ME449HW3 – Demo:

Matt Collins

---

2)

The end-effector configuration M ∈ SE(3) for the UR5 robot is $\begin{pmatrix} 1 & 0 & 0 & -L1-L2 \\ 0 & 0 & -1 & -W1-W2 \\ 0 & 1 & 0 & H1-H2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$.

a)
The six screw axes expressed in the end-effector frame (Bi) are:

$$B1 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ W1+W2 \\ 0 \\ L1+L2 \end{pmatrix} \quad B2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ H2 \\ -L1-L2 \\ 0 \end{pmatrix} \quad B3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ H2 \\ -L2 \\ 0 \end{pmatrix} \quad B4 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ H2 \\ 0 \\ 0 \end{pmatrix} \quad B5 = \begin{pmatrix} 0 \\ -1 \\ 0 \\ -W2 \\ 0 \\ 0 \end{pmatrix} \quad B6 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

b)
The six screw axes expressed in the fixed frame (Si) are:

$$S1 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad S2 = \begin{pmatrix} 0 \\ -1 \\ 0 \\ H1 \\ 0 \\ 0 \end{pmatrix} \quad S3 = \begin{pmatrix} 0 \\ -1 \\ 0 \\ H1 \\ 0 \\ L1 \end{pmatrix} \quad S4 = \begin{pmatrix} 0 \\ -1 \\ 0 \\ H1 \\ 0 \\ L1+L2 \end{pmatrix} \quad S5 = \begin{pmatrix} 0 \\ 0 \\ -1 \\ W1 \\ -L1-L2 \\ 0 \end{pmatrix} \quad S6 = \begin{pmatrix} 0 \\ -1 \\ 0 \\ H1-H2 \\ 0 \\ L1+L2 \end{pmatrix}$$

---

3)

The end-effector configuration M ∈ SE(3) for the WAM robot is $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L1+L2+L3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$.

a)
The six screw axes expressed in the end-effector frame (Bi) are:

$$B1 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad B2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ L1 + L2 + L3 \\ 0 \\ 0 \end{pmatrix} \quad B3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad B4 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ L2 + L3 \\ 0 \\ W1 \end{pmatrix} \quad B5 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad B6 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ L3 \\ 0 \\ 0 \end{pmatrix} \quad B7 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

b)
The six screw axes expressed in the fixed frame (Si) are:

$$S1 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad S2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad S3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad S4 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ -L1 \\ 0 \\ W1 \end{pmatrix} \quad S5 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad S6 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ -L1 - L2 \\ 0 \\ 0 \end{pmatrix} \quad S7 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Notes on 4, 5, & 6

The zero configuration for both the UR5 and WAM robots occured at a singularity. To get over this so the inverse kinematics code would work, a slight disturbance was made to some of the joints so that the robot started slightly away from the singularity. Since it was still close, however, there were rapid changes in the inverse kinematics code close to the singularities and led to very high angle measurements for some of the joints in the process of converging to a solution, that did indeed get to the desired end-effector configuration. These rapid angle changes can be seen in the answers to 4 and 5, as well as in the images of the robots in 6.

Another solution to get over the singularity problem is to use the built in *Mathematica* pseudoinverse function, which uses SVD for the pseudoinverse and can therefore find a solution even for the zero configuration of these robots.

