x_4 - b_6 v_6 - F \qquad m_5 a_5 = -k_2 x_2 - b_3 v_3 - k_4 x_4$$

- Deduct 2 points per missing force
- Deduct 1 point per sign error

} unless these are consistent with prior step.

- Do not "double-deduct" for earlier errors

- Max of 10 pts deducted

1.C: (**10 points**): Define all kinematic constraints/geometric continuity equations (4 are required to solve this problem, though other relationships may exist).

$V_2 = V_3$

$V_2 = V_5$

$V_4 = V_5 - V_1$

$V_1 = V_6$

Other Options

$V_3 = V_5$

$V_1 + V_4 = V_5$

$V_6 + V_4 = V_5$

$V_6 + V_4 = V_2$

... there are more

- Any 4 correct equations are acceptable
- Deduct 3 points for each missing equation
- Deduct 3 points for any inaccurate equations
- Max deductions of 10 pts

1.D (**5 points**): Define the state variables and state equations.

SEs	SVs
V_2	x_2
V_4	x_4
a_1	V_1
a_5	V_5

- Deduct 1 point for each missing/incorrect SE or SV
- F may be included as an SV; no deduction
- Deduct 2 points if definitions are flipped
  - If SEs are listed as SVs
- SEs may be written as functions (e.g. $V_2 = f(x_2, x_4, V_1, V_5)$)
- Max deductions of 5 points

1.E (**15 points**): Solve for the state equations (equations of motion), in terms of your state variables and any additional constants.

$V_2 = V_5$

$V_4 = V_5 - V_1$

$$a_1 = \frac{k_4}{m_1} x_4 - \frac{b_6}{m_1} V_1 - \frac{F}{m_1}$$

$$a_5 = -\frac{k_2}{m_5} x_2 - \frac{b_3}{m_5} V_5 - \frac{k_4}{m_5} x_4$$

- Deduct 3 points for any inclusion of $V_6, V_3, V_2, V_4, a_1, a_5$ in any final equation of motion
- Max 15 points deduction
- Again, do not "double-deduct" if an error from 1.A-1.C propagated to a wrong result here.
- Deduct 1 point for any stray sign/algebraic errors

1.F (**10 points**): Solve for the value of each state variable at time=0.1 second using the Forward Euler's Method with time steps of 1 second, given the following initial conditions.

$$k_2 = k_4 = 2\frac{N}{m} \qquad x_2(0) = 1\,m$$
$$b_3 = b_6 = 4\frac{Ns}{m} \qquad x_4(0) = 1\,m$$
$$m_1 = 1\,kg \qquad v_1(0) = 1\frac{m}{s}$$
$$m_5 = 2\,kg \qquad v_5(0) = 1\frac{m}{s}$$
$$F = 3N$$

$V_2 = V_5$

$V_4 = V_5 - V_1$

$a_1 = 2x_4 - 4v_1 - 3$

$a_5 = -x_2 - 2v_5 - x_4$

$V_2(0) = 1\ m/s$

$V_4(0) = 0\ m/s$

$a_1(0) = -5\ m/s^2$

$a_5(0) = -4\ m/s^2$

$x_2(0.1) = V_2(0)\Delta t + x_2(0) = 1(0.1) + 1 = 1.1$

$x_4(0.1) = V_4(0)\Delta t + x_4(0) = 0 + 1 = 1.$

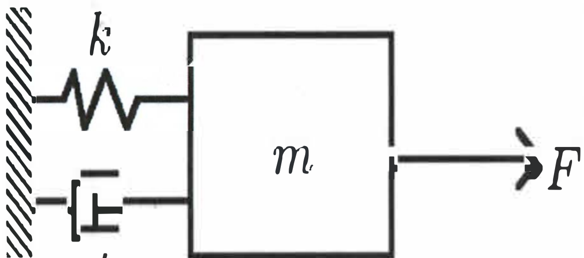
$V_1(0.1) = a_1(0)\Delta t + V_1(0) = -5(0.1) + 1 = 0.5\ m/s$

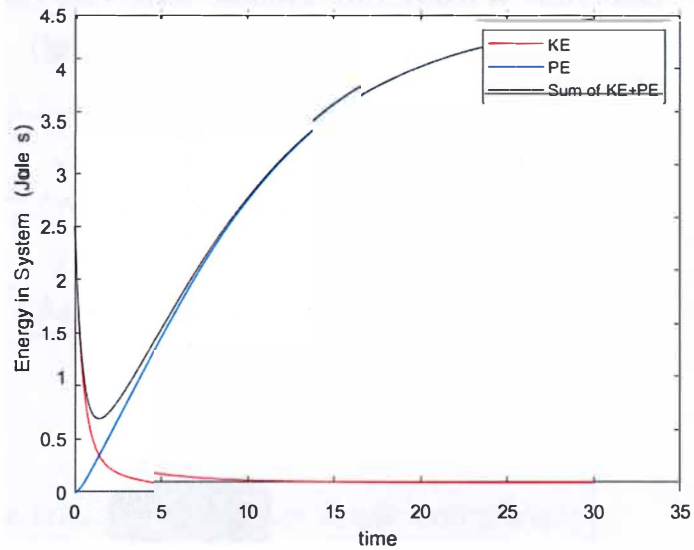
$V_5(0.1) = a_5(0)\Delta t + V_5(0) = -4(0.1) + 1 = 0.6\ m/s$

- 5 points if state equations are not solved first
- 5 points for improper form of Eulers Equation ~~[illegible]~~
- 2 for each missed equation
- 1 point for each wrong answer

Max deduction of 10 points

Problem 2: The following graph presents the energy (kinetic and elastic potential) within a spring-mass-damper system. At the initial condition, the system has 2.5 Joules of energy, and at steady state (t=∞) the system has 4.5 Joules of energy. **(15 points)**





2.A (5 points): Note the trend of the total energy in the system (black trace). Why does the energy initially decrease? (Describe in terms of Power; please answer in 1 brief sentence)

Because the damper is initially moving fast, it is dissipating energy at a greater rate (higher power) than the external force is adding energy back in.

