Embedded Computing and Mechatronics with the PIC32 Microcontroller

Kevin M. Lynch
Nicholas Marchuk
Matthew L. Elwin

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Dedication

To Yuko, Erin, and Patrick.

—Kevin M. Lynch

To Mark and Liz.

—Nicholas Marchuk

To Hannah.

—Matthew L. Elwin

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Preface

This book is about the Microchip 32-bit PIC32 microcontroller, its hardware, programming it in C, and interfacing it to sensors and actuators. This book also covers related mechatronics topics such as motor theory, choosing motor gearing, and practical introductions to digital signal processing and feedback control. This book is written for:

- **Anyone starting out with the Microchip PIC32 32-bit microcontroller.** Microchip documentation can be hard to navigate; this is the book we wish we had when we started!
- **The hobbyist ready to explore beyond Arduino.** Arduino software and its large user support community allow you to be up and running quickly with Atmel microcontrollers. But reliance on Arduino software prevents you from fully exploiting or understanding the capability of the microcontroller.
- **Teachers and students in mechatronics.** The exercises, online material, and associated kit are designed to support introductory, advanced, and flipped or online courses in mechatronics.
- **Anyone interested in mechatronics, actuators, sensors, and practical embedded control.**

Contents

This book was written based on the two-quarter mechatronics sequence at Northwestern University, ME 333 Introduction to Mechatronics and ME 433 Advanced Mechatronics. In ME 333, students learn about PIC32 hardware, fundamentals of programming the PIC32 in C, the use of some basic peripherals, and interfacing the PIC32 with sensors and actuators. In ME 433, material from the rest of the book is used as reference by groups working on projects. Students taking the sequence range from sophomores to graduate students. The only prerequisite is introductory circuit analysis and design; experience in C programming is not required. While experience in C would allow faster progression through the material, we decided not to require it, to make the course available to the broad set of students interested in the material. To partially compensate for the wide range of experience in C (from expert to none), we begin ME 333 with an intensive two-week introduction to fundamental C concepts and syntax using the "Crash Course in C" in Appendix A. We also take advantage of student expertise by facilitating peer mentoring.

The goals of this book mirror those of the Northwestern mechatronics sequence:

- to provide the beginner a sound introduction to microcontrollers using the example of the PIC32, a modern 32-bit architecture;
- to do so by first providing an overview of microcontroller hardware, firm in the belief that microcontroller programming is much more grounded when tightly connected to the hardware that implements it;
- to provide a clear understanding of the fundamentals of professional PIC32 programming in C, which builds a foundation for further exploration of the PIC32's capabilities using Microchip documentation and other advanced references;
- to provide reference material and sample code on the major peripherals and special features of the PIC32;
- to instill an understanding of the theory of motor operation and control; and
- to teach how microcontroller peripherals can be used to interface with sensors and motors.

To achieve these goals, the book is divided into five main parts:

1. **Quickstart.** This part (Chapter 1) allows the student to get up and running with the PIC32 quickly.
2. **Fundamentals.** After achieving some early success with the quickstart, the five chapters in Fundamentals (Chapters 2 to 6) examine the PIC32 hardware, the build process in C and the connection of the code to the hardware, the use of libraries, and two important topics for real-time embedded computing: interrupts and the time and space efficiency of code. The time investment in these chapters provides the foundation needed to move quickly through later chapters and to profit from other reference material, like Microchip's PIC32 Reference Manual, Data Sheets, and XC32 C/C++ Compiler User's Guide.
3. **Peripheral Reference.** This part (Chapters 7 to 20) gives details on the operation of the various peripherals on the PIC32, as well as sample code and applications. It is primarily reference material that can be read in any order, though we recommend the first few chapters (digital I/O, counter/timers, output compare, and analog input) be covered in order. The peripheral reference concludes with an introduction to Harmony, Microchip's recent framework for high-level programming of PIC32s.
4. **Mechatronics.** This part (Chapters 21 to 29) focuses on interfacing sensors to a microcontroller, digital signal processing, feedback control, brushed DC motor theory, motor sizing and gearing, control by a microcontroller, and other actuators such as brushless motors, stepper motors, and servo motors.
5. **Appendixes.** The appendixes cover background topics such as analysis of simple circuits and an introduction to programming in C. We have our students first get used to writing C programs on their laptops, and compiling with `gcc`, before moving on to programming a microcontroller.

In ME 333, we cover the crash course in C; the Quickstart; the Fundamentals; select topics from the Peripheral Reference (digital I/O, counter/timers, output compare/PWM, and analog input); and simple sensor interfacing, DC motor theory, motor sizing and gearing, and control of a DC motor from the Mechatronics part. Other chapters are used for reference in ME 433 and other projects that students undertake.

Choices made in this book

We made several choices about how to teach mechatronics in ME 333, and those choices are reflected in this book. Our choices are based on the desire to expose our students to the topics they will need to integrate sensors and actuators and microcontrollers professionally, subject to the constraint that most students will take only one or two courses in mechatronics. Our choices are based on what we believe to be the smallest building blocks that a mechatronics engineer needs to know about. For example, we do not attempt to teach microcontroller architecture at the level that a computer engineer might learn it, since a mechatronics engineer is not likely to design a microcontroller. On the other hand, we also do not rely on software and hardware abstractions that keep the budding mechatronics engineer at arm's length from concepts needed to progress beyond the level of a hobbyist. With that philosophy in mind, here are some of the choices made for ME 333 and this book:

- *Microcontrollers vs. sensors and actuators.* Mechatronics engineering integrates sensors, actuators, and microcontrollers. Handing a student a microcontroller development board and sample code potentially allows the course to focus on the sensors and actuators part. In ME 333, however, we opted to make understanding the hardware and software of the microcontroller approximately 50% of the course. This choice recognizes the fundamental role microcontrollers play in mechatronics, and that mechatronics engineers must be comfortable with programming.
- *Choice of microcontroller manufacturer.* There are many microcontrollers on the market, with a wide variety of features. Manufacturers include Microchip, Atmel, Freescale, Texas Instruments, STMicroelectronics, and many others. In particular, Atmel microcontrollers are used in Arduino boards. Arduinos are heavily used by hobbyists and in K-12 and university courses in large part due to the large online user support community and the wide variety of add-on boards and user-developed software libraries. In this book, we opt for the commercially popular Microchip PIC microcontrollers, and we avoid the high-level software abstractions synonymous with Arduino. (Arduinos are used in other Northwestern courses, particularly those focusing on rapid product prototyping with little mechatronics design.)
- *Choice of a particular microcontroller model.* Microchip's microcontroller line consists of hundreds of different models, including 8-bit, 16-bit, and 32-bit architectures. We have chosen a modern 32-bit architecture. And instead of trying to write a book that deals with all PIC32 models, which includes six different families of PIC32s as of this writing (see

Appendix C), we focus on one particular model: the PIC32MX795F512H. The reasons for this choice are (a) it is a powerful chip with plenty of peripherals and memory (128 KB data RAM and 512 KB program flash), and, more importantly, (b) focusing on a single chip allows us to be concrete in the details of its operation. This is especially important when learning how the hardware relates to the software. (One of the reasons Microchip's documentation is difficult to read, and is so full of exceptions and special cases, is that it is written to be general to all PIC32s in the case of the Reference Manual, or all PIC32s in a specific family in the case of the Data Sheets.) Once the reader has learned about the operation of a specific PIC32, it is not too difficult to learn about the differences for a different PIC32 model.

- *Programming language: C++ vs. C vs. assembly.* C++ is a relatively high-level language, C is lower level, and assembly is lower still. We choose to program in C because of the portability of the language, while staying relatively close to the assembly language level and minimizing abstractions introduced by C++.
- *Integrated Development Environment vs. command line.* MPLAB X is Microchip's Integrated Development Environment (IDE) for developing software for PICs. So why do we avoid using it in this book? Because we feel that it hides key steps in understanding how the code you write turns into an executable for the PIC32. In this book, code is written in a text editor and the C compiler is invoked at the command line. There are no hidden steps. Once the reader has mastered the material in the first few chapters of this book, MPLAB will no longer be mysterious.
- *Use of the Harmony software vs. ignoring it.* Microchip provides an extensive library of middleware, device drivers, system services, and other software to support all of their PIC32 models. One goal of this software is to allow you to write programs that are portable across different PIC32 models. To achieve this, however, a significant amount of abstraction is introduced, separating the code you write from the actual hardware implementation. This is bad pedagogically as you learn about the PIC32. Instead, we use low-level software commands to control the PIC32's peripherals, reinforcing the hardware documentation in this book and in the Data Sheet and Reference Manual. Only with the more complicated peripherals do we use the Harmony software, specifically for USB, in Chapter 20.
- *Sample code vs. writing it yourself.* The usual way to learn to program PIC32s is to take some working sample code and try to modify it to do something else. This is natural, except that if your modified code fails, you are often left with no idea what to do. In this book we provide plenty of sample code, but we also focus on the fundamentals of programming the PIC32 so that you learn to write code from scratch as well as strategies to debug if things go wrong (Figure 0.1).

The philosophy represented by the choices above can be summed up succinctly: There should be no magic steps! You should know how and why the code you write works, and how it

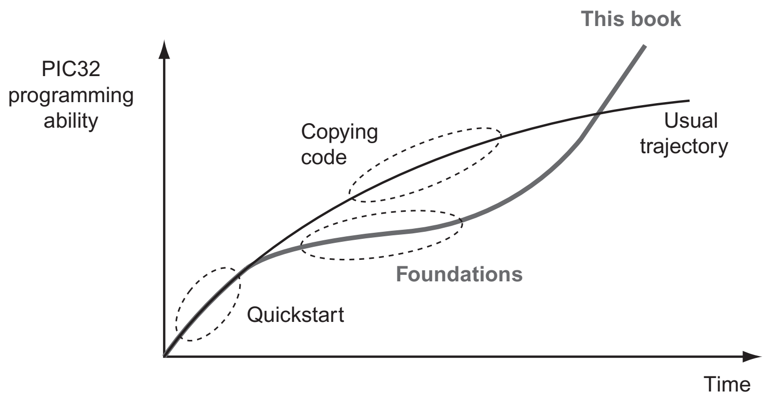

Figure 0.1
The trajectory of PIC32 programming ability vs. time for the usual "copy and modify" approach vs. the foundational approach in this book. The crossover should occur at only a few weeks!

connects to the hardware. You should not be simply modifying opaque and abstract code, compiling with a mysterious IDE, and hoping for the best.

The NU32 development board

The NU32 development board was created to support this book. If you do not have the board, you can still learn a lot about how a PIC32 works from reading this book. We highly recommend that you get the NU32 board and the kit of mechatronics parts, however, to allow you to work through the examples in the book.

In keeping with the "no magic" philosophy, the primary function of the NU32 is to break out the pins of the PIC32MX795F512H to a solderless prototyping breadboard, to allow easy wiring to the pins. Otherwise we try to keep the board as bare bones and inexpensive as possible, leaving external circuits to the reader. To allow you to get up and running as quickly as possible, though, the board does provide a few devices external to the PIC32: two LEDs and two buttons for simple user interaction; a 3.3 V regulator (to provide power to the PIC32) and a 5 V regulator (to provide a commonly needed voltage); a resonator to provide a clock signal; and a USB-to-UART chip that simplifies communication between the user's computer and the PIC32.

The PIC32 on the NU32 comes with a bootloader program pre-installed, allowing you to program the PIC32 with just a USB cable. The NU32 can also be programmed directly using a programmer device, like the PICkit 3. This is covered in Chapter 3.6.

How to use this book in a course

Mechatronics is fundamentally an integrative discipline, requiring knowledge of microcontrollers, programming, circuit design, sensors, signal processing, feedback control, and motors. This book contains a practical introduction to these topics.

Recognizing that most students take no more than one or two courses in mechatronics, however, this book does not delve deeply into the mathematical theory underlying topics such as linear systems, circuit analysis, signal processing, or control theory.[1] Instead, a course based on this book is meant to motivate further theoretical study in these disciplines by exposing students to their practical applications.

As a result, students need only a basic background in circuits and programming to be able to take a course based on this book. At Northwestern, this means that students take ME 333 as early as their sophomore year. ME 333 is an intense 11-week quarter, covering, in order:

- Appendix A, a Crash Course in C. (Approximately 2 weeks.)
- Chapters 1–6, fundamentals of hardware and software of the PIC32 microcontroller. (Approximately 3 weeks.)
- Chapters 7–10, covering digital input and output, counter/timers, output compare/PWM, and analog input. These chapters are primarily used as reference in the context of the following assignment.
- Chapters 23 and 24, on feedback control and PI control of the brightness of an LED using a phototransistor for feedback. This project is the students' first significant project using the PIC32 for embedded control. It also serves as a warmup for the final project. (Approximately 2 weeks.)
- Chapter 25 on theory and experimental characterization of a brushed DC motor. (Approximately 1 week.)
- Introduction to encoders and current sensing in Chapter 21 and all of Chapters 27 and 28 on DC motor control. Chapter 27 introduces all the hardware and software elements of a professional DC motor control system, including a nested-loop control system with an outer-loop motion controller and an inner-loop current controller. Chapter 28 is a chapter-long project that applies the ideas, leading the student through a significant software design project to develop a motor control system that interfaces with a menu system in MATLAB. This "capstone" project is motivated by professional motor amplifier design and integrates the student's knowledge of the PIC32, C programming, brushed DC motors, feedback control, and the interfacing of sensors and actuators with a PIC32. (Approximately 3 weeks.)

[1] Because other courses generally do not cover the operation of motors, this book goes into greater detail on motor theory.

This is a very full quarter, which would be less intense if students were required to know C before taking the course.

ME 333 at Northwestern is taught as a flipped class. Students watch videos that support the text on their own time, then work on assignments and projects during class time while the instructor and TAs circulate to help answer questions. Students bring their laptops and portable mechatronics kits to every class. This kit includes an inexpensive function generator and oscilloscope, the nScope, that uses their laptop as the display. Thus ME 333 does not use a lab facility; students use the classroom and their own dorm rooms. Students work and learn together during classes, but each student completes her own assignment individually. The follow-on course ME 433 focuses on more open-ended mechatronics projects in teams and makes extensive use of a mechatronics lab that is open to students 24/7.

For a 15-week semester, good additions to the course would be two weeks on different sensor technologies (Chapter 21) and digital signal processing of sensor data (Chapter 22). Another week should also be devoted to the final motor control project (Chapter 28), to allow students to experiment with various extensions. Time permitting, other common actuators (e.g., steppers, RC servos, and brushless motors) could be covered in Chapter 29.

For a two-quarter or two-semester sequence, the second course could focus on open-ended team design projects, similar to ME 433 at Northwestern. The book then serves as a reference. Other appropriate material includes chapters on communication protocols and supporting PIC32 peripherals (e.g., UART, SPI, I^{2}C, USB, and CAN).

Website, videos, and flipped classrooms

The book's website, `www.nu32.org`, has links to downloadable data sheets, sample code, PCB layouts and schematics, chapter extensions, errata, and other useful information and updates. This website also links to short videos that summarize many of the chapters. These videos can be used to flip a traditional classroom, as in ME 333, allowing students to watch the lectures at home and to use class time to ask questions and work on projects.

Other PIC32 references

One goal of this book is to organize Microchip reference material in a logical way, for the beginner. Another goal is to equip the reader to be able to parse Microchip documentation. This ability allows the reader to continue to develop her PIC32 programming abilities beyond the limits of this book. The reader should download and have at the ready the first two references below; the others are optional. The readings are summarized in Figure 0.2.