4)

```
Quit[];
```

```
Tsd = {{0, 1, 0, -.6}, {0, 0, -1, .1}, {-1, 0, 0, .1}, {0, 0, 0, 1}};
L1 = 425 / 1000;
L2 = 392 / 1000;
W1 = 109 / 1000;
W2 = 82 / 1000;
H1 = 89 / 1000;
H2 = 95 / 1000;
M = {{1, 0, 0, -L1 - L2}, {0, 0, -1, -W1 - W2}, {0, 1, 0, H1 - H2}, {0, 0, 0, 1}};
B1 = {{0, 1, 0, W1 + W2, 0, L1 + L2}}^T;
B2 = {{0, 0, 1, H2, -L1 - L2, 0}}^T;
B3 = {{0, 0, 1, H2, -L2, 0}}^T;
B4 = {{0, 0, 1, H2, 0, 0}}^T;
B5 = {{0, -1, 0, -W2, 0, 0}}^T;
B6 = {{0, 0, 1, 0, 0, 0}}^T;
B = {B1, B2, B3, B4, B5, B6};

IKinBody[B, M, Tsd, {0, 0, -.01, 0, .01, 0}, .01, .001]

{{0, 0, -0.01, 0, 0.01, 0},
 {-0.355641, 55.9432, -117.243, 60.1539, -0.353506, -0.424769},
 {-0.479119, 55.0686, -117.41, 62.4227, -0.200544, -1.67196},
 {-0.548124, 55.6983, -117.736, 61.8845, -0.545956, -1.41769},
 {-0.459065, 55.6853, -117.948, 62.2405, -0.454013, -1.5447},
 {-0.469499, 55.7147, -117.985, 62.2707, -0.469368, -1.57159}}
```

The joint angles θ_d that satisfy the inverse kinematics are {-0.469499, 55.7147, -117.985, 62.2707, -0.469368, -1.57159}. The resultant configuration can be seen below and closely matches the desired.

```
FKinBody[{{1, 0, 0, -L1 - L2}, {0, 0, -1, -W1 - W2}, {0, 1, 0, H1 - H2}, {0, 0, 0, 1}},
  B, {-0.46949945826843015`, 55.714685537265055`, -117.98469417226988`,
   62.27074872739645`, -0.4693679098674825`, -1.571587945198965`}] // MatrixForm
```

$$\begin{pmatrix} -0.000131608 & 1. & -0.000131659 & -0.599778 \\ -0.000334743 & -0.000131703 & -1. & 0.100058 \\ -1. & -0.000131564 & 0.000334761 & 0.0999508 \\ 0. & 0. & 0. & 1. \end{pmatrix}$$

5)

```
Tsd = {{1, 0, 0, .4}, {0, 1, 0, 0}, {0, 0, 1, .4}, {0, 0, 0, 1}};
L1 = 550 / 1000;
L2 = 300 / 1000;
L3 = 60 / 1000;
W1 = 45 / 1000;
M = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, L1 + L2 + L3}, {0, 0, 0, 1}};
S1 = {{0, 0, 1, 0, 0, 0}}ᵀ;
S2 = {{0, 1, 0, 0, 0, 0}}ᵀ;
S3 = {{0, 0, 1, 0, 0, 0}}ᵀ;
S4 = {{0, 1, 0, -L1, 0, W1}}ᵀ;
S5 = {{0, 0, 1, 0, 0, 0}}ᵀ;
S6 = {{0, 1, 0, -L1 - L2, 0, 0}}ᵀ;
S7 = {{0, 0, 1, 0, 0, 0}}ᵀ;
S = {S1, S2, S3, S4, S5, S6, S7};
```

```
IKinFixed[S, M, Tsd, {0, 0, 0, .01, 0, .01, .01}, .01, .001]
```

```
{{0, 0, 0, 0.01, 0, 0.01, 0.01},
 {0.333423, 4.66974, 0.333423, -11.9042, -1.34362, 7.23442, 0.676846},
 {1.30676, 1.02457, -2.17981, 0.840588, -4.29037, 3.63807, 10.3345},
 {3.2592, 2.2921, -2.1766, 5.73829, 3.30573, 5.74059, 20.2712},
 {2.35611, -0.29569, -1.91777, 1.38233, 1.49008, 2.63967, 21.4708},
 {2.44549, -0.634731, -2.00431, 1.74333, 3.00061, 1.78509, 22.2305},
 {2.28329, -0.27023, -2.20563, 2.1913, 2.94506, 2.40234, 21.8072},
 {2.50196, -0.278387, -2.29227, 2.17556, 2.82812, 2.35331, 21.5779},
 {2.50022, -0.291184, -2.28969, 2.17087, 2.83569, 2.342, 21.5854}}
```

The joint angles θ_d that satisfy the inverse kinematics are {2.50022, -0.291184 ,-2.28969, 2.17087, 2.83569, 2.342, 21.5854}. The resultant configuration can be seen below and closely matches the desired.

