ME 449 Assignment 3
Due 1:30 pm November 6, 2026

Part 1. Starting from `IKinBody` in the MR code library, write a new function, `IKinBodyIterates`. This function prints out a report for each iteration of the Newton-Raphson process, for iterates 0 (the initial guess) to the final answer. Each iteration reports the iteration number i, the joint vector θ^i, the end-effector configuration $T_{sb}(\theta^i)$, the error twist $\mathcal{V}_b$, and the angular and linear error magnitudes, $\|\omega_b\|$ and $\|v_b\|$ (something like the table at the end of Chapter 6.2.2). For a four-joint robot, an iterate might look like:

```
Iteration 3:

joint vector:
0.221, 0.375, 2.233, 1.414

SE(3) end-effector config:
1.000  0.000  0.000  3.275
0.000  1.000  0.000  4.162
0.000  0.000  1.000 -5.732
0      0      0      1

          error twist V_b:  (0.232, 0.171, 0.211, 0.345, 1.367, -0.222)
angular error ||omega_b||:  0.357
     linear error ||v_b||:  1.427
```

The function should also save the joint vector of each iteration as a row in a matrix. For a four-joint robot with three iterates (including the initial guess), the matrix would be

$$\begin{bmatrix} \theta_1^0 & \theta_2^0 & \theta_3^0 & \theta_4^0 \\ \theta_1^1 & \theta_2^1 & \theta_3^1 & \theta_4^1 \\ \theta_1^2 & \theta_2^2 & \theta_3^2 & \theta_4^2 \end{bmatrix}.$$

When your function completes (the angular error is less than ϵ_ω and the linear error is less than ϵ_v), it should save the matrix as a .csv file, where each row of the text file consists of the comma separated joint values for that iterate. You can learn more about generating .csv files here.

Test your new function for the UR5 robot of Example 4.5 of Chapter 4.1.2 (Figure 4.6). **(Note: The UR5 admits *analytical* inverse kinematics, but here we will be practicing with numerical inverse kinematics.)** The home configuration of the end-effector M is given in the book, as well as the numerical values of the constants $L_1, L_2, H_1, H_2, W_1, W_2$. The screw axes $\mathcal{B}_i$ in the end-effector frame are

$$\begin{array}{ll} \text{joint 1:} & (0, 1, 0, W_1 + W_2, 0, L_1 + L_2) \\ \text{joint 2:} & (0, 0, 1, H_2, -L_1 - L_2, 0) \\ \text{joint 3:} & (0, 0, 1, H_2, -L_2, 0) \\ \text{joint 4:} & (0, 0, 1, H_2, 0, 0) \\ \text{joint 5:} & (0, -1, 0, -W_2, 0, 0) \\ \text{joint 6:} & (0, 0, 1, 0, 0, 0) \end{array}$$

The desired end-effector configuration is

$$T_{sd} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0.5 \\ -1 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where linear distances are in meters. Use $\epsilon_\omega = 0.001$ rad ($0.057°$) and $\epsilon_v = 0.0001$ (0.1 mm).

You will use the Newton-Raphson method with two different initial guesses θ^0 to determine how the initial guess affects convergence to a set of joint angles that satisfies the desired end-effector configuration. Choose initial guesses θ^0 so that the numerical inverse kinematics

1. converges after 2–4 Newton-Raphson steps (short_iterates), not more or less, and
2. converges after 10 Newton-Raphson steps, or never converges (long_iterates).

You can use the sliders in the CoppeliaSim UR5 interactive scene (Scene 1) to choose initial guesses. (Check out http://hades.mech.northwestern.edu/index.php/CoppeliaSim_Introduction for more information.) You can type the final joint angles found by your function into the UR5 interactive scene to confirm that your function works properly and the desired end-effector configuration is achieved. (Keep in mind that the numerical IK procedure treats all joint values as real numbers, so it will happily compute joint angles of tens, hundreds, thousands, etc., radians, particularly if the process is thrashing because you chose an initial guess far from a solution. But CoppeliaSim uses the joint limits of the UR5, which are limited to the range $[-2\pi, 2\pi]$ for each joint, so if you say a joint angle is 7 radians, for example, CoppeliaSim will convert it to the maximum joint angle, 2π radians. For that reason, I suggest that at each iteration of the IK process, each angle guess θ be converted to $\text{atan2}(\sin\theta, \cos\theta)$, which will convert θ to something in the range $(-\pi, \pi]$.)

Your pdf report should provide the following figures:

1. A 3D plot that shows the (x, y, z) position of the end-effector at each iterate, with a line between successive iterations. (See Figure 1 for an example.)
2. A plot of the magnitude of the linear error (on the y-axis) as a function of the iterate number (on the x-axis).
3. A plot of the magnitude of the angular error as a function of the iterate number.

The results from both initial guesses should be shown in each figure. (Three figures total.) Label all axes and figures, and provide a legend that labels the two different initial guesses.

Part 2. Imagine the robot is powered up at a random configuration θ^0 such that $T_{sb}(\theta^0) = T^0$, and its first task is to move to a θ^* satisfying $T_{sb}(\theta^*) = T_{sd}$. Part 1 of this assignment deals with calculating an appropriate θ^*. In this part, you will consider how to actually control the robot to move from rest at θ^0 to rest at θ^* in t_f seconds.

(a) If you decide to move at constant joint speeds, what joint speed vector $\dot{\theta}$ do you command to the joints? Your answer should be symbolic in terms of relevant variables.