- **The PIC32 Reference Manual.** The Reference Manual sections describe software and hardware for all PIC32 families and models, so they can sometimes be confusing in their

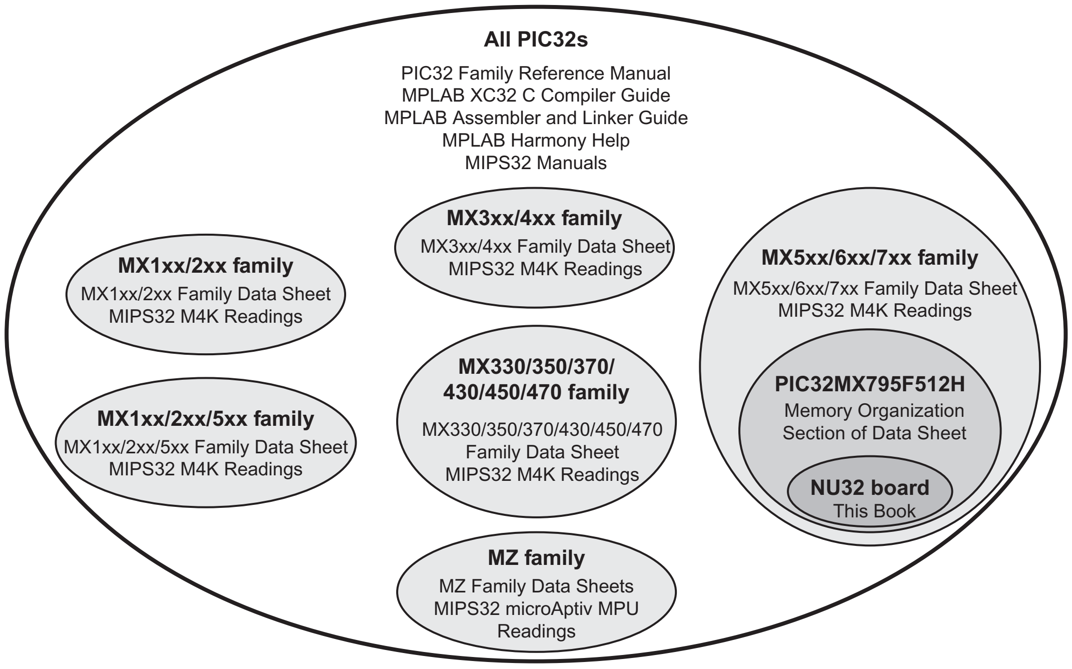
All PIC32s
PIC32 Family Reference Manual
MPLAB XC32 C Compiler Guide
MPLAB Assembler and Linker Guide
MPLAB Harmony Help
MIPS32 Manuals
MX1xx/2xx family
MX1xx/2xx Family Data Sheet
MIPS32 M4K Readings
MX1xx/2xx/5xx family
MX1xx/2xx/5xx Family Data Sheet
MIPS32 M4K Readings
MX3xx/4xx family
MX3xx/4xx Family Data Sheet
MIPS32 M4K Readings
MX330/350/370/430/450/470 family
MX330/350/370/430/450/470 Family Data Sheet
MIPS32 M4K Readings
MZ family
MZ Family Data Sheets
MIPS32 microAptiv MPU Readings
MX5xx/6xx/7xx family
MX5xx/6xx/7xx Family Data Sheet
MIPS32 M4K Readings
PIC32MX795F512H
Memory Organization Section of Data Sheet
NU32 board
This Book

Figure 0.2
Other reference reading and the PIC32s they apply to.

generality. Nevertheless, they are a good source for understanding the functions of the PIC32 in more detail. Some of the sections, particularly the later ones, focus on the PIC32MZ family and are not relevant to the PIC32MX795F512H.

- **The PIC32MX5xx/6xx/7xx Family Data Sheet.** This Data Sheet provides details specific to the PIC32MX5xx/6xx/7xx family. In particular, the Memory Organization section of the Data Sheet clarifies which special function registers (SFRs) are included on the PIC32MX795F512H, and therefore which Reference Manual functions are available for that model.
- **(Optional) The Microchip MPLAB XC32 C Compiler User's Guide** and **The Assembler, Linker, and Utilities User's Guide.** These come with your XC32 C compiler installation, so no need to download separately.
- **(Optional) MPLAB Harmony Help.** This documentation, which comes with the Harmony installation, can be helpful once you start writing more complex code that uses the Harmony software.
- **(Optional) MIPS32 Architecture for Programmers manuals and other MIPS32 documentation.** If you are curious about the MIPS32 M4K CPU, which is used on the PIC32MX795F512H, and its assembly language instruction set, you can find references online.

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This book has benefited from the feedback of many students and teaching assistants in ME 333 at Northwestern over the years, particularly Alex Ansari, Philip Dames, Mike Hwang, Andy Long, David Meyer, Ben Richardson, Nelson Rosa, Jarvis Schultz, Jian Shi, Craig Shultz, Matt Turpin, and Zack Woodruff.

CHAPTER 1

Quickstart

Edit, compile, run, repeat: familiar to generations of C programmers, this mantra applies to programming in C, regardless of platform. Architecture, program loading, input and output: these details differ between your computer and the PIC32. Architecture refers to processor type: your computer's x86-64 CPU and the PIC32's MIPS32 CPU understand different machine code and therefore require different compilers. Your computer's operating system allows you to seamlessly run programs; the PIC32's *bootloader* writes programs it receives from your computer to flash memory and executes them when the PIC32 resets.[1] You interact directly with your computer via the screen and keyboard; you interact indirectly with the PIC32 using a *terminal emulator* to relay information between your computer and the microcontroller. As you can see, programming the PIC32 requires attention to details that you probably ignore when programming your computer.

Armed with an overview of the differences between computer programming and microcontroller programming, you are ready to get your hands dirty. The rest of this chapter will guide you through gathering the hardware and installing the software necessary to program the PIC32. You will then verify your setup by running two programs on the PIC32. By the end of the chapter, you will be able to compile and run programs for the PIC32 (almost) as easily as you compile and run programs for your computer!

Throughout this book, we will refer to "the PIC32." Although there are many PIC32 models, for us "the PIC32" is shorthand for the PIC32MX795F512H. While most of the concepts in this book apply to many PIC32 models, you should be aware that some of the details differ between models. (See Appendix C for a discussion of the differences.) Further, we refer to the PIC32MX795F512H as it is configured on the NU32 development board; in particular, it is powered by 3.3 V and is clocked by a system clock and a peripheral bus clock at 80 MHz. You will learn more about these details in Chapter 2.

[1] Your computer also has a bootloader. It runs when you turn the computer on and loads the operating system. Also, operating systems are available for the PIC32, but we will not use them in this book.

Embedded Computing and Mechatronics with the PIC32 Microcontroller. http://dx.doi.org/10.1016/B978-0-12-420165-1.00001-9

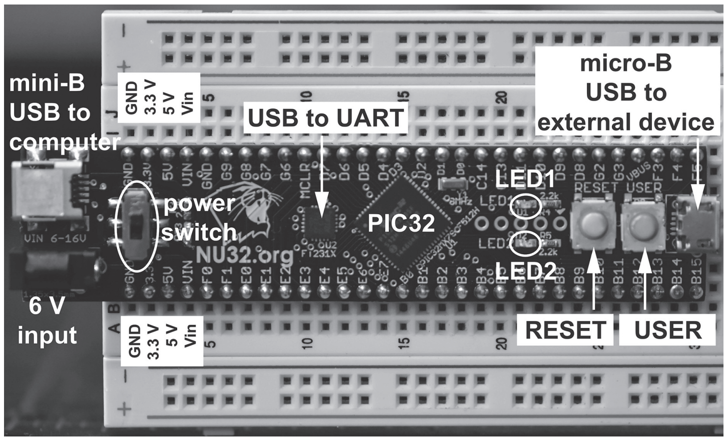


Figure 1.1
A photo of the NU32 development board mounted on a solderless breadboard.

1.1 What You Need

This section explains the hardware and software that you need to program the PIC32. Links to purchase the hardware and download the free software are provided at the book's website, `www.nu32.org`.

1.1.1 Hardware

Although PIC32 microcontrollers integrate many devices on a single chip, they also require external circuitry to function. The NU32 development board, shown in Figure 1.1, provides this circuitry and more: buttons, LEDs, breakout pins, a USB port, and a virtual USB serial port. The examples in this book assume that you use this board. You will also need the following hardware:

1. **Computer with a USB port.** The host computer is used to create PIC32 programs. The examples in this book work with the Linux, Windows, and Mac operating systems.
2. **USB A to mini-B cable.** This cable carries signals between the NU32 board and your computer.
3. **AC/DC adapter (6 V).** This cable provides power to the PIC32 and NU32 board.

1.1.2 Software

Programming the PIC32 requires various software. You should be familiar with some of the software from programming your computer in C; if not, refer to Appendix A.

For your convenience, we have aggregated the software you need at the book's website. You should download and install all of the following software.

1. **The command prompt** allows you to control your computer using a text-based interface. This program, `cmd.exe` on Windows, `Terminal` on Mac, and `bash` on Linux, comes with your operating system so you should not need to install it. See Appendix A for more information about the command line.
2. **A text editor** allows you to create text files, such as those containing C source code. See Appendix A for more information.
3. **A native C compiler** converts human-readable C source code files into machine code that your computer can execute. We suggest the free GNU compiler, `gcc`, which is available for Windows, Mac, and Linux. See Appendix A for more information.
4. **Make** simplifies the build process by automatically executing the instructions required to convert source code into executables. After manually typing all of the commands necessary create your first program, you will appreciate `make`.
5. **The Microchip XC32 compiler** converts C source files into machine code that the PIC32 understands. This compiler is known as a *cross compiler* because it runs on one processor architecture (e.g., x86-64 CPU) and creates machine code for another (e.g., MIPS32). This compiler installation also includes C libraries to help you control PIC32-specific features. Note where you install the compiler; we will refer to this directory as `<xc32dir>`. If you are asked during installation whether you would like to add XC32 to your path variable, do so.
6. **MPLAB Harmony** is Microchip's collection of libraries and drivers that simplify the task of writing code targeting multiple PIC32 models. We will use this library only in Chapter 20; however, you should install it now. Note the installation directory, which we will refer to as `<harmony>`.
7. **The FTDI Virtual COM Port Driver** allows you to use a USB port as a "virtual serial communication (COM) port" to talk to the NU32 board. This driver is already included with most Linux distributions, but Windows and Mac users may need to install it.
8. **A terminal emulator** provides a simple interface to a COM port on your computer, sending keyboard input to the PIC32 and displaying output from the PIC32. For Linux/Mac, you can use the built-in `screen` program. For Windows, we recommend you download `PuTTY`. Remember where you install `PuTTY`; we refer to this directory as `<puttyPath>`.
9. **The PIC32 quickstart code** contains source code and other support files to help you program the PIC32. Download `PIC32quickstart.zip` from the book's website, extract it, and put it in a directory that you create. We will refer to this directory as `<PIC32>`. In `<PIC32>` you will keep the quickstart code, plus all of the PIC32 code you write, so make sure the directory name makes sense to you. For example, depending on your operating system, `<PIC32>` could be `/Users/kevin/PIC32` or `C:\Users\kevin\Documents\PIC32`. In `<PIC32>`, you should have the following three files and one directory:

- `nu32utility.c`: a program for your computer, used to load PIC32 executable programs from your computer to the PIC32
- `simplePIC.c, talkingPIC.c`: PIC32 sample programs that we will test in this chapter
- `skeleton`: a directory containing
    - `Makefile`: a file that will help us compile future PIC32 programs
    - `NU32.c`, `NU32.h`: a library of useful functions for the NU32 board
    - `NU32bootloaded.ld`: a linker script used when compiling programs for the PIC32

We will learn more about each of these shortly.

You should now have code in the following directories (if you are a Windows user, you will also have `PuTTY` in the directory `<puttyPath>`):

- `<xc32dir>`. The Microchip XC32 compiler. **You will never modify code in this directory.** Microchip wrote this code, and there is no reason for you to change it. Depending on your operating system, your `<xc32dir>` could look something like the following:
    - `/Applications/microchip/xc32`
    - `C:\Program Files (x86)\Microchip\xc32`
- `<harmony>`. Microchip Harmony. **You will never modify code in this directory.** Depending on your operating system, your `<harmony>` could look something like the following:
    - `/Users/kevin/microchip/harmony`
    - `C:\microchip\harmony`
- `<PIC32>`. Where PIC32 quickstart code, and code you will write, is stored, as described above.

Now that you have installed all of the necessary software, it is time to program the PIC32. By following these instructions, not only will you run your first PIC32 program, you will also verify that all of the software and hardware is functioning properly. Do not worry too much about what all the commands mean, we will explain the details in subsequent chapters.

Notation: Wherever we write `<something>`, replace it with the value relevant to your computer. On Windows, use a backslash (`\`) and on Linux/Mac use a slash (`/`) to separate the directories in a path. At the command line, place paths that contain spaces between quotation marks (i.e., "`C:\Program Files`"). Enter the text following a `>` at the command line. Use a single line, even if the command spans multiple lines in the book.

1.2 Compiling the Bootloader Utility

The bootloader utility, located at `<PIC32>/nu32utility.c`, sends compiled code to the PIC32. To use the bootloader utility you must compile it. Navigate to the `<PIC32>` directory by typing:

```
> cd <PIC32>
```

Verify that `<PIC32>/nu32utility.c` exists by executing the following command, which lists all the files in a directory:

- **Windows**
  ```
  > dir
  ```
- **Linux/Mac**
  ```
  > ls
  ```

Next, compile the bootloader utility using the native C compiler `gcc`:

- **Windows**
  ```
  > gcc nu32utility.c -o nu32utility -lwinmm
  ```
- **Linux/Mac**
  ```
  > gcc nu32utility.c -o nu32utility
  ```

When you successfully complete this step the executable file `nu32utility` will be created. Verify that it exists by listing the files in `<PIC32>`.

1.3 Compiling Your First Program

The first program you will load onto your PIC32 is `<PIC32>/simplePIC.c`, which is listed below. We will scrutinize the source code in Chapter 3, but reading it now will help you understand how it works. Essentially, after some setup, the code enters an infinite loop that alternates between delaying and toggling two LEDs. The delay loops infinitely while the USER button is pressed, stopping the toggling.

Code Sample 1.1 `simplePIC.c`**. Blinking Lights on the NU32, Unless the** `USER` **Button Is Pressed.**

```
#include <xc.h>          // Load the proper header for the processor

void delay(void);

int main(void) {
  TRISF = 0xFFFC;        // Pins 0 and 1 of Port F are LED1 and LED2.  Clear
                         // bits 0 and 1 to zero, for output.  Others are inputs.
  LATFbits.LATF0 = 0;    // Turn LED1 on and LED2 off.  These pins sink current
  LATFbits.LATF1 = 1;    // on the NU32, so "high" (1) = "off" and "low" (0) = "on"

  while(1) {
    delay();
    LATFINV = 0x0003;    // toggle LED1 and LED2; same as LATFINV = 0x3;
  }
  return 0;
}
```

```
void delay(void) {
  int j;
  for (j = 0; j < 1000000; j++) { // number is 1 million
    while(!PORTDbits.RD7) {
        ;   // Pin D7 is the USER switch, low (FALSE) if pressed.
    }
  }
}
```

To compile this program you will use the `xc32-gcc` cross compiler, which compiles code for the PIC32's MIPS32 processor. This compiler and other Microchip tools are located at `<xc32dir>/<xc32ver>/bin`, where `<xc32ver>` refers to the XC32 version (e.g., v1.40). To find `<xc32ver>` list the contents of the Microchip XC32 directory, e.g.,

```
> ls <xc32dir>
```

The subdirectory displayed is your `<xc32ver>` value. If you happen to have installed two or more versions of XC32, you will always use the most recent version (the largest version number).

Next you will compile `simplePIC.c` and create the executable *hex file*. To do this, you first create the `simplePIC.elf` file and then you create the `simplePIC.hex` file. (This two-step process will be discussed in greater detail in Chapter 3.) Issue the following commands from your `<PIC32>` directory (where `simplePIC.c` is), being sure to replace the text between the `<>` with the values appropriate to your system. Remember, if the paths contain spaces, you must surround them with quotes (i.e., `"C:\Program Files\xc32\v1.40\bin\xc32-gcc"`).

```
> <xc32dir>/<xc32ver>/bin/xc32-gcc -mprocessor=32MX795F512H
    -o simplePIC.elf -Wl,--script=skeleton/NU32bootloaded.ld simplePIC.c
> <xc32dir>/<xc32ver>/bin/xc32-bin2hex simplePIC.elf
```

The `-Wl` is "-W ell" not "-W one." You can list the contents of `<PIC32>` to make sure both `simplePIC.elf` and `simplePIC.hex` were created. The hex file contains MIPS32 machine code in a format that the bootloader understands, allowing it to load your program onto the PIC32.