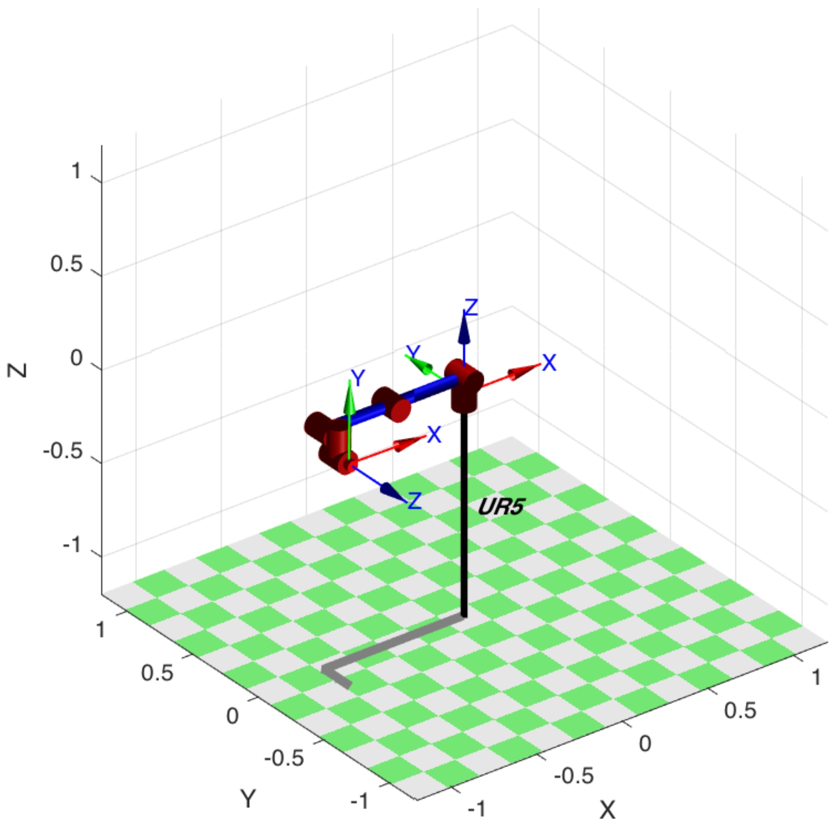
```
FKinFixed[{{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, L1 + L2 + L3}, {0, 0, 0, 1}},
  S, {2.5002238615346886`, -0.29118355879486074`,
   -2.289687416725524`, 2.170866432490594`, 2.8356907857498936`,
   2.3419957310863833`, 21.585379501151866`}] // MatrixForm
```

$$\begin{pmatrix} 1. & -0.000059129 & -0.0000112785 & 0.400007 \\ 0.0000591279 & 1. & -0.0000993917 & -0.0000241917 \\ 0.0000112843 & 0.000099391 & 1. & 0.400022 \\ 0. & 0. & 0. & 1. \end{pmatrix}$$