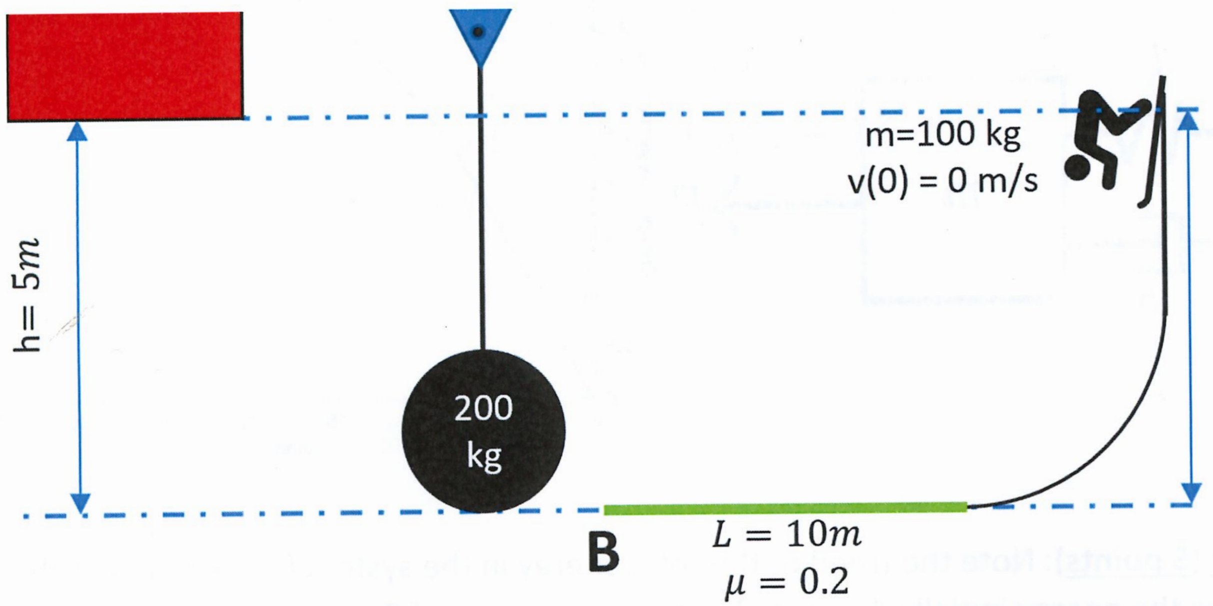
- Answer must relate damper speed to power draw
    - If not, -5

2.B (5 points): Following the initial decrease, why does the energy begin to increase? (Describe in terms of Power; please answer in 1 brief sentence)

As the damper velocity slows, the power of the external force exceeds the dissipative power of the damper.

- Answer must indicate that power draw from damper dropped as velocity slowed
- If not, -5 & that external force adds energy to system

Problem 3: A skier (m=100 kg) slides 5 meters down a frictionless slope onto a horizontal, frictional surface ($\mu = 0.2$). After sliding across the surface, they collide with and stick to a wrecking ball (m=200kg) which is free to swing in either direction. A fixed, red "ceiling" exists to the left of the wrecking ball, 5 meters above the frictional surface. (**30 pts total**)



3.A (10 pts): What is the velocity of the skier at the left end of the frictional surface (Point "B")?

$mgh = \frac{1}{2}mv_B^2 + \mu mgL$

$v_B = \sqrt{2g(h-\mu L)} = 7.75\ m/s$

-3 for missing / inaccurate terms (each)
-1 for inaccurate signs (one deduction)

-2 for inaccurate answer form
- Exact expression may vary
- Numbers may be substituted at this point

Max Deductions: -10

3.B (10 pts): What is the velocity of the wrecking ball immediately after colliding with the skier?

$m_s v_B = (m_s + m_{wB})v$

$v = \frac{m_s}{m_s + m_{wB}} v_B$

$v = 2.58\ m/s$

-10 if energy conservation is used

-10 if skier & wrecking ball have different velocities after collision

-5 for inaccurate form
- exact expresion may vary

Max deductions: -10

3.C (5 pts): In the swinging motion that follows the collision between the skier and the wrecking ball, does the wrecking ball collide with the red ceiling? If not, how high does the wrecking ball swing? (*treat the wrecking ball as a point mass)*

No, it will not : If wrong, -5

$\frac{1}{2}(m_s+m_{wb})v^2 = (m_s+m_{wb})gh_2$: set-up must include kinetic & potential only

-2 if other energies are added

-2 if either is missing

$h_2 = \frac{v^2}{2g} = 0.33\ m$

-2 if form is incorrect

Max Deductions: -5

3.D (5 pts): If the frictional surface were smoothed out (i.e. made completely frictionless), would the wrecking ball collide with the red ceiling? No calculations necessary; please justify your answer in one, brief sentence.

If wrong: -5

No, it will not because energy is lost during the perfectly inelastic collision between the skier and wrecking ball.

-5 if energy lost in collision is not mentioned

Do not deduct if they choose to prove this numerically / algebraically

Max Deductions: -5