If, when you installed XC32, you selected to have XC32 added to your path, then in the two commands above you could have simply typed

```
> xc32-gcc -mprocessor=32MX795F512H
   -o simplePIC.elf -Wl,--script=skeleton/NU32bootloaded.ld simplePIC.c
> xc32-bin2hex simplePIC.elf
```

and your operating system would be able to find `xc32-gcc` and `xc32-bin2hex` without needing the full paths to them.

1.4 Loading Your First Program

Loading a program onto the PIC32 from your computer requires communication between the two devices. When the PIC32 is powered and connected to a USB port, your computer creates a new serial communication (COM) port. Depending on your specific system setup, this COM port will have different names. Therefore, we will determine the name of your COM port through experimentation. First, with the PIC32 unplugged, execute the following command to enumerate the current COM ports, and note the names that are listed:

- **Windows**:
  ```
  > mode
  ```
- **Mac**:
  ```
  > ls /dev/tty.*
  ```
- **Linux**:
  ```
  > ls /dev/ttyUSB*
  ```

Next, plug the NU32 board into the wall using the AC adapter, turn the power switch on, and verify that the red “power” LED illuminates. Connect the USB cable from the NU32’s mini-B USB jack (next to the power jack) to a USB port on the host computer. Repeat the steps above, and note that a new COM port appears. If it does not appear, make sure that you installed the FTDI driver from Section 1.1.2. The name of the port will differ depending on the operating system; therefore we have listed some typical names:

- **Windows**: `COM4`
- **Mac**: `/dev/tty.usbserial-DJ00DV5V`
- **Linux**: `/dev/ttyUSB0`

Your computer, upon detecting the NU32 board, has created this port. Your programs and the bootloader use this port to communicate with your computer.

After identifying the COM port, place the PIC32 into *program receive mode*. Locate the RESET button and the USER button on the NU32 board (Figure 1.1). The RESET button is immediately above the USER button on the bottom of the board (the power jack is the board’s top). Press and hold both buttons, release RESET, and then release USER. After completing this sequence, the PIC32 will flash LED1, indicating that it has entered program receive mode.

Assuming that you are still in the `<PIC32>` directory, start the loading process by typing

- **Windows**
  ```
  nu32utility <COM> simplePIC.hex
  ```
- **Linux/Mac**
  ```
  > ./nu32utility <COM> simplePIC.hex
  ```

where `<COM>` is the name of your COM port.[2] After the utility finishes, LED1 and LED2 will flash back and forth. Hold USER and notice that the LEDs stop flashing. Release USER and watch the flashing resume. Turn the PIC32 off and then on. The LEDs resume blinking because you have written the program to the PIC32's nonvolatile flash memory. Congratulations, you have successfully programmed the PIC32!

1.5 Using `make`

As you just witnessed, building an executable for the PIC32 requires several steps. Fortunately, you can use `make` to simplify this otherwise tedious and error-prone procedure. Using `make` requires a `Makefile`, which contains instructions for building the executable. We have provided a `Makefile` in `<PIC32>/skeleton`. Prior to using `make`, you need to modify `<PIC32>/skeleton/Makefile` so that it contains the paths and COM port specific to your system.

Aside from the paths you have already used, you need your terminal emulator's location, `<termEmu>`, and the Harmony version, `<harmVer>`. On Windows, `<termEmu>` is `<puttyPath>/putty.exe` and for Linux/Mac, `<termEmu>` is `screen`. To find Harmony's version, `<harmVer>`, list the contents of the `<harmony>` directory. Edit `<PIC32>/skeleton/Makefile` and update the first five lines as indicated below.

```
XC32PATH=<xc32dir>/<xc32ver>/bin
HARMONYPATH=<harmony>/<harmVer>
NU32PATH=<PIC32>
PORT=<COM>
TERMEMU=<termEmu>
```

In the `Makefile`, do not surround paths with quotation marks, even if they contain spaces.

If your computer has more than one USB port, you should always use the same USB port to connect your NU32. Otherwise, the name of the COM port may change, requiring you to edit the `Makefile` again.

After saving the `Makefile`, you can use the skeleton directory to easily create new PIC32 programs. The skeleton directory contains not only the `Makefile`, but also the NU32 library (`NU32.h and NU32.c`), and the linker script `NU32bootloaded.ld`, all of which will be used extensively throughout the book. The `Makefile` automatically compiles and links every `.c` file in the directory into a single executable; therefore, your project directory should contain all the C files you need and none that you do not want!

[2] Windows: Write the ports as `\\.\COMx` rather than `COMx`. Linux: To avoid needing to execute commands as root, add yourself to the group that owns the COM port (e.g., uucp).

Each new project you create will have its own directory in `<PIC32>`, e.g., `<PIC32>/<projectdir>`. We now explain how to use the `<PIC32>/skeleton` directory to create a new project, using `<PIC32>/talkingPIC.c` as an example. For this example, we will name the project `talkingPIC`, so `<projectdir>` is `talkingPIC`. By following this procedure, you will have access to the NU32 library and will be able to avoid repeating the previous setup steps. Make sure you are in the `<PIC32>` directory, then copy the `<PIC32>/skeleton` directory to the new project directory:

- **Windows**
  ```
  > mkdir <projectdir>
  > copy skeleton\*.* <projectdir>
  ```
- **Linux/Mac**
  ```
  > cp -R skeleton <projectdir>
  ```

Now copy the project source files, in this case just `talkingPIC.c`, to `<PIC32>/<projectdir>`, and change to that directory:

- **Windows**
  ```
  > copy talkingPIC.c <projectdir>
  > cd <projectdir>
  ```
- **Linux/Mac**
  ```
  > cp talkingPIC.c <projectdir>
  > cd <projectdir>
  ```

Before explaining how to use `make`, we will examine `talkingPIC.c`, which accepts input from and prints output to a terminal emulator running on the host computer. These capabilities facilitate user interaction and debugging. The source code for `talkingPIC.c` is listed below:

Code Sample 1.2 `talkingPIC.c`**. The PIC32 Echoes Any Messages Sent to It from the Host Keyboard Back to the Host Screen.**

```
#include "NU32.h"          // constants, funcs for startup and UART

#define MAX_MESSAGE_LENGTH 200

int main(void) {
  char message[MAX_MESSAGE_LENGTH];

  NU32_Startup(); // cache on, interrupts on, LED/button init, UART init
  while (1) {
    NU32_ReadUART3(message, MAX_MESSAGE_LENGTH);  // get message from computer
    NU32_WriteUART3(message);                     // send message back
    NU32_WriteUART3("\r\n");                      // carriage return and newline
    NU32_LED1 = !NU32_LED1;                       // toggle the LEDs
    NU32_LED2 = !NU32_LED2;
  }
  return 0;
}
```

The NU32 library function `NU32_ReadUART3` allows the PIC32 to read data sent from your computer's terminal emulator. The function `NU32_WriteUART3` sends data from your PIC32 to be displayed by the terminal emulator.

Now that you know how `talkingPIC.c` works, it is time see it in action. First, make sure you are in the `<projectdir>`. Next, build the project using `make`.

```
> make
```

This command compiles and assembles all `.c` files into `.o` object files, links them into a single `out.elf` file, and turns that `out.elf` file into an executable `out.hex` file. You can do a directory listing to see all of these files.

Next, put the PIC32 into program receive mode (use the RESET and USER buttons) and execute

```
> make write
```

to invoke the bootloader utility `nu32utility` and program the PIC32 with `out.hex`. When LED1 stops flashing, the PIC32 has been programmed.

In summary, to create a new project and program the PIC32, you (1) create the project directory `<PIC32>/<projectdir>`; (2) copy the contents of `<PIC32>/skeleton` to this new directory; (3) create the source code (`talkingPIC.c` in this case) in `<projectdir>`; (4) build the executable by executing `make` in `<projectdir>`; and (5) use the RESET and USER buttons to put the PIC32 in program receive mode and execute `make write` from `<projectdir>`. To modify the program, you simply edit the source code and repeat steps (4) and (5) above. In fact, you can skip step (4), since `make write` also builds the executable before loading it onto the PIC32.

Now, to communicate with `talkingPIC`, you must connect to the PIC32 using your terminal emulator. Recall that the terminal emulator communicates with the PIC32 using `<COM>`. Enter the following command:

- **Windows**
  ```
  <puttyPath>\putty -serial <COM> -sercfg 230400,R
  ```
- **Linux/Mac**
  ```
  screen <COM> 230400,crtscts
  ```

`PuTTY` will launch in a new window, whereas `screen` will use the command prompt window. The number 230400 in the above commands is the baud, the speed at which the PIC32 and computer communicate, and the other parameter enables hardware flow control (see Chapter 11 for details).

After connecting, press RESET to restart the program. Start typing, and notice that no characters appear until you hit `ENTER`. This behavior may seem strange, but it occurs because the terminal emulator only displays the text it receives from the PIC32. The PIC32 does not send any text to your computer until it receives a special *control character*, which you generate by pressing `ENTER`.[3]

For example, if you type `Hello! ENTER`, the PIC32 will receive `Hello!\r`, write `Hello!\r\n` to the terminal emulator, and wait for more input.

When you are done conversing with the PIC32, you can exit the terminal emulator. To exit `screen` type

```
CTRL-a k y
```

Note that `CTRL` and `a` should be pressed simultaneously. To exit `PuTTY` make sure the command prompt window is focused and type

```
CTRL-c
```

Rather than memorizing these rather long commands to connect to the serial port, you can use the `Makefile`. To connect `PuTTY` to the PIC32 type

```
> make putty
```

To use `screen` type

```
> make screen
```

Your system is now configured for PIC32 programming. Although the build process may seem opaque, do not worry. For now it is only important that you can successfully compile programs and load them onto the PIC32. Later chapters will explain the details of the build process.

1.6 Chapter Summary

- To start a new project, copy the `<PIC32>/skeleton` directory to a new location, `<projectdir>`, and add your source code.
- From the directory `<projectdir>`, use `make` to build the executable.

[3] Depending on the terminal emulator, `ENTER` may generate a carriage return (`\r`), newline (`\n`) or both. The terminal emulator typically moves the cursor to the leftmost column when it receives a `\r` and to the next line when it receives a `\n`.

- Put the PIC32 into program receive mode by pressing the USER and RESET buttons simultaneously, then releasing the RESET button, and finally releasing the USER button. Then use `make write` to load your program.
- Use a terminal emulator to communicate with programs running on the PIC32. Typing `make putty` or `make screen` from `<projectdir>` will launch the appropriate terminal emulator and connect it to the PIC32.

Further Reading

Embedded computing and mechatronics with the PIC32 microcontroller website. `http://www.nu32.org`.

CHAPTER 25

Brushed Permanent Magnet DC Motors

Most electric motors operate on the principle that current flowing through a magnetic field creates a force. Because of this relationship between current and force, electric motors can be used to convert electrical power to mechanical power. They can also be used to convert mechanical power to electrical power; as with, for example, generators in hydroelectric dams or regenerative braking in electric and hybrid cars.

In this chapter we study perhaps the simplest, cheapest, most common, and arguably most useful electrical motor: the brushed permanent magnet direct current (DC) motor. For brevity, we refer to these simply as DC motors. A DC motor has two input terminals, and a voltage applied across these terminals causes the motor shaft to spin. For a constant load or resistance at the motor shaft, the motor shaft achieves a speed proportional to the input voltage. Positive voltage causes spinning in one direction, and negative voltage causes spinning in the other.

Depending on the specifications, DC motors cost anywhere from tens of cents up to thousands of dollars. For most small-scale or hobby applications, appropriate DC motors typically cost a few dollars. DC motors are often outfitted with a sensing device, most commonly an encoder, to track the position and speed of the motor, and a gearhead to reduce the output speed and increase the output torque.

25.1 Motor Physics

DC motors exploit the *Lorentz force law*,

$$\mathbf{F} = \ell \mathbf{I} \times \mathbf{B}, \tag{25.1}$$

where $\mathbf{F}, \mathbf{I}$, and $\mathbf{B}$ are three-vectors, $\mathbf{B}$ describes the magnetic field created by permanent magnets, $\mathbf{I}$ is the current vector (including the magnitude and direction of the current flow through the conductor), ℓ is the length of the conductor in the magnetic field, and $\mathbf{F}$ is the force on the conductor. For the case of a current perpendicular to the magnetic field, the force is easily understood using the right-hand rule for cross-products: with your right hand, point your index finger along the current direction and your middle finger along the magnetic field flux lines. Your thumb will then point in the direction of the force (see Figure 25.1).

Embedded Computing and Mechatronics with the PIC32 Microcontroller. http://dx.doi.org/10.1016/B978-0-12-420165-1.00025-1

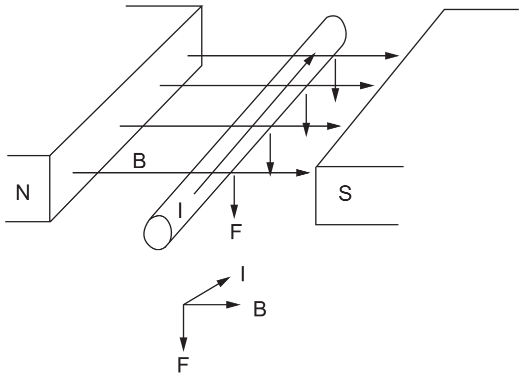


Figure 25.1
Two magnets create a magnetic field **B**, and a current **I** along the conductor causes a force **F** on the conductor.

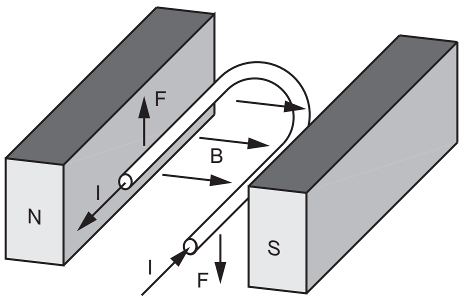


Figure 25.2
A current-carrying loop of wire in a magnetic field.

Now let us replace the conductor by a loop of wire, and constrain that loop to rotate about its center. See Figures 25.2 and 25.3. In one half of the loop, the current flows into the page, and in the other half of the loop the current flows out of the page. This creates forces of opposite directions on the loop. Referring to Figure 25.3, let the magnitude of the force acting on each half of the loop be f, and let d be the distance from the halves of the loop to the center of the loop. Then the total torque acting on the loop about its center can be written

$$\tau = 2df \cos\theta,$$

where θ is the angle of the loop. The torque changes as a function of θ. For $-90° < \theta < 90°$, the torque is positive, and it is maximum at $\theta = 0$. A plot of the torque on the loop as a function of θ is shown in Figure 25.4(a). The torque is zero at $\theta = -90°$ and $90°$, and of these two, $\theta = 90°$ is a stable equilibrium while $\theta = -90°$ is an unstable equilibrium. Therefore, if we send a constant current through the loop, it will likely come to rest at $\theta = 90°$.

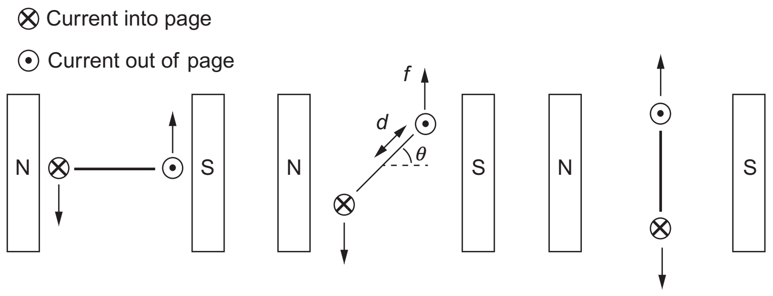


Figure 25.3
A loop of wire in a magnetic field, viewed end-on. Current flows into the page on one side of the loop and out of the page on the other, creating forces of opposite directions on the two halves of the loop. These opposite forces create torque on the loop about its center at most angles θ of the loop.