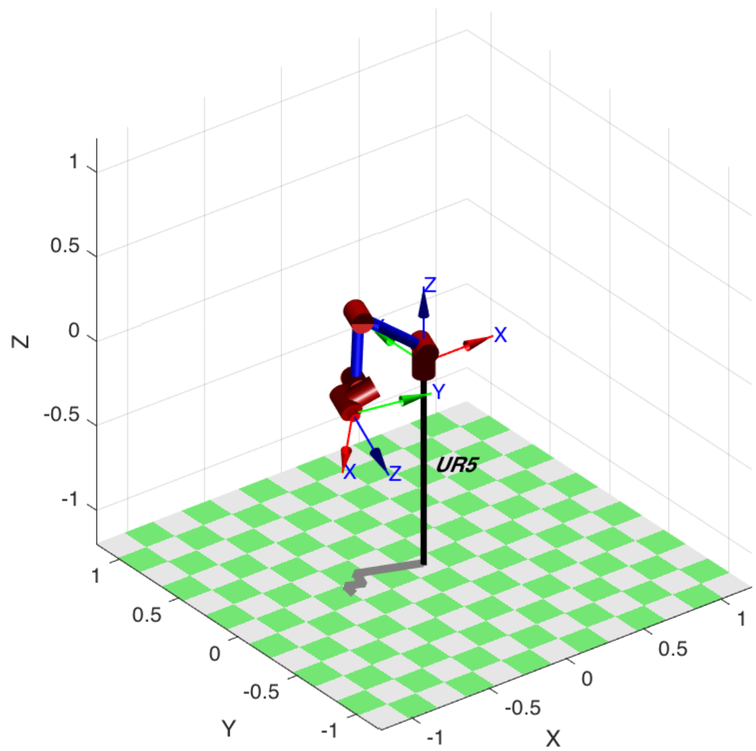
6)

UR5)