(b) If you decide to make the last link of the robot follow a single constant twist from T^0 to T_{sd}, what joint speed vector $\dot{\theta}(0)$ do you command to the joints at time $t = 0$? What is the commanded joint speed vector $\dot{\theta}(t_f/2)$, halfway through the motion? Your answers should be symbolic in terms of relevant variables. Use the function $\text{vec}([\mathcal{V}])$ to convert $[\mathcal{V}] \in se(3)$ to the twist $\mathcal{V} \in \mathbb{R}^6$, and assume you have access to the body Jacobian $J_b(\theta)$.

(c) Do you see any advantages to using the approach in (a) relative to (b), or vice-versa?

Submission. Assemble your documents for submission. You will submit one zip file, FamilyName_GivenName_asst3.zip; for me, it would be Lynch_Kevin_asst3.zip.

- The zip file should contain:

  1. The pdf file FamilyName_GivenName_asst3.pdf. (See below for a detailed description of the pdf file.)
  2. Your commented code from Part 1 in a directory (folder) called "code." You only need to provide your modified function(s), not the rest of the MR code.
  3. A text file called "short_iterates.csv" created by your `IKinBodyIterates` function for the example that takes 2–4 iterations to converge.
  4. A text file called "long_iterates.csv," created by your `IKinBodyIterates` function for the example that takes 10 or more iterations to converge, or never converges.
  5. Two CoppeliaSim videos animating the Newton-Raphson iterations, **one for each initial guess**. Use the CoppeliaSim csv animation scene for the UR5 (Scene 2). The videos should show CoppeliaSim "playing" your .csv file. (Go to `http://hades.mech.northwestern.edu/index.php/CoppeliaSim_Introduction` to learn about making videos with CoppeliaSim.) The video is just a sequence of configurations of the robot, equal to the number of iterates in your .csv file. (**Keep in mind that, in real numerical IK, the robot does not actually move to the configurations computed before the IK computation has converged!** It only moves to the computed configuration after the process has converged. This video is just to visualize the solution process.) You should uncheck the "Interpolate" checkbox in the CoppeliaSim UR5 animation scene to make this video. Your video should be a "reasonable" size (e.g., a few MB, less than 10 MB) and use a standard codec (e.g., some variant of .mp4) that common video viewers, in Mac OS, Windows, or Linux, can view. Your video should be taken from a virtual camera angle that makes it easy to see the end-effector configuration. **The video file names should follow the convention FamilyName_GivenName_short.mp4 and FamilyName_GivenName_long.mp4 (or possibly a different common file extension).**

- The pdf file should contain:

  1. Any information that will help the grader understand your entire submission.
  2. One screen log for each of your two guesses in Part 1. This log should show how your code is called and the text output, i.e., the Newton-Raphson iterates that are printed to the screen when you call `IKinBodyIterates` with your initial guess.
  3. A CoppeliaSim screenshot showing the UR5 at the solution configuration calculated after 2–4 iterations for your "good" initial guess. This screenshot should clearly show the UR5's end-effector configuration as well as the $SE(3)$ configuration reported by the scene's interface, confirming that your code calculated a good solution.
  4. A figure showing the progression of end-effector (x, y, z) positions during the solution process. (Plots of both initial guesses in the same figure.)
  5. A figure showing the magnitudes of the linear error as a function of iterations (both initial guesses).

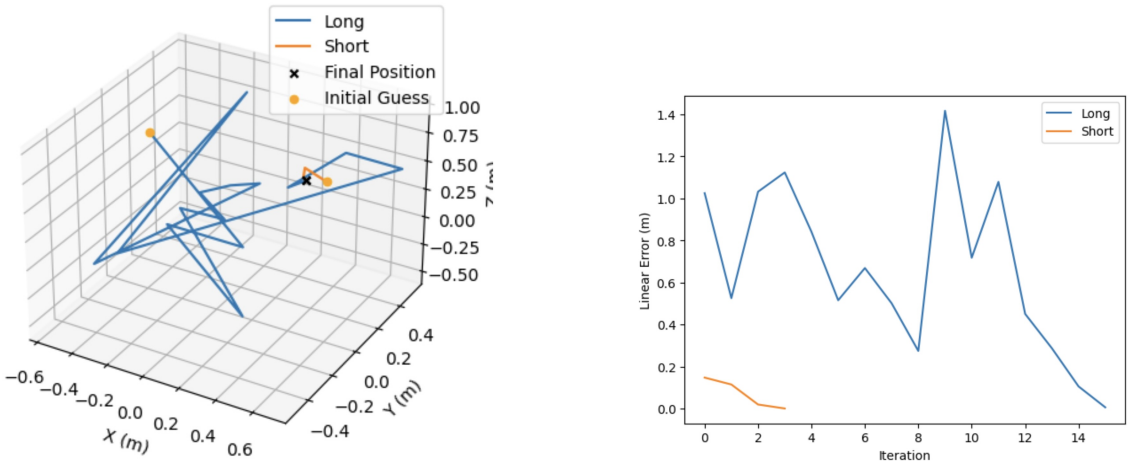


Figure 1 (Left) A plot showing the progression of the {b} frame location of the iterates during the IK solving process. (Right) A plot showing the linear error magnitude of the iterates during the IK process. (Not shown: A plot of the angular error magnitude.)

6. A figure showing the magnitude of the angular error as a function of iterations (both initial guesses).
7. An explanation why convergence is difficult from the long_iterates initial guess.
8. The answers to the questions in Part 2.