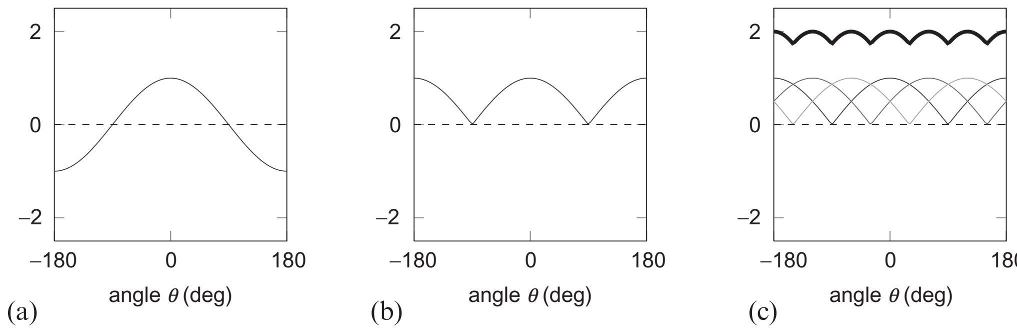


Figure 25.4
(a) The torque on the loop of Figure 25.3 as a function of its angle for a constant current. (b) If we reverse the current direction at the angles $\theta = -90°$ and $\theta = 90°$, we can make the torque nonnegative at all θ. (c) If we use several loops offset from each other, the sum of their torques (the thick curve) becomes more constant as a function of angle. The remaining variation contributes to torque ripple.

To make a more useful motor, we can reverse the direction of current at $\theta = -90°$ and $\theta = 90°$, which makes the torque nonnegative at all angles (Figure 25.4(b)). The torque is still zero at $\theta = -90°$ and $\theta = 90°$, however, and it undergoes a large variation as a function of θ. To make the torque more constant as a function of θ, we can introduce more loops of wire, each offset from the others in angle, and each reversing their current direction at appropriate angles. Figure 25.4(c) shows an example with three loops of wire offset from each other by 120°. Their component torques sum to give a more constant torque as a function of angle. The remaining variation in torque contributes to angle-dependent *torque ripple*.

Finally, to increase the torque generated, each loop of wire is replaced by a coil of wire (also called a winding) that loops back and forth through the magnetic field many times. If the coil consists of 100 loops, it generates 100 times the torque of the single loop for the same current. Wire used to create coils in motors, like magnet wire, is very thin, so there is resistance from one end of a coil to the other, typically from fractions of an ohm up to hundreds of ohms.

As stated previously, the current in the coils must switch direction at the appropriate angle to maintain non-negative torque. Figure 25.5 shows how brushed DC motors accomplish this current reversal. The two input terminals are connected to *brushes*, typically made of a soft conducting material like graphite, which are spring-loaded to press against the *commutator*, which is connected to the motor coils. As the motor rotates, the brushes slide over the commutator and switch between commutator *segments*, each of which is electrically connected to the end of one or more coils. This switching changes the direction of current through the coils. This process of switching the current through the coils as a function of the angle of the motor is called *commutation*. Figure 25.5 shows a schematic of a minimal motor design with three commutator segments and a coil between each pair of segments. Most high quality motors have more commutator segments and coils.

Unlike the simplified example in Figure 25.4, the brush-commutator geometry means that each coil in a real brushed motor is only energized at a subset of angles of the motor. Apart

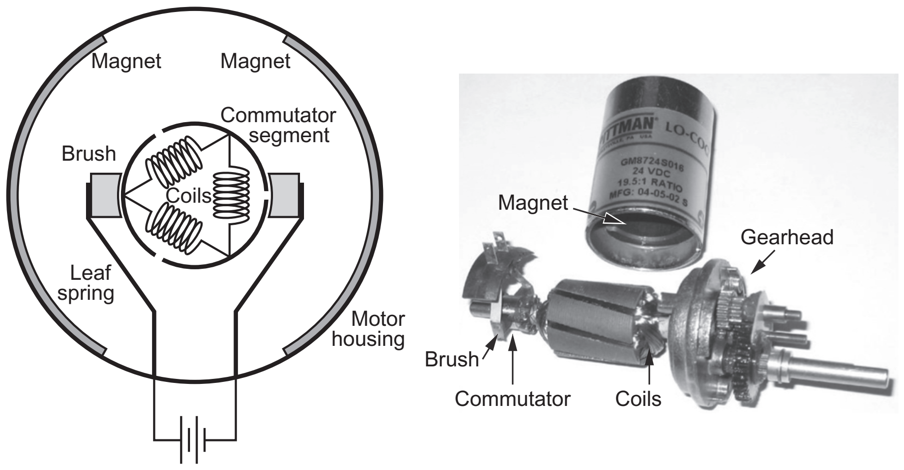


Figure 25.5
(Left) A schematic end-on view of a simple DC motor. The two brushes are held against the commutator by leaf springs which are electrically connected to the external motor terminals. This commutator has three segments and there are coils between each segment pair. The stator magnets are epoxied to the inside of the motor housing. (Right) This disassembled Pittman motor has seven commutator segments. The two brushes are attached to the motor housing, which has otherwise been removed. One of the two permanent magnets is visible inside the housing. The coils are wrapped around a ferromagnetic core to increase magnetic permeability. This motor has a gearhead on the output.

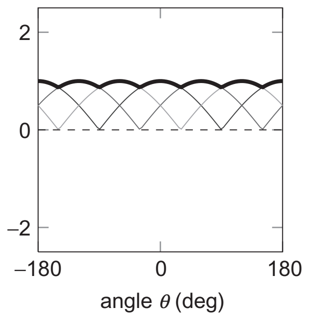


Figure 25.6

Figure 25.4(c) illustrates the sum of the torque of three coils offset by 120° if they are all energized at the same time. The geometry of the brushes and commutator ensure that not all coils are energized simultaneously, however. This figure shows the angle-dependent torque of a three-coil brushed motor that has only one coil energized at a time, which is approximately what happens if the brushes in Figure 25.5 are small. The energized coil is the one at the best angle to create a torque. The result is a motor torque as indicated by the thick curve; the thinner curves are the torques that would be provided by the other coils if they were energized. Comparing this figure to Figure 25.4(c) shows that this more realistic motor produces half the torque, but uses only one-third of the electrical power, since only one of the three coils is energized. Power is not wasted by putting current through coils that would generate little torque.

from being a consequence of the geometry, this has the added benefit of avoiding wasting power when current through a coil would provide little torque. Figure 25.6 is a more realistic version of Figure 25.4(c).

The stationary portion of the motor attached to the housing is called the stator, and the rotating portion of the motor is called the rotor.

Figure 25.7 shows a cutaway of a Maxon brushed motor, exposing the brushes, commutator, magnets, and windings. The figure also shows other elements of a typical motor application: an encoder attached to one end of the motor shaft to provide feedback on the angle and a gearhead attached to the other end of the motor shaft. The output shaft of the gearhead provides lower speed but higher torque than the output shaft of the motor.

Brushless motors are a variant that use electronic commutation as opposed to brushed commutation. For more on brushless DC motors, see Chapter 29.5.

25.2 Governing Equations

To derive an equation to model the motor's behavior, we ignore the details of the commutation and focus instead on electrical and mechanical power. The electrical power into the motor is IV, where I is the current through the motor and V is the voltage across the motor. We know that the motor converts some of this input power to mechanical power $\tau\omega$, where τ

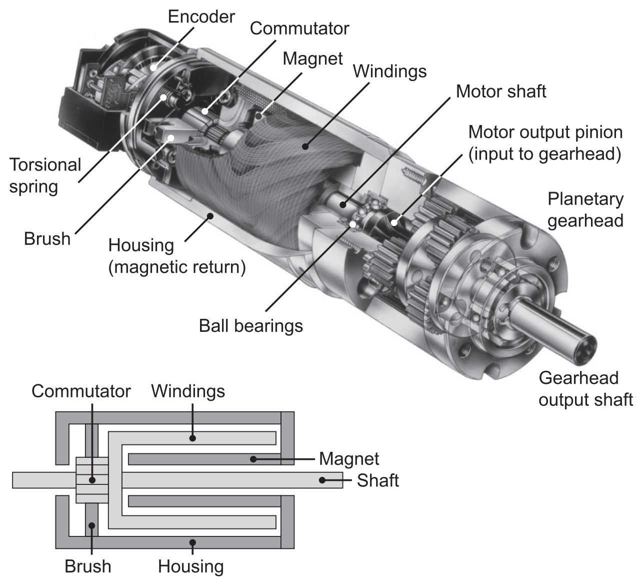


Figure 25.7
A cutaway of a Maxon brushed motor with an encoder and a planetary gearhead. The brushes are spring-loaded against the commutator. The bottom left schematic is a simplified cross-section showing the stationary parts of the motor (the stator) in dark gray and the rotating parts of the motor (the rotor) in light gray. In this "coreless" motor geometry, the windings spin in a gap between the permanent magnets and the housing. (Cutaway image courtesy of Maxon Precision Motors, Inc., maxonmotorusa.com.)

and ω are the torque and velocity of the output shaft, respectively. Electrically, the motor is described by a resistance R between the two terminals as well as an inductance L due to the coils. The resistance of the motor coils dissipates power I^2R as heat. The motor also stores energy $\frac{1}{2}LI^2$ in the inductor's magnetic field, and the time rate of change of this is $LI(\mathrm{d}I/\mathrm{d}t)$, the power into (charging) or out of (discharging) the inductor. Finally, power is dissipated as sound, heat due to friction at the brush-commutator interface and at the bearings between the motor shaft and the housing, etc. In SI units, all these power components are expressed in watts. Combining all of these factors provides a full accounting for the electrical power put into the motor:

$$IV = \tau\omega + I^2R + LI\frac{\mathrm{d}I}{\mathrm{d}t} + \text{ power dissipated due to friction, sound, etc.}$$

Ignoring the last term, we have our simple motor model, written in terms of power:

$$IV = \tau\omega + I^2R + LI\frac{\mathrm{d}I}{\mathrm{d}t}. \tag{25.2}$$

From (25.2) we can derive all other relationships of interest. For example, dividing both sides of (25.2) by I yields

$$V = \frac{\tau}{I}\omega + IR + L\frac{\mathrm{d}I}{\mathrm{d}t}. \tag{25.3}$$

The ratio τ/I is a constant, an expression of the Lorentz force law for the particular motor design. This constant, relating current to torque, is called the *torque constant* k_t. The torque constant is one of the most important properties of the motor:

$$k_t = \frac{\tau}{I} \quad \text{or} \quad \tau = k_t I. \tag{25.4}$$

The SI units of k_t are Nm/A. (In this chapter, we only use SI units, but you should be aware that many different units are used by different manufacturers, as on the speed-torque curve and data sheet in Figure 25.16 in the Exercises.) Equation (25.3) also shows that the SI units for k_t can be written equivalently as Vs/rad, or simply Vs. When using these units, we sometimes call the motor constant the *electrical constant* k_e. The inverse is sometimes called the *speed constant*. You should recognize that these terms all refer to the same property of the motor. For consistency, we usually refer to the torque constant k_t.

We now express the motor model in terms of voltage as

$$V = k_t\omega + IR + L\frac{\mathrm{d}I}{\mathrm{d}t}. \tag{25.5}$$

You should remember, or be able to quickly derive, the power equation (25.2), the torque constant (25.4), and the voltage equation (25.5).

The term $k_t\omega$, with units of voltage, is called the *back-emf*, where emf is short for *electromotive force*. We could also call this "back-voltage." Back-emf is the voltage generated by a spinning motor to "oppose" the input voltage generating the motion. For example, assume that the motor's terminals are not connected to anything (open circuit). Then $I = 0$ and $\frac{\mathrm{d}I}{\mathrm{d}t} = 0$, so (25.5) reduces to

$$V = k_t\omega.$$

This equation indicates that back-driving the motor (e.g., spinning it by hand) will generate a voltage at the terminals. If we were to connect a capacitor across the motor terminals, then spinning the motor by hand would charge the capacitor, storing some of the mechanical energy we put in as electrical energy in the capacitor. In this situation, the motor acts as a generator, converting mechanical energy to electrical energy.

The existence of this back-emf term also means that if we put a constant voltage V across a free-spinning frictionless motor (i.e., the motor shaft is not connected to anything), after some time it will reach a constant speed V/k_t. At this speed, by (25.5), the current I drops to zero, meaning there is no more torque τ to accelerate the motor. This happens because as the motor accelerates, the back-emf increases, countering the applied voltage until no current flows (and hence there is no torque or acceleration).

25.3 The Speed-Torque Curve

Consider a motor spinning a boat's propeller at constant velocity. The torque τ provided by the motor can be written

$$\tau = \tau_{\text{fric}} + \tau_{\text{pushing water}},$$

where τ_{fric} is the torque the motor has to generate to overcome friction and begin to spin, while $\tau_{\text{pushing water}}$ is the torque needed for the propeller to displace water when the motor is spinning at velocity ω. In this section we assume $\tau_{\text{fric}} = 0$, so $\tau = \tau_{\text{pushing water}}$ in this example. In Section 25.4 we consider nonzero friction.

For a motor spinning at constant speed ω and providing constant torque τ (as in the propeller example above), the current I is constant and therefore $\mathrm{d}I/\mathrm{d}t = 0$. Under these assumptions, (25.5) reduces to

$$V = k_t\omega + IR. \tag{25.6}$$

Using the definition of the torque constant, we get the equivalent form

$$\omega = \frac{1}{k_t}V - \frac{R}{k_t^2}\tau. \tag{25.7}$$

Equation (25.7) gives ω as a linear function of τ for a given constant V. This line, of slope $-R/k_t^2$, is called the *speed-torque curve* for the voltage V.

The speed-torque curve plots all the possible constant-current operating conditions with voltage V across the motor. Assuming friction torque is zero, the line intercepts the $\tau = 0$ axis at

$$\omega_0 = V/k_t = \text{no load speed}.$$

The line intercepts the $\omega = 0$ axis at

$$\tau_{\text{stall}} = \frac{k_t V}{R} = \text{Stall torque}.$$

At the no-load condition, $\tau = I = 0$; the motor rotates at maximum speed with no current or torque. At the stall condition, the shaft is blocked from rotating, and the current ($I_{\text{stall}} = \tau_{\text{stall}}/k_t = V/R$) and output torque are maximized due to the lack of back-emf. Which point along the speed-torque curve the motor actually operates at is determined by the load attached to the motor shaft.

An example speed-torque curve is shown in Figure 25.8. This motor has $\omega_0 = 500$ rad/s and $\tau_{\text{stall}} = 0.1067$ Nm for a nominal voltage of $V_{\text{nom}} = 12$ V. The *operating region* is any point below the speed-torque curve, corresponding to voltages less than or equal to 12 V. If the motor is operated at a different voltage cV_{nom}, the intercepts of the speed-torque curve are linearly scaled to $c\omega_0$ and $c\tau_{\text{stall}}$.

Imagine squeezing the shaft of a motor powered by a voltage V and spinning at a constant velocity. Your hand is applying a small torque to the shaft. Since the motor is not accelerating and we are neglecting friction in the motor, the torque created by the motor's coils must be equal and opposite the torque applied by your hand. Thus the motor operates at a specific point on the speed-torque curve. If you slowly squeeze the shaft harder, increasing the torque you apply to the rotor, the motor will slow down and increase the torque it applies, to balance your hand's torque. Assuming the motor's current changes slowly (i.e., $L\mathrm{d}I/\mathrm{d}t$ is negligible), then the operating point of the motor moves down and to the right on the speed-torque curve as you increase your squeeze force. When you squeeze hard enough that the motor can no longer move, the operating point is at the stall condition, the bottom-right point on the speed-torque curve.