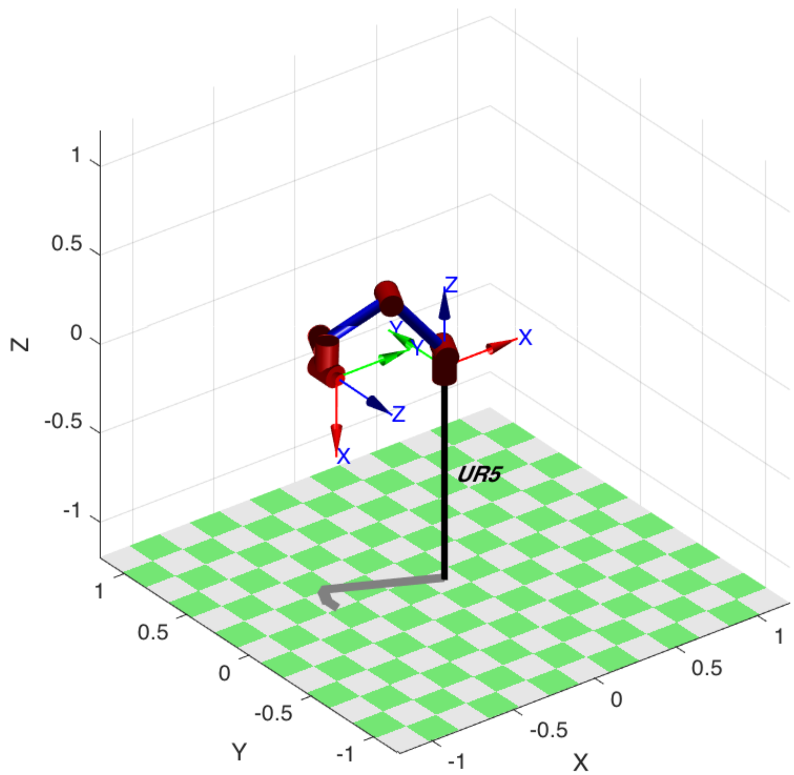
Initial Configuration



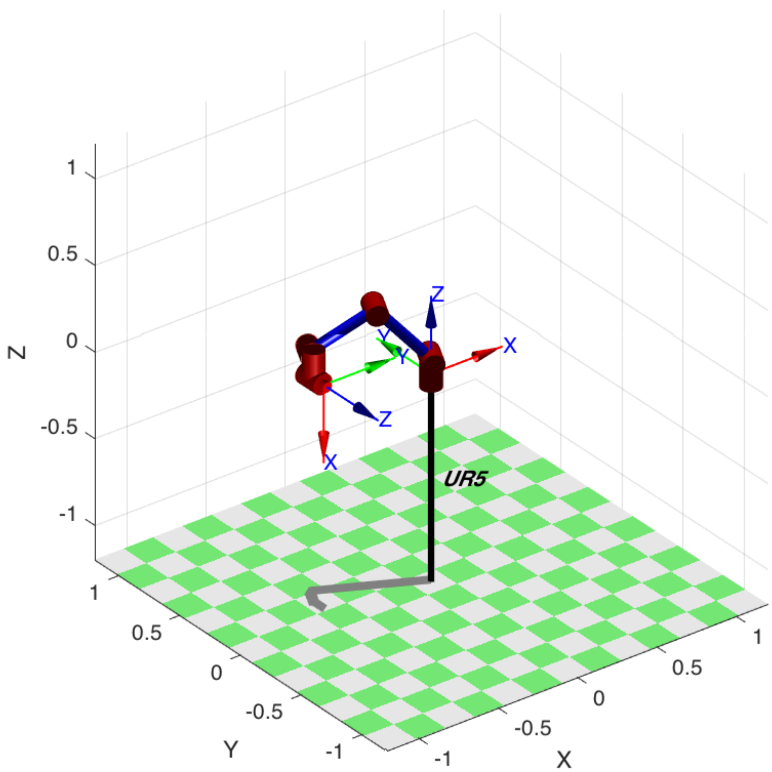
Intermediate 1



Intermediate 2

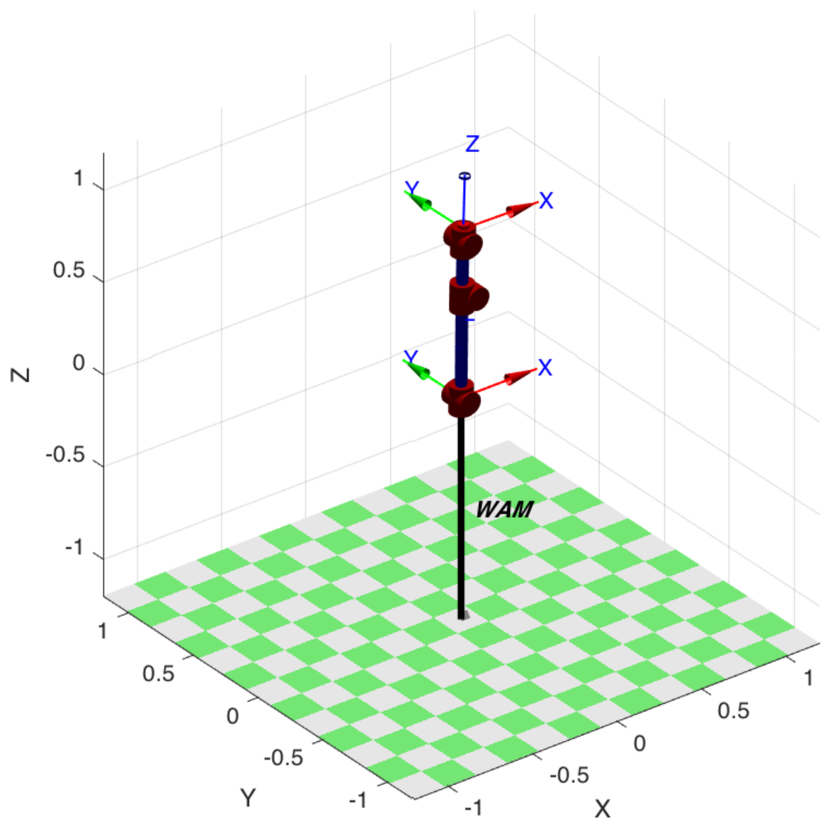