The speed-torque curve corresponds to constant V, but not to constant input power $P_{\text{in}} = IV$. The current I is linear with τ, so the input electrical power increases linearly with τ. The output mechanical power is $P_{\text{out}} = \tau\omega$, and the *efficiency* in converting electrical to mechanical power is $\eta = P_{\text{out}}/P_{\text{in}} = \tau\omega/IV$. We return to efficiency in Section 25.4.

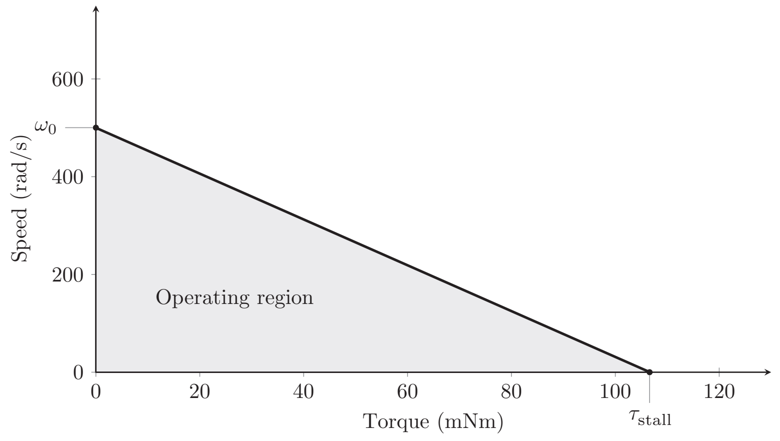


Figure 25.8
A speed-torque curve. Many speed-torque curves use rpm for speed, but we prefer SI units.

To find the point on the speed-torque curve that maximizes the mechanical output power, we can write points on the curve as $(\tau, \omega) = (c\tau_{\text{stall}}, (1-c)\omega_0)$ for $0 \le c \le 1$, so the output power is expressed as

$$P_{\text{out}} = \tau\omega = (c - c^2)\tau_{\text{stall}}\omega_0,$$

and the value of c that maximizes the power output is found by solving

$$\frac{\mathrm{d}}{\mathrm{d}c}\left((c - c^2)\tau_{\text{stall}}\omega_0\right) = (1 - 2c)\tau_{\text{stall}}\omega_0 = 0 \quad \rightarrow \quad c = \frac{1}{2}.$$

Thus the mechanical output power is maximized at $\tau = \tau_{\text{stall}}/2$ and $\omega = \omega_0/2$. This maximum output power is

$$P_{\max} = \left(\frac{1}{2}\tau_{\text{stall}}\right)\left(\frac{1}{2}\omega_0\right) = \frac{1}{4}\tau_{\text{stall}}\omega_0.$$

See Figure 25.9.

Motor current is proportional to motor torque, so operating at high torques means large coil heating power loss I^2R, sometimes called *ohmic heating*. For that reason, motor manufacturers specify a *maximum continuous current* I_{cont}, the largest continuous current such that the coils' steady-state temperature remains below a critical point.[1] The maximum continuous current has a corresponding *maximum continuous torque* τ_{cont}. Points to the left of this torque and under the speed-torque curve are called the *continuous operating region*. The motor can be

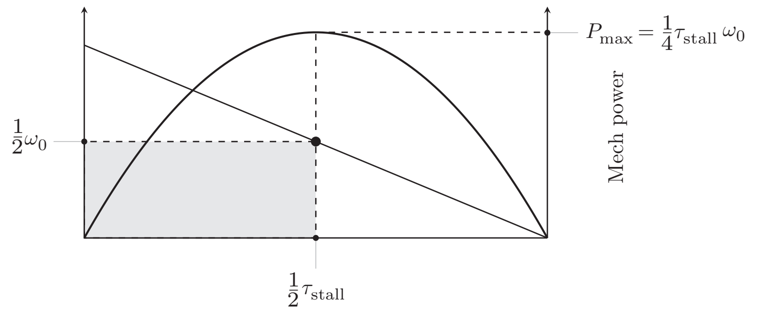


Figure 25.9
The quadratic mechanical power plot $P = \tau\omega$ plotted alongside the speed-torque curve. The area of the speed-torque rectangle below and to the left of the operating point is the mechanical power.

[1] The maximum continuous current depends on thermal properties governing how fast coil heat can be transferred to the environment. This depends on the environment temperature, typically considered to be room temperature. The maximum continuous current can be increased by cooling the motor.

operated intermittently outside of the continuous operating region, in the *intermittent operating region*, provided the motor is allowed to cool sufficiently between uses in this region. Motors are commonly rated with a nominal voltage that places the maximum mechanical power operating point (at $\tau_{\text{stall}}/2$) outside the continuous operating region.

Given thermal characteristics of the motor of Figure 25.8, the speed-torque curve can be refined to Figure 25.10, showing the continuous and intermittent operating regions of the motor. The point on the speed-torque curve at τ_{cont} is the *rated* or *nominal operating point*, and the mechanical power output at this point is called the motor's *power rating*. For the motor of Figure 25.10, $\tau_{\text{cont}} = 26.67$ mNm, which occurs at $\omega = 375$ rad/s, for a power rating of

$$0.02667 \text{ Nm} \times 375 \text{ rad/s} = 10.0 \text{ W}.$$

Figure 25.10 also shows the constant output power hyperbola $\tau\omega = 10$ W passing through the nominal operating point.

The speed-torque curve for a motor is drawn based on a nominal voltage. This is a "safe" voltage that the manufacturer recommends. It is possible to overvolt the motor, however, provided it is not continuously operated beyond the maximum continuous current. A motor also may have a specified *maximum permissible speed* ω_{max}, which creates a horizontal line constraint on the permissible operating range. This speed is determined by allowable brush wear, or possibly properties of the shaft bearings, and it is typically larger than the no-load speed ω_0. The shaft and bearings may also have a maximum torque rating $\tau_{\text{max}} > \tau_{\text{stall}}$. These

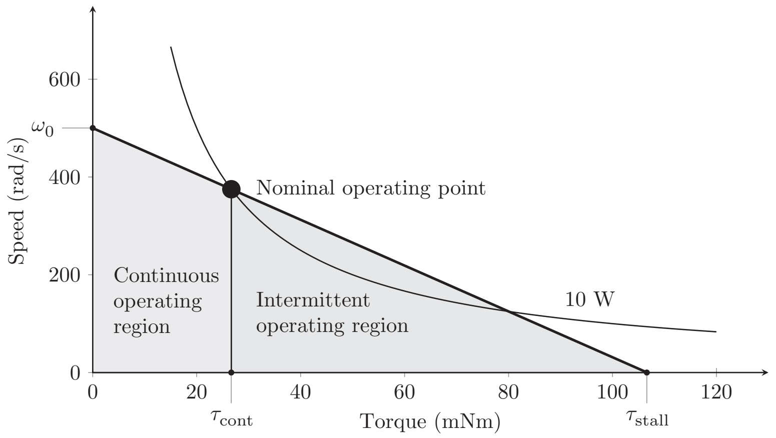


Figure 25.10
The continuous operating region (under the speed-torque curve and left of τ_{cont}) and the intermittent operating region (the rest of the area under the speed-torque curve). The 10 W mechanical power hyperbola is indicated, including the nominal operating point at τ_{cont}.

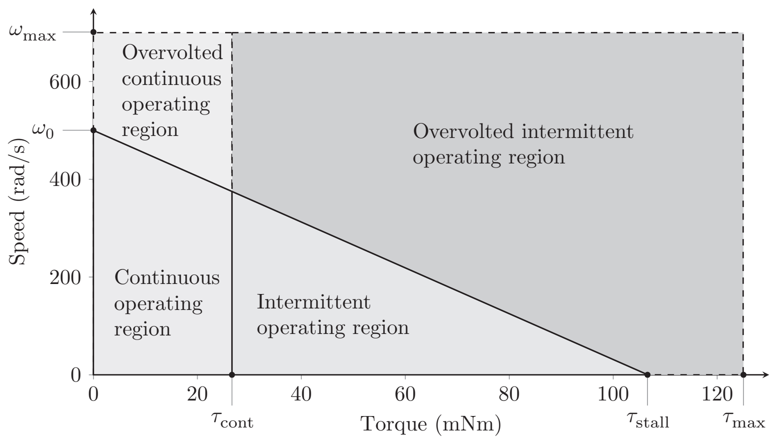


Figure 25.11
It is possible to exceed the nominal operating voltage, provided the constraints $\omega < \omega_{\max}$ and $\tau < \tau_{\max}$ are respected and τ_{cont} is only intermittently exceeded.

limits allow the definition of overvolted continuous and intermittent operating regions, as shown in Figure 25.11.

25.4 Friction and Motor Efficiency

Until now we have been assuming that the full torque $\tau = k_t I$ generated by the windings is available at the output shaft. In practice, some torque is lost due to friction at the brushes and the shaft bearings. Let us use a simple model of friction: assume a torque $\tau \geq \tau_{\text{fric}} > 0$ must be generated to overcome friction and initiate motion, and any torque beyond τ_{fric} is available at the output shaft regardless of the motor speed (e.g., no friction that depends on speed magnitude). When the motor is spinning, the torque available at the output shaft is

$$\tau_{\text{out}} = \tau - \tau_{\text{fric}}.$$

Nonzero friction results in a nonzero *no-load current* $I_0 = \tau_{\text{fric}}/k_t$ and a no-load speed ω_0 less than V/k_t. The speed-torque curve of Figure 25.11 is modified to show a small friction torque in Figure 25.12. The torque actually delivered to the load is reduced by τ_{fric}.

Taking friction into account, the motor's efficiency in converting electrical to mechanical power is

$$\eta = \frac{\tau_{\text{out}}\omega}{IV}. \tag{25.8}$$

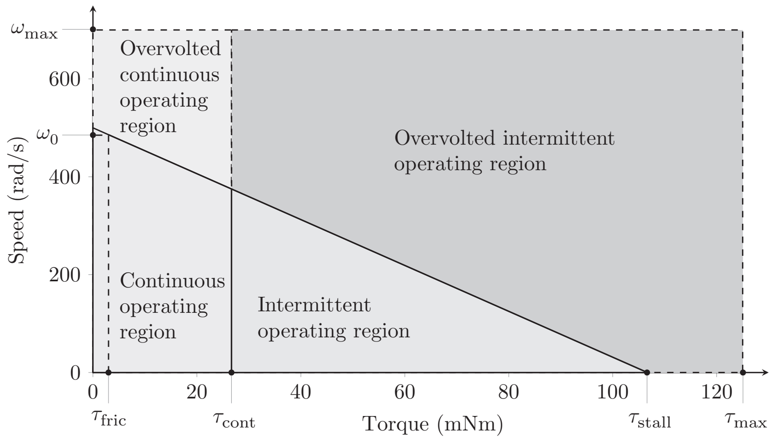


Figure 25.12
The speed-torque curve of Figure 25.11 modified to show a nonzero friction torque τ_{fric} and the resulting reduced no-load speed ω_0.

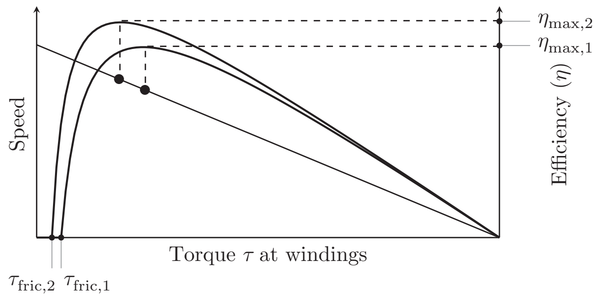


Figure 25.13
The speed-torque curve for a motor and two efficiency plots, one for high friction torque (case 1) and one for low friction torque (case 2). For each case, efficiency is zero for all τ below the level needed to overcome friction. The low friction version of the motor (case 2) achieves a higher maximum efficiency, at a higher speed and lower torque, than the high friction version (case 1).

The efficiency depends on the operating point on the speed-torque curve, and it is zero when either τ_{out} or ω is zero, as there is no mechanical power output. Maximum efficiency generally occurs at high speed and low torque, approaching the limit of 100% efficiency at $\tau = \tau_{\text{out}} = 0$ and $\omega = \omega_0$ as τ_{fric} approaches zero. As an example, Figure 25.13 plots efficiency vs. torque for the same motor with two different values of τ_{fric}. Lower friction results in a higher maximum efficiency $\eta_{\max}$, occurring at a higher speed and lower torque.

To derive the maximally efficient operating point and the maximum efficiency $\eta_{\max}$ for a given motor, we can express the motor current as

$$I = I_0 + I_a,$$

where I_0 is the no-load current necessary to overcome friction and I_a is the added current to create torque to drive the load. Recognizing that $\tau_{\text{out}} = k_t I_a$, $V = I_{\text{stall}} R$, and $\omega = R(I_{\text{stall}} - I_a - I_0)/k_t$ by the linearity of the speed-torque curve, we can rewrite the efficiency (25.8) as

$$\eta = \frac{I_a(I_{\text{stall}} - I_0 - I_a)}{(I_0 + I_a)I_{\text{stall}}}. \tag{25.9}$$

To find the operating point I_a^* maximizing η, we solve $\mathrm{d}\eta/\mathrm{d}I_a = 0$ for I_a^*, and recognizing that I_0 and I_{stall} are nonnegative, the solution is

$$I_a^* = \sqrt{I_{\text{stall}} I_0} - I_0.$$

In other words, as the no-load current I_0 goes to zero, the maximally efficient current (and therefore τ) goes to zero.

Plugging I_a^* into (25.9), we find

$$\eta_{\max} = \left(1 - \sqrt{\frac{I_0}{I_{\text{stall}}}}\right)^2.$$

This answer has the form we would expect: maximum efficiency approaches 100% as the friction torque approaches zero, and maximum efficiency approaches 0% as the friction torque approaches the stall torque.

Choosing an operating point that maximizes motor efficiency can be important when trying to maximize battery life in mobile applications. For the majority of analysis and motor selection problems, however, ignoring friction is a good first approximation.

25.5 Motor Windings and the Motor Constant

It is possible to build two different versions of the same motor by simply changing the windings while keeping everything else the same. For example, imagine a coil of resistance R with N loops of wire of cross-sectional area A. The coil carries a current I and therefore has a voltage drop IR. Now we replace that coil with a new coil with N/c loops of wire with cross-sectional area cA. This preserves the volume occupied by the coil, fitting in the same form factor with similar thermal properties. Without loss of generality, let us assume that the new coil has fewer loops and uses thicker wire ($c > 1$).

The resistance of the new coil is reduced to R/c^2 (a factor of c due to the shorter coil and another factor of c due to the thicker wire). To keep the torque of the motor the same, the new coil would have to carry a larger current cI to make up for the fewer loops, so that the current times the pathlength through the magnetic field is unchanged. The voltage drop across the new coil is $(cI)(R/c^2) = IR/c$.

Replacing the coils allows us to create two versions of the motor: a many-loop, thin wire version that operates at low current and high voltage, and a fewer-loop, thick wire version that operates at high current and low voltage. Since the two motors create the same torque with different currents, they have different torque constants. Each motor has the same *motor constant* k_m, however, where

$$k_m = \frac{\tau}{\sqrt{I^2R}} = \frac{k_t}{\sqrt{R}}$$

with units of $\mathrm{Nm}/\sqrt{\mathrm{W}}$. The motor constant defines the torque generated per square root of the power dissipated by coil resistance. In the example above, the new coil dissipates $(cI)^2(R/c^2) = I^2R$ power as heat, just as the original coil does, while generating the same torque.

Figure 25.16 shows the data sheet for a motor that comes in several different versions, each identical in every way except for the winding. Each version of the motor has a similar stall torque and motor constant but different nominal voltage, resistance, and torque constant.

25.6 Other Motor Characteristics

Electrical time constant

When the motor is subject to a step in the voltage across it, the *electrical time constant* T_e measures the time it takes for the unloaded current to reach 63% of its final value. The motor's voltage equation is

$$V = k_t\omega + IR + L\frac{\mathrm{d}I}{\mathrm{d}t}.$$

Ignoring back-emf (because the motor speed does not change significantly over one electrical time constant), assuming an initial current through the motor of I_0, and an instantaneous drop in the motor voltage to 0, we get the differential equation

$$0 = I_0 R + L\frac{\mathrm{d}I}{\mathrm{d}t}$$

or

$$\frac{\mathrm{d}I}{\mathrm{d}t} = -\frac{R}{L} I_0$$

with solution

$$I(t) = I_0\,\mathrm{e}^{-tR/L} = I_0\,\mathrm{e}^{-t/T_e}.$$

The time constant of this first-order decay of current is the motor's electrical time constant, $T_e = L/R$.

Mechanical time constant

When the motor is subject to a step voltage across it, the *mechanical time constant* T_m measures the time it takes for the unloaded motor speed to reach 63% of its final value. Beginning from the voltage equation

$$V = k_t\omega + IR + L\frac{\mathrm{d}I}{\mathrm{d}t},$$

ignoring the inductive term, and assuming an initial speed ω_0 at the moment the voltage drops to zero, we get the differential equation

$$0 = IR + k_t\omega_0 = \frac{R}{k_t}\tau + k_t\omega_0 = \frac{JR}{k_t}\frac{\mathrm{d}\omega}{\mathrm{d}t} + k_t\omega_0,$$

where we used $\tau = Jd\omega/dt$, where J is the inertia of the motor. We can rewrite this equation as

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = -\frac{k_t^2}{JR}\omega_0$$

with solution

$$\omega(t) = \omega_0\,\mathrm{e}^{-t/T_m},$$

with a time constant of $T_m = JR/k_t^2$. If the motor is attached to a load that increases the inertia J, the mechanical time constant increases.

Short-circuit damping

When the terminals of the motor are shorted together, the voltage equation (ignoring inductance) becomes

$$0 = k_t\omega + IR = k_t\omega + \frac{\tau}{k_t}R$$

or

$$\tau = -B\omega = -\frac{k_t^2}{R}\omega,$$

where $B = k_t^2/R$ is the short-circuit damping. A spinning motor is slowed more quickly by shorting its terminals together, compared to leaving the terminals open circuit, due to this damping.

25.7 Motor Data Sheet

Motor manufacturers summarize motor properties described above in a speed-torque curve and in a data sheet similar to the one in Figure 25.14. When you buy a motor second-hand or surplus, you may need to measure these properties yourself. We will use all SI units, which is not the case on most motor data sheets.

Many of these properties have been introduced already. Below we describe some methods for estimating them.

Experimentally Characterizing a Brushed DC Motor

Given a mystery motor with an encoder, you can use a function generator, oscilloscope, multimeter and perhaps some resistors and capacitors to estimate most of the important properties of the motor. Below are some suggested methods; you may be able to devise others.

Terminal resistance R

You can measure R with a multimeter. The resistance may change as you rotate the shaft by hand, as the brushes move to new positions on the commutator. You should record the minimum resistance you can reliably find. A better choice, however, may be to measure the current when the motor is stalled.

Torque constant k_t

You can measure this by spinning the shaft of the motor, measuring the back-emf at the motor terminals, and measuring the rotation rate ω using the encoder. Or, if friction losses are

Motor Characteristic	Symbol	Value	Units	Comments
Terminal resistance	R		Ω	Resistance of the motor windings. May change as brushes slide over commutator segments. Increases with heat.
Torque constant	k_t		Nm/A	The constant ratio of torque produced to current through the motor.
Electrical constant	k_e		Vs/rad	Same numerical value as the torque constant (in SI units). Also called voltage or back-emf constant.
Speed constant	k_s		rad/(Vs)	Inverse of electrical constant.
Motor constant	k_m		Nm/$\sqrt{\text{W}}$	Torque produced per square root of power dissipated by the coils.
Max continuous current	I_{cont}		A	Max continuous current without overheating.
Max continuous torque	τ_{cont}		Nm	Max continuous torque without overheating.
Short-circuit damping	B		Nms/rad	Not included in most data sheets, but useful for motor braking (and haptics).
Terminal inductance	L		H	Inductance due to the coils.
Electrical time constant	Te		s	The time for the motor current to reach 63% of its final value. Equal to L/R.
Rotor inertia	J		kgm^2	Often given in units gcm^2.
Mechanical time constant	Tm		s	The time for the motor to go from rest to 63% of its final speed under constant voltage and no load. Equal to JR/kt^2.
Friction				Not included in most data sheets. See explanation.
Values at Nominal Voltage				
Nominal voltage	V_{nom}		V	Should be chosen so the no-load speed is safe for brushes, commutator, and bearings.
Power rating	P		W	Output power at the nominal operating point (max continuous torque).
No-load speed	ω_0		rad/s	Speed when no load and powered by $V\text{nom}$. Usually given in rpm (revs/min, sometimes m^{-1}).
No-load current	I_0		A	The current required to spin the motor at the no-load condition. Nonzero because of friction torque.
Stall current	I		A	Same as starting current, $V\text{nom}/R$.
Stall torque	τ_{stall}		Nm	The torque achieved at the nominal voltage when the motor is stalled.
Max mechanical power	$P_{\max}$		W	The max mechanical power output at the nominal voltage (including short-term operation).
Max efficiency	η		%	The maximum efficiency achievable in converting electrical to mechanical power.

Figure 25.14
A sample motor data sheet, with values to be filled in.

negligible, a good approximation is V_{nom}/ω_0. This eliminates the need to spin the motor externally.

Electrical constant k_e

Identical to the torque constant in SI units. The torque constant k_t is often expressed in units of Nm/A or mNm/A or English units like oz-in/A, and often k_e is given in V/rpm, but k_t and k_e have identical numerical values when expressed in Nm/A and Vs/rad, respectively.

Speed constant k_s

Just the inverse of the electrical constant.

Motor constant k_m

The motor constant is calculated as $k_m = k_t/\sqrt{R}$.

Max continuous current I_{cont}

This is determined by thermal considerations, which are not easy to measure. It is typically less than half the stall current.

Max continuous torque τ_{cont}

This is determined by thermal considerations, which are not easy to measure. It is typically less than half the stall torque.

Short-circuit damping B

This is most easily calculated from estimates of R and k_t: $B = k_t^2/R$.

Terminal inductance L

There are several ways to measure inductance. One approach is to add a capacitor in parallel with the motor and measure the oscillation frequency of the resulting RLC circuit. For example, you could build the circuit shown in Figure 25.15, where a good choice for C may be 0.01 or 0.1 μF. The motor acts as a resistor and inductor in series; back-emf will not be an issue, because the motor will be powered by tiny currents at high frequency and therefore will not move.

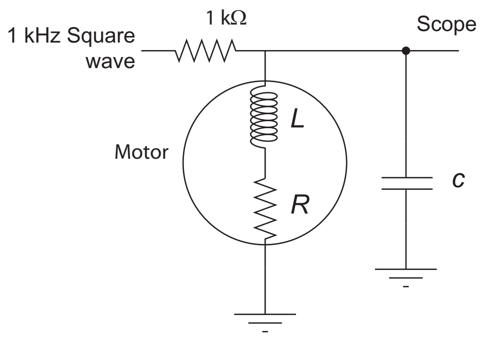


Figure 25.15
Using a capacitor to create an RLC circuit to measure motor inductance.

Use a function generator to put a 1 kHz square wave between 0 and 5 V at the point indicated. The 1 kΩ resistor limits the current from the function generator. Measure the voltage with an oscilloscope where indicated. You should be able to see a decaying oscillatory response to the square wave input when you choose the right scales on your scope. Measure the frequency of the oscillatory response. Knowing C and that the natural frequency of an RLC circuit is $\omega_n = 1/\sqrt{LC}$ in rad/s, estimate L.

Let us think about why we see this response. Say the input to the circuit has been at 0 V for a long time. Then your scope will also read 0 V. Now the input steps up to 5 V. After some time, in steady state, the capacitor will be an open circuit and the inductor will be a closed circuit (wire), so the voltage on the scope will settle to $5\text{ V} \times (R/(1000 + R))$—the two resistors in the circuit set the final voltage. Right after the voltage step, however, all current goes to charge the capacitor (as the zero current through the inductor cannot change discontinuously). If the inductor continued to enforce zero current, the capacitor would charge to 5 V. As the voltage across the capacitor grows, however, so does voltage across the inductor, and therefore so does the rate of change of current that must flow through the inductor (by the relation $V_L + V_R = V_C$ and the constitutive law $V_L = L\,\mathrm{d}I/\mathrm{d}t$). Eventually the integral of this rate of change dictates that all current is redirected to the inductor, and in fact the capacitor will have to provide current to the inductor, discharging itself. As the voltage across the capacitor drops, though, the voltage across the inductor will eventually become negative, and therefore the rate of change of current through the inductor will become negative. And so on, to create the oscillation. If R were large, i.e., if the circuit were heavily damped, the oscillation would die quickly, but you should be able to see it.

Note that you are seeing a damped oscillation, so you are actually measuring a damped natural frequency. But the damping is low if you are seeing at least a couple of cycles of oscillation, so the damped natural frequency is nearly indistinguishable from the undamped natural frequency.

Electrical time constant T_e

The electrical time constant can be calculated from L and R as $T_e = L/R$.

Rotor inertia J

The rotor inertia can be estimated from measurements of the mechanical time constant T_m, the torque constant k_t, and the resistance R. Alternatively, a ballpark estimate can be made based on the mass of the motor, a guess at the portion of the mass that belongs to the spinning rotor, a guess at the radius of the rotor, and a formula for the inertia of a uniform density cylinder. Or, more simply, consult a data sheet for a motor of similar size and mass.

Mechanical time constant T_m

The time constant can be measured by applying a constant voltage to the motor, measuring the velocity, and determining the time it takes to reach 63% of final speed. Alternatively, you could make a reasonable estimate of the rotor inertia J and calculate $T_m = JR/k_t^2$.

Friction

Friction torque arises from the brushes sliding on the commutator and the motor shaft spinning in its bearings, and it may depend on external loads. A typical model of friction includes both Coulomb friction and viscous friction, written

$$\tau_{\text{fric}} = b_0 \,\text{sgn}(\omega) + b_1\omega,$$

where b_0 is the Coulomb friction torque (sgn(ω) just returns the sign of ω) and b_1 is a viscous friction coefficient. At no load, $\tau_{\text{fric}} = k_t I_0$. An estimate of each of b_0 and b_1 can be made by running the motor at two different voltages with no load.

Nominal voltage V_{nom}

This is the specification you are most likely to know for an otherwise unknown motor. It is sometimes printed right on the motor itself. This voltage is just a recommendation; the real issue is to avoid overheating the motor or spinning it at speeds beyond the recommended value for the brushes or bearings. Nominal voltage cannot be measured, but a typical no-load speed for a brushed DC motor is between 3000 and 10,000 rpm, so the nominal voltage will often give a no-load speed in this range.

Power rating P

The power rating is the mechanical power output at the max continuous torque.

No-load speed ω_0

You can determine ω_0 by measuring the unloaded motor speed when powered with the nominal voltage. The amount that this is less than V_{nom}/k_t can be attributed to friction torque.

No-load current I_0

You can determine I_0 by using a multimeter in current measurement mode.

Stall current I_{stall}

Stall current is sometimes called starting current. You can estimate this using your estimate of R. Since R may be difficult to measure with a multimeter, you can instead stall the motor shaft and use your multimeter in current sensing mode, provided the multimeter can handle the current.

Stall torque τ_{stall}

This can be obtained from k_t and I_{stall}.

Max mechanical power P_{max}

The max mechanical power occurs at $\frac{1}{2}\tau_{\text{stall}}$ and $\frac{1}{2}\omega_0$. For most motor data sheets, the max mechanical power occurs outside the continuous operation region.

Max efficiency η_{max}

Efficiency is defined as the power out divided by the power in, $\tau_{\text{out}}\omega/(IV)$. The wasted power is due to coil heating and friction losses. Maximum efficiency can be estimated using the no-load current I_0 and the stall current I_{stall}, as discussed in Section 25.4.

25.8 Chapter Summary

- The Lorentz force law says that a current-carrying conductor in a constant magnetic field feels a net force according to

$$\mathbf{F} = \ell \mathbf{I} \times \mathbf{B},$$

  where ℓ is the length of the conductor in the field, **I** is the current vector, and **B** is the (constant) magnetic field vector.
- A brushed DC motor consists of multiple current-carrying coils attached to a rotor, and magnets on the stator to create a magnetic field. Current is transmitted to the coils by two brushes connected to the stator sliding over a commutator ring attached to the rotor. Each coil attaches to two different commutator segments.
- The voltage across a motor's terminals can be expressed as

$$V = k_t\omega + IR + L\frac{\mathrm{d}I}{\mathrm{d}t},$$

  where k_t is the torque constant and $k_t\omega$ is the back-emf.
- The speed-torque curve is obtained by plotting the steady-state speed as a function of torque for a given motor voltage V,

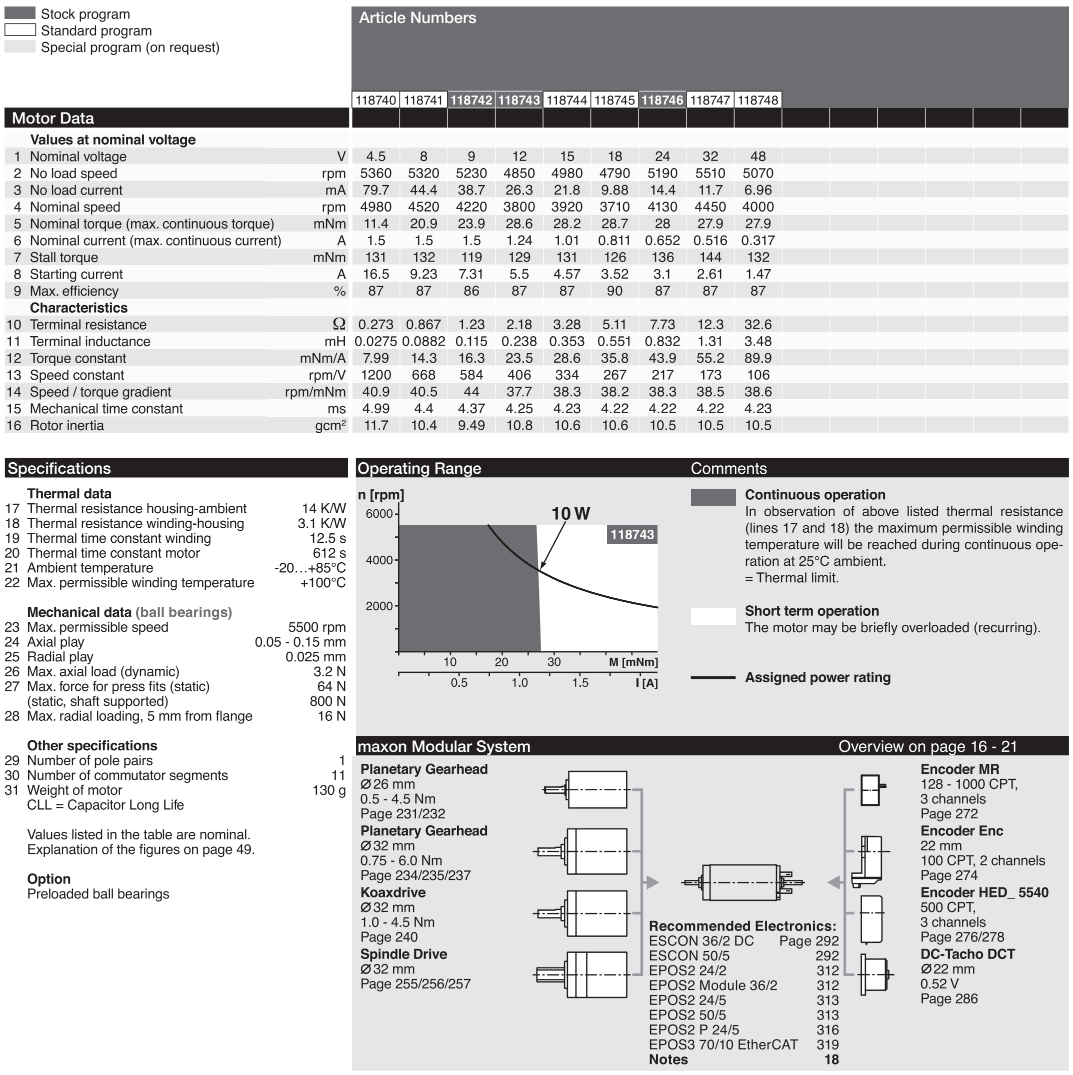
$$\omega = \frac{1}{k_t} V - \frac{R}{k_t^2} \tau.$$

The maximum speed (at $\tau = 0$) is called the no-load speed ω_0 and the maximum torque (at $\omega = 0$) is called the stall torque τ_{stall}.