Final Configuration

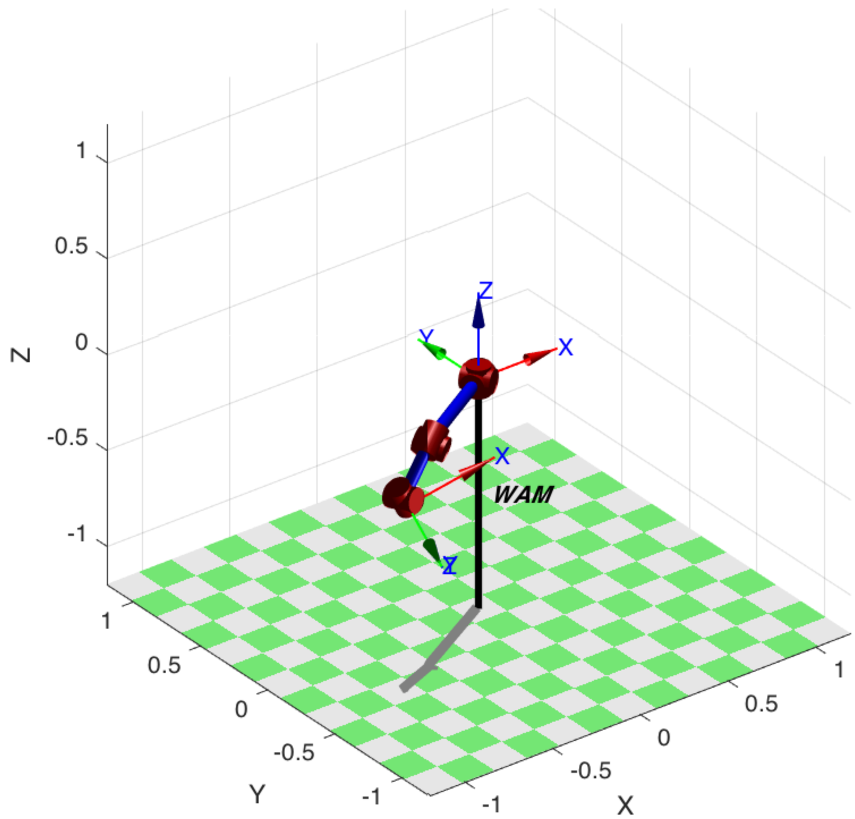


WAM)

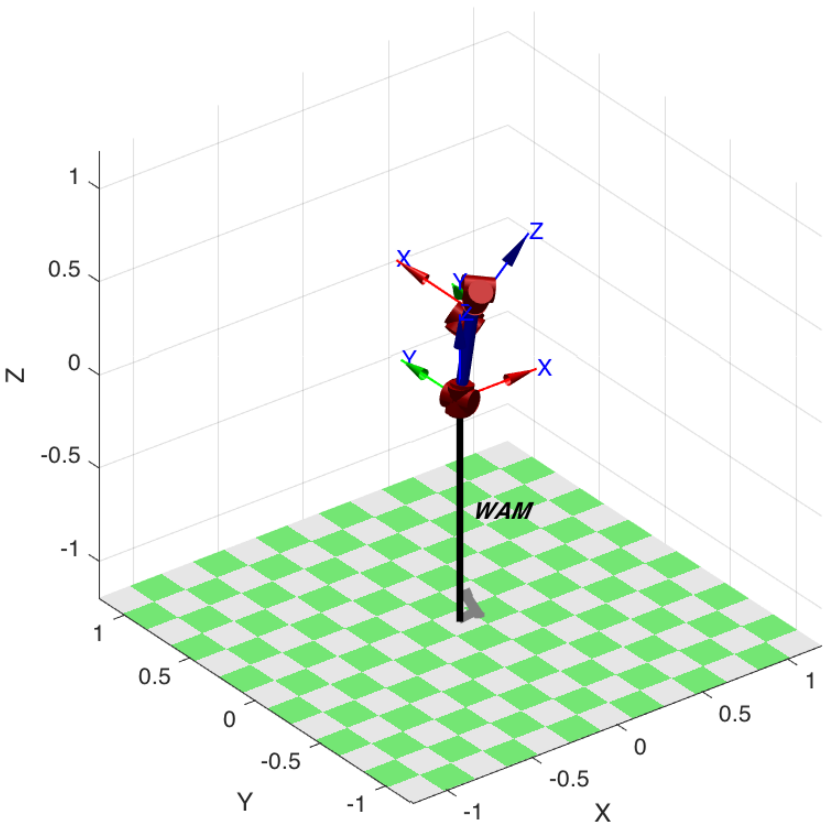
Initial Configuration



Intermediate 1



Intermediate 2



Final Configuration