- The continuous operating region of a motor is defined by the maximum current I the motor coils can conduct continuously without overheating due to I^2R power dissipation.
- The mechanical power $\tau\omega$ delivered by a motor is maximized at half the stall torque and half the no-load speed, $P_{\max} = \frac{1}{4}\tau_{\text{stall}}\omega_0$.
- A motor's electrical time constant $T_e = L/R$ is the time needed for current to reach 63% of its final value in response to a step input in voltage.
- A motor's mechanical time constant $T_m = JR/k_t^2$ is the time needed for the motor speed to reach 63% of its final value in response to a step change in voltage.

25.9 Exercises

1. Assume a DC motor with a five-segment commutator. Each segment covers 70° of the circumference of the commutator circle. The two brushes are positioned at opposite ends of the commutator circle, and each makes contact with 10° of the commutator circle.
   a. How many separate coils does this motor likely have? Explain.
   b. Choose one of the motor coils. As the rotor rotates 360°, what is the total angle over which that coil is energized? (For example, an answer of 360° means that the coil is energized at all angles; an answer of 180° means that the coil is energized at half of the motor positions.)
2. Figure 25.16 gives the data sheet for the 10 W Maxon RE 25 motor. The columns correspond to different windings.
   a. Draw the speed-torque curve for the 12 V version of the motor, indicating the no-load speed (in rad/s), the stall torque, the nominal operating point, and the rated power of the motor.
   b. Explain why the torque constant is different for the different versions of the motor.
   c. Using other entries in the table, calculate the maximum efficiency $\eta_{\max}$ of the 12 V motor and compare to the value listed.
   d. Calculate the electrical time constant T_e of the 12 V motor. What is the ratio to the mechanical time constant T_m?
   e. Calculate the short-circuit damping B for the 12 V motor.
   f. Calculate the motor constant k_m for the 12 V motor.
   g. How many commutator segments do these motors have?
   h. Which versions of these motors are likely to be in stock?



▨ Stock program
☐ Standard program
░ Special program (on request)

	Motor Data		118740	118741	118742	118743	118744	118745	118746	118747	118748
	Values at nominal voltage										
1	Nominal voltage	V	4.5	8	9	12	15	18	24	32	48
2	No load speed	rpm	5360	5320	5230	4850	4980	4790	5190	5510	5070
3	No load current	mA	79.7	44.4	38.7	26.3	21.8	9.88	14.4	11.7	6.96
4	Nominal speed	rpm	4980	4520	4220	3800	3920	3710	4130	4450	4000
5	Nominal torque (max. continuous torque)	mNm	11.4	20.9	23.9	28.6	28.2	28.7	28	27.9	27.9
6	Nominal current (max. continuous current)	A	1.5	1.5	1.5	1.24	1.01	0.811	0.652	0.516	0.317
7	Stall torque	mNm	131	132	119	129	131	126	136	144	132
8	Starting current	A	16.5	9.23	7.31	5.5	4.57	3.52	3.1	2.61	1.47
9	Max. efficiency	%	87	87	86	87	87	90	87	87	87
	Characteristics										
10	Terminal resistance	Ω	0.273	0.867	1.23	2.18	3.28	5.11	7.73	12.3	32.6
11	Terminal inductance	mH	0.0275	0.0882	0.115	0.238	0.353	0.551	0.832	1.31	3.48
12	Torque constant	mNm/A	7.99	14.3	16.3	23.5	28.6	35.8	43.9	55.2	89.9
13	Speed constant	rpm/V	1200	668	584	406	334	267	217	173	106
14	Speed / torque gradient	rpm/mNm	40.9	40.5	44	37.7	38.3	38.2	38.3	38.5	38.6
15	Mechanical time constant	ms	4.99	4.4	4.37	4.25	4.23	4.22	4.22	4.22	4.23
16	Rotor inertia	gcm^2	11.7	10.4	9.49	10.8	10.6	10.6	10.5	10.5	10.5

Specifications

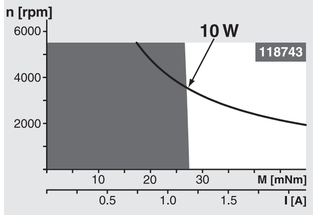
	Specifications	
	Thermal data	
17	Thermal resistance housing-ambient	14 K/W
18	Thermal resistance winding-housing	3.1 K/W
19	Thermal time constant winding	12.5 s
20	Thermal time constant motor	612 s
21	Ambient temperature	-20…+85°C
22	Max. permissible winding temperature	+100°C
	Mechanical data (ball bearings)	
23	Max. permissible speed	5500 rpm
24	Axial play	0.05 - 0.15 mm
25	Radial play	0.025 mm
26	Max. axial load (dynamic)	3.2 N
27	Max. force for press fits (static)	64 N
	(static, shaft supported)	800 N
28	Max. radial loading, 5 mm from flange	16 N
	Other specifications	
29	Number of pole pairs	1
30	Number of commutator segments	11
31	Weight of motor	130 g

CLL = Capacitor Long Life

Values listed in the table are nominal.
Explanation of the figures on page 49.

Option
Preloaded ball bearings

Operating Range



Comments

▨ **Continuous operation**
In observation of above listed thermal resistance (lines 17 and 18) the maximum permissible winding temperature will be reached during continuous operation at 25°C ambient.
= Thermal limit.

☐ **Short term operation**
The motor may be briefly overloaded (recurring).

— **Assigned power rating**

maxon Modular System Overview on page 16 - 21

Planetary Gearhead
Ø26 mm
0.5 - 4.5 Nm
Page 231/232
Planetary Gearhead
Ø32 mm
0.75 - 6.0 Nm
Page 234/235/237
Koaxdrive
Ø32 mm
1.0 - 4.5 Nm
Page 240
Spindle Drive
Ø32 mm
Page 255/256/257

Recommended Electronics:	
ESCON 36/2 DC	Page 292
ESCON 50/5	292
EPOS2 24/2	312
EPOS2 Module 36/2	312
EPOS2 24/5	313
EPOS2 50/5	313
EPOS2 P 24/5	316
EPOS3 70/10 EtherCAT	319
Notes	**18**

Encoder MR
128 - 1000 CPT,
3 channels
Page 272
Encoder Enc
22 mm
100 CPT, 2 channels
Page 274
Encoder HED_ 5540
500 CPT,
3 channels
Page 276/278
DC-Tacho DCT
Ø22 mm
0.52 V
Page 286

Figure 25.16
The data sheet for the Maxon RE 25 motor. The columns correspond to different windings for different nominal voltages. (Image courtesy of Maxon Precision Motors. Motor data is subject to change at any time; consult maxonmotorusa.com for the latest data sheets.)

i. (Optional) Motor manufacturers may specify slightly different continuous and intermittent operating regions than the ones described in this chapter. For example, the limit of the continuous operating region is not quite vertical in the speed-torque plot of Figure 25.16. Come up with a possible explanation, perhaps using online resources.

3. There are 21 entries on the motor data sheet from Section 25.7. Let us assume zero friction, so we ignore the last entry. To avoid thermal tests, you may also assume a maximum continuous power that the motor coils can dissipate as heat before overheating.

Of the 20 remaining entries, under the assumption of zero friction, how many independent entries are there? That is, what is the minimum number N of entries you need to be able to fill in the rest of the entries? Give a set of N independent entries from which you can derive the other $20 - N$ dependent entries. For each of the $20 - N$ dependent entries, give the equation in terms of the N independent entries. For example, V_{nom} and R will be two of the N independent entries, from which we can calculate the dependent entry $I_{\mathrm{stall}} = V_{\mathrm{nom}}/R$.

4. This exercise is an experimental characterization of a motor. For this exercise, you need a low-power motor (preferably without a gearhead to avoid high friction) with an encoder. You also need a multimeter, oscilloscope, and a power source for the encoder and motor. Make sure the power source for the motor can provide enough current when the motor is stalled. A low-voltage battery pack is a good choice.
   a. Spin the motor shaft by hand. Get a feel for the rotor inertia and friction. Try to spin the shaft fast enough that it continues spinning briefly after you let go of it.
   b. Now short the motor terminals by electrically connecting them. Spin again by hand, and try to spin the shaft fast enough that it continues spinning briefly after you let go of it. Do you notice the short-circuit damping?
   c. Try measuring your motor's resistance using your multimeter. It may vary with the angle of the shaft, and it may not be easy to get a steady reading. What is the minimum value you can get reliably? To double-check your answer, you can power your motor and use your multimeter to measure the current as you stall the motor's shaft by hand.
   d. Attach one of your motor's terminals to scope ground and the other to a scope input. Spin the motor shaft by hand and observe the motor's back-emf.
   e. Power the motor's encoder, attach the A and B encoder channels to your oscilloscope, and make sure the encoder ground and scope ground are connected together. Do *not* power the motor. (The motor inputs should be disconnected from anything.) Spin the motor shaft by hand and observe the encoder pulses, including their relative phase.
   f. Now power your motor with a low-voltage battery pack. Given the number of lines per revolution of the encoder, and the rate of the encoder pulses you observe on your scope, calculate the motor's no-load speed for the voltage you are using.
   g. Work with a partner. Couple your two motor shafts together by tape or flexible tubing. (This may only work if your motor has no gearhead.) Now plug one terminal of one of the motors (we shall call it the *passive* motor) into one channel of a scope, and plug the other terminal of the passive motor into GND of the same scope. Now power the other motor (the *driving* motor) with a battery pack so that both motors spin. Measure the speed of the passive motor by looking at its encoder count rate on your scope. Also measure its back-emf. With this information, calculate the passive motor's torque constant k_t.
5. Using techniques discussed in this chapter, or techniques you come up with on your own, create a data sheet with all 21 entries for your nominal voltage. Indicate how you

calculated the entry. (Did you do an experiment for it? Did you calculate it from other entries? Or did you estimate by more than one method to cross-check your answer?) For the friction entry, you can assume Coulomb friction only—the friction torque opposes the rotation direction ($b_0 \neq 0$), but is independent of the speed of rotation ($b_1 = 0$). For your measurement of inductance, turn in an image of the scope trace you used to estimate ω_n and L, and indicate the value of C that you used.

If there are any entries you are unable to estimate experimentally, approximate, or calculate from other values, simply say so and leave that entry blank.

6. Based on your data sheet from above, draw the speed-torque curves described below, and answer the associated questions. Do not do any experiments for this exercise; just extrapolate your previous results.
   a. Draw the speed-torque curve for your motor. Indicate the stall torque and no-load speed. Assume a maximum power the motor coils can dissipate continuously before overheating and indicate the continuous operating regime. Given this, what is the power rating P for this motor? What is the max mechanical power $P_{\max}$?
   b. Draw the speed-torque curve for your motor assuming a nominal voltage four times larger than in Exercise 6a. Indicate the stall torque and no-load speed. What is the max mechanical power $P_{\max}$?
7. You are choosing a motor for the last joint of a new direct-drive robot arm design. (A direct-drive robot does not use gearheads on the motors, creating high speeds with low friction.) Since it is the last joint of the robot, and it has to be carried by all the other joints, you want it to be as light as possible. From the line of motors you are considering from your favorite motor manufacturer, you know that the mass increases with the motor's power rating. Therefore you are looking for the lowest power motor that works for your specifications. Your specifications are that the motor should have a stall torque of at least 0.1 Nm, should be able to rotate at least 5 revolutions per second when providing 0.01 Nm, and the motor should be able to operate continuously while providing 0.02 Nm. Which motor do you choose from Table 25.1? Give a justification for your answer.
8. The speed-torque curve of Figure 25.8 is drawn for the positive speed and positive torque quadrant of the speed-torque plane. In this exercise, we will draw the motor's operating region for all four quadrants. The power supply used to drive the motor is 24 V, and assume the H-bridge motor controller (discussed in Chapter 27) can use that power supply to create any average voltage across the motor between −24 and 24 V. The motor's resistance is 1 Ω and the torque constant is 0.1 Nm/A. Assume the motor has zero friction.

   a. Draw the four-quadrant speed-torque operating region for the motor assuming the 24 V power supply (and the H-bridge driver) has no limit on current. Indicate the torque and speed values where the boundaries of the operating region intersect the $\omega = 0$ and $\tau = 0$ axes. Assume there are no other speed or torque constraints on the motor except for the one due to the 24 V limit of the power supply. (Hint: the operating region is unbounded in both speed and torque!)

Table 25.1: Motors to choose from

Assigned power rating	W	3	10	20	90
Nominal voltage	V	15	15	15	15
No load speed	rpm	13,400	4980	9660	7180
No load current	mA	36.8	21.8	60.8	247
Nominal speed	rpm	10,400	3920	8430	6500
Max continuous torque	mNm	2.31	28.2	20.5	73.1
Max continuous current	mA	259	1010	1500	4000
Stall torque	mNm	10.5	131	225	929
Stall current	mA	1030	4570	15,800	47,800
Max efficiency	%	65	87	82	83
Terminal resistance	Ohm	14.6	3.28	0.952	0.314
Terminal inductance	mH	0.486	0.353	0.088	0.085
Torque constant	mNm/A	10.2	28.6	14.3	19.4
Speed constant	rpm/V	932	334	670	491
Mechanical time constant	ms	7.51	4.23	4.87	5.65
Rotor inertia	gcm^2	0.541	10.6	10.4	68.1
Max permissible speed	rpm	16,000	5500	14,000	12,000
Cost	USD	88	228	236	239

Note that sometimes the "Assigned power rating" is different from the mechanical power output at the nominal operating point, for manufacturer-specific reasons. The meanings of the other terms in the table are unambiguous.

b. Update the operating region with the constraint that the power supply can provide a maximum current of 30 A. What is the maximum torque that can be generated using this power supply, and what are the maximum and minimum motor speeds possible at this maximum torque? What is the largest back-emf voltage that can be achieved?

c. Update the operating region with the constraint that the maximum recommended speed for the motor brushes and shaft bearings is 250 rad/s.

d. Update the operating region with the constraint that the maximum recommended torque at the motor shaft is 5 Nm.

e. Update the operating region to show the continuous operating region, assuming the maximum continuous current is 10 A.

f. We typically think of a motor as consuming electrical power ($IV > 0$, or "motoring") and converting it to mechanical power, but it can also convert mechanical power to electrical power ($IV < 0$, or "regenerating"). This occurs in electric car braking systems, for example. Update the operating region to show the portion where the motor is consuming electrical power and the portion where the motor is generating electrical power.

Further Reading

Hughes, A., & Drury, B. (2013). *Electric motors and drives: Fundamentals, types and applications* (4th ed.). Amsterdam: Elsevier.

Maxon DC motor RE 25, ø 25 mm, graphite brushes, 20 Watt. (2015). Maxon.

APPENDIX A

A Crash Course in C

This appendix provides an introduction to C for readers with no C experience but some experience in another programming language. It is not intended as a complete reference; plenty of great C resources already exist and provide a more complete picture. This appendix applies to C in general, not just C on the Microchip PIC32. We recommend that you start by programming your computer so you can experiment with C without needing extra equipment or complication.

A.1 Quick Start in C

To start with C, you need a computer, a text editor, and a C compiler. You use the text editor to write your C program, a plain text file with a name ending with the extension `.c` (e.g., `myprog.c`). The C compiler converts this program into machine code that your computer can execute. There are many free C compilers available; we recommend the `gcc` C compiler, which is part of the GNU Compiler Collection (GCC, found at `http://gcc.gnu.org`). GCC is available for Windows, Mac OS, and Linux. For Windows, you can download the GCC collection in MinGW.[1] If the installation asks you about what tools to install, make sure to include the `make` tools. For Mac OS, you can download the full Xcode environment from the Apple Developers website. This installation is multiple gigabytes; however, you can opt to install only the "Command Line Tools for Xcode," which is smaller and more than sufficient for getting started with C (and for this appendix).

Many C installations come with an Integrated Development Environment (IDE) complete with text editor, menus, and graphical tools to assist you with your programming projects. Every IDE is different, and what we cover in this appendix does not require a sophisticated IDE. We therefore use only *command line tools*, meaning that we initiate compilation and run the program by typing at the command line. In Mac OS, the command line can be accessed from the Terminal program. In Windows, you can access the command line by searching for `cmd` or `command prompt`. Linux users should run a shell such as bash.

[1] You are also welcome to use Visual C from Microsoft. The command line compile command will look a bit different than what you see in this appendix.

To use the command line, you must learn some command line instructions. The Mac operating system is built on top of Unix, which is similar to Linux, so Mac/Unix/Linux use the same syntax. Windows uses slightly different commands. See the table of a few useful commands below. You can find more information on how to use these commands as well as others by searching for command line commands in Unix, Linux, or Windows.

Function	Mac/Unix/Linux	Windows
Show current directory	`pwd`	`cd`
List directory contents	`ls`	`dir`
Make subdirectory `newdir`	`mkdir newdir`	`mkdir newdir`
Change to subdirectory `newdir`	`cd newdir`	`cd newdir`
Move "up" to parent directory	`cd ..`	`cd ..`
Copy `file` to `filenew`	`cp file filenew`	`copy file filenew`
Delete file `file`	`rm file`	`del file`
Delete directory `dir`	`rmdir dir`	`rmdir dir`
Help on using command `cmd`	`man cmd`	`cmd /?`

Following the long-established programming tradition, your first C program will simply print "Hello world!" to the screen. Use a text editor to create the file `HelloWorld.c`:

```
#include <stdio.h>
int main(void) {
  printf("Hello world!\n");
  return 0;
}
```

Possible text editors include Notepad++ for Windows, TextWrangler for Mac OS, and Gedit for Linux. You can also try vim or emacs, though they are not easy to get started with! Whichever editor you use, you should save your file as plain text, not rich text or any other formatted text.

To compile your program, navigate from the command line to the directory where the program sits. Then, assuming your command prompt appears as `>`, type the following at the prompt:

```
> gcc HelloWorld.c -o HelloWorld
```

This command should create the executable file `HelloWorld` in the same directory. (The argument after the `-o` output flag is the name of the executable file to be created from `HelloWorld.c`.) Now, to execute the program, type

Windows: `> HelloWorld`
Linux/MacOS: `> ./HelloWorld`

For Linux/MacOS users, the "`.`" is shorthand for "current directory," and the `./` tells your computer to look in the current directory for `HelloWorld`. Windows implicitly searches the current directory for executables, so you need not explicitly specify it.

If you have succeeded in getting this far, your C installation works and you can proceed. If not, you may need to get help from friends or the internet.

A.2 Overview

If you are familiar with a high-level language like MATLAB or Python, you may know about loops, functions, and other programming constructs. You will see that although C's syntax is different, the same concepts translate to C. Rather than starting with basic loops, if statements, and functions, we begin by focusing on important concepts that you must master in C but which you probably have not dealt with in a language such as MATLAB or Python.

- **Memory, addresses, and pointers.** Variables are stored at particular *addresses* in memory as bits (0's and 1's). In C, unlike in MATLAB or Python, it is often useful to access a variable's memory address. Special variables called *pointers* contain the address of another variable and can be used to access the contents of that address. Although powerful, pointers can also be dangerous; misusing them can cause all sorts of bugs, which is why many higher-level languages forgo them completely.
- **Data types.** In MATLAB, for example, you can simply type `a = 1; b = [1.2 3.1416]; c = [1 2; 3 4]; s = 'a string'`. MATLAB determines that `a` is a scalar, `b` is a vector with two elements, `c` is a 2 × 2 matrix, and `s` is a string; automatically tracks the variable's type (e.g., a list of numbers for a vector or a list of characters for a string); and sets aside, or *allocates*, memory to store them. In C, on the other hand, you must first *define* the variable before you ever use it. To use a vector, for example, you must specify the number and *data type* of its elements—integers or decimal numbers (floating point). The variable definition tells the C compiler how much memory it needs to store the vector, the address of each element, and how to interpret the bits of each element (as integers or floating point numbers, for example).
- **Compiling.** MATLAB programs are typically *interpreted*: the commands are converted to machine code and executed while the program is running. C programs, on the other hand, are *compiled*, i.e., converted to machine-code in advance. This process consists of several steps whose purpose is to turn your C program into machine-code before it ever runs. The compiler can identify some errors and warn you about other questionable code. Compiled code typically runs faster than interpreted code, since the translation to machine code is done in advance.[2]

Each of these concepts is described in Section A.3 without going into detail on C syntax. In Section A.4 we look at sample programs to introduce syntax, then offer more detailed explanations.

[2] The distinction between compiled and interpreted programs is narrowing: many interpreted languages are actually just-in-time (JIT) compiled, that is program chunks are compiled in advance right before they are needed.

A.3 Important Concepts in C

We begin our discussion of C with this caveat:

C consists of an evolving set of standards for a programming language, and any specific C installation is an "implementation" of C. While C standards require certain behavior from all implementations, some details are implementation-dependent. For example, the number of bytes used for some data types is non-standard. We sometimes ignore these details in favor of clarity and succinctness. Platform- and compiler-specific results are from gcc 4.9.2 running on an x86_64 compatible processor.

A.3.1 Data Types

Binary and hexadecimal

On a computer, programs and data are represented by sequences of 0's and 1's. A 0 or 1 may be represented by two different voltages (low and high) held and controlled by a logic circuit, for example. Each of these units of memory is called a **bit**.

A sequence of bits may be interpreted as a base-2 or **binary** number, just as a sequence of digits in the range 0 to 9 is commonly treated as a base-10 or **decimal** number.[3] In the decimal numbering system, a multi-digit number like 793 is interpreted as $7 * 10^2 + 9 * 10^1 + 3 * 10^0$; the rightmost column is the 10^0 (or 1's) column, the next column to the left is the 10^1 (or 10's) column, the next column to the left is the 10^2 (or 100's) column, and so on. Similarly, the rightmost column of a binary number is the 2^0 column, the next column to the left is the 2^1 column, etc. Converting the binary number `00111011` to its decimal representation, we get

$$0*2^7+0*2^6+1*2^5+1*2^4+1*2^3+0*2^2+1*2^1+1*2^0 = 32+16+8+2+1 = 59.$$

The leftmost digit in a multi-digit number is called the **most significant digit**, and the rightmost digit, corresponding to the 1's column, is called the **least significant digit**. For binary representations, these are often called the **most significant bit (msb)** and **least significant bit (lsb)**, respectively.

We specify that a sequence of numbers is base-2 by writing it as $\texttt{00111011}_2$ or `0b00111011`, where the `b` stands for "binary."

To convert a base-10 number x to binary:

1. Initialize the binary result to all zeros and k to a large integer, such that 2^k is known to be larger than x.

[3] Bit is a portmanteau of binary and digit.

2. If $2^k \le x$, place a 1 in the 2^k column of the binary number and set x to $x - 2^k$.
3. If $x = 0$ or $k = 0$, we are finished. Else set k to $k - 1$ and go to line 2.

An alternative base-10 to binary conversion algorithm builds the binary number from the rightmost to leftmost bit.

1. Divide x by 2.
2. The next digit (from right to left) is the remainder (so 1 if x is odd, 0 if x is even).
3. x = the quotient. (So if x were 5, the new x is 2, and if x were 190 the new x is 95).
4. Repeat process until $x = 0$.

Compared to base-10, base-2 has a closer connection to actual hardware. Binary can be inconvenient for human reading and writing, however, due to the large number of digits. Therefore we often group four binary digits together (taking values `0b0000` to `0b1111`, or 0 to 15 in base-10) and represent them with a single character using the numbers 0 to 9 or the letters A to F. This base-16 representation is called **hexadecimal** or hex for short:Thus we can

base-2 (binary)	base-16 (hex)	base-10 (decimal)	base-2 (binary)	base-16 (hex)	base-10 (decimal)
0000	0	0	1000	8	8
0001	1	1	1001	9	9
0010	2	2	1010	A	10
0011	3	3	1011	B	11
0100	4	4	1100	C	12
0101	5	5	1101	D	13
0110	6	6	1110	E	14
0111	7	7	1111	F	15

write the eight-digit binary number `0b00111011`, or `0011 1011`, more succinctly in hex as `3B`, or $3B_{16}$ or `0x3B` to clarify that it is a hex number. The corresponding decimal number is $3 * 16^1 + 11 * 16^0 = 59$.

Bits, bytes, and data types

Bits of memory are grouped together in groups of eight called **bytes**. A byte can be written in binary or hexadecimal (e.g., `0b00111011` or `0x3B`), and can represent values between 0 and $2^8 - 1 = 255$. Sometimes the four bits represented by a single hex digit are referred to as a **nibble**. (Get it?)

A **word** is a grouping of multiple bytes. The number of bytes in a word depends on the processor, but four-byte words are common, as with the PIC32. A word `01001101 11111010 10000011 11000111` in binary can be written in hexadecimal as `0x4DFA83C7`. The **most significant byte (MSB)** is the leftmost byte, `0x4D` in this case, and the **least significant byte**

(LSB) is the rightmost byte `0xC7`. The msb is the leftmost bit of the MSB, a `0` in this case, and the lsb is the rightmost bit of the LSB, a `1` in this case.

A byte is the smallest unit of memory that has its own **address**. The address of the byte is a number that represents the byte's location in memory. Suppose your computer has 4 gigabytes (GB), or $4 \times 2^{30} = 2^{32}$ bytes, of RAM.[4] Then to find the value stored in a particular byte, you need at least 32 binary digits (8 hex digits or 4 bytes) to specify the address.

An example showing the first eight addresses in memory is given below. Here we show the lowest address on the right, but we could have made the opposite choice.

...	7	6	5	4	3	2	1	0	Address
	11001101	00100111	01110001	01010111	01010011	00011110	10111011	01100010	Value

Assume that the byte at address 4 is part of the representation of a variable. Do these 0's and 1's represent an integer, or part of an integer? A number with a fractional component? Something else?

The answer lies in the **type** of the variable at that address. In C, before you use a variable, you must *define* it and its type, telling the compiler how many bytes to allocate for the variable (its size) and how to interpret the bits.[5]

The most common data types come in two flavors: integers and floating point numbers (numbers with a decimal point). Of the integers, the two most common types are `char`, often used to represent keyboard characters, and `int`.[6] Of the floating point numbers, the two most common types are `float` and `double`. As we will see shortly, a `char` uses 1 byte and an `int` usually uses 4, so two possible interpretations of the data held in the eight memory addresses could be

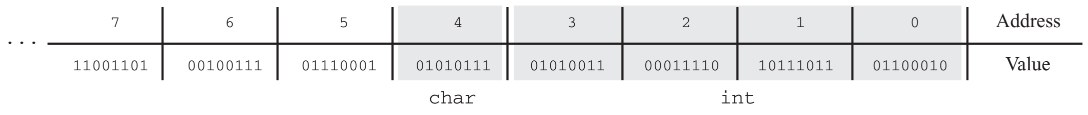
...	7	6	5	4	3	2	1	0	Address
	11001101	00100111	01110001	01010111	01010011	00011110	10111011	01100010	Value
		int						char	

[4] In common usage, a kilobyte (KB) is $2^{10} = 1024$ bytes, a megabyte (MB) is $2^{20} = 1,048,576$ bytes, a gigabyte is $2^{30} = 1,073,741,824$ bytes, and a terabyte (TB) is $2^{40} = 1,099,511,627,776$ bytes. To remove confusion with the common SI prefixes that use powers of 10 instead of powers of 2, these are sometimes referred to instead as kibibyte, mebibyte, gibibyte, and tebibyte, where the "bi" refers to "binary."

[5] In C you can *declare* or *define* a variable. They use similar syntax, but a declaration simply gives the name and the type of the variable, while a definition also allocates the memory to hold it. We avoid using the distinction for now and just call everything a definition.

[6] `char` is derived from the word "character." People pronounce `char` variously as "car" (as in "driving the car"), "care" (a shortening of "character"), and "char" (as in charcoal), and some just punt and say "character."

where byte 0 is used to represent a `char` and bytes 4-7 are used to represent an `int`, or



where bytes 0-3 are used to represent an `int` and byte 4 represents a `char`. Fortunately we do not usually have to worry about how variables are packed into memory.

Below we describe the common data types. Although the number of bytes used for each type is not the same for every processor, the numbers given are common on modern computers. (Differences for the PIC32 are noted in Table A.1.) Example syntax for defining variables is also given. Note that most C statements end with a semicolon.

`char`
Example definition:
`char ch;`

This syntax defines a variable named `ch` to be of type `char`. `char`s are the smallest data type, using only one byte. They are often interpreted according to the "ASCII table" (pronounced "ask-key"), the American Standard Code for Information Interchange, which maps the values 0 to 127 to letters, numbers, and other characters (Figure A.1). (The values 128 to 255 map to an "extended" ASCII table.) For example, the values 48 to 57 map to the characters '0' to '9', 65 to 90 map to the uppercase letters 'A' to 'Z', and 97 to 122 map to the lowercase letters 'a' to 'z'. The assignments

```
ch = 'a';
```

and

```
ch = 97;
```

are equivalent, as C equates characters inside single quotation marks with their ASCII table numerical value.

Depending on the C implementation, `char` may be treated by default as `unsigned`, i.e., taking values from 0 to 255, or `signed`, taking values from −128 to 127. If you use the `char` to represent a standard ASCII character, the distinction does not matter. If, however, you use the `char` data type for integer math on small integers, you should use the specifier `signed` or `unsigned`, as appropriate. For example, we could use the following definitions, where everything after `//` is a comment:

ASCII Table

0 NULL	16 DLE	32 space	48 0	64 @	80 P	96 ‘	112 p
1 SOH	17 DC1	33 !	49 1	65 A	81 Q	97 a	113 q
2 STX	18 DC2	34 "	50 2	66 B	82 R	98 b	114 r
3 ETX	19 DC3	35 #	51 3	67 C	83 S	99 c	115 s
4 EOT	20 DC4	36 $	52 4	68 D	84 T	100 d	116 t
5 ENQ	21 NAK	37 %	53 5	69 E	85 U	101 e	117 u
6 ACK	22 SYN	38 &	54 6	70 F	86 V	102 f	118 v
7 BELL	23 ETB	39 ’	55 7	71 G	87 W	103 g	119 w
8 BACKSPACE	24 CAN	40 (	56 8	72 H	88 X	104 h	120 x
9 TAB	25 EM	41)	57 9	73 I	89 Y	105 i	121 y
10 NEWLINE	26 SUB	42 *	58 :	74 J	90 Z	106 j	122 z
11 VT	27 ESC	43 +	59 ;	75 K	91 [	107 k	123 {
12 FORMFEED	28 FS	44 ,	60 <	76 L	92 \	108 l	124 |
13 RETURN	29 GS	45 -	61 =	77 M	93]	109 m	125 }
14 SO	30 RS	46 .	62 >	78 N	94 ^	110 n	126 ~
15 SI	31 US	47 /	63 ?	79 O	95 _	111 o	127 DEL

Figure A.1
The 128 standard ASCII characters. The first 32 characters are non-printing characters and the names of most of them are obscure. Values 128 to 255 (or −128 to −1) correspond to the extended ASCII table.

```
unsigned char ch1; // ch1 can take values 0 to 255
signed char ch2;   // ch2 can take values -128 to 127
```

`int` (also known as `signed int` or `signed`)
Example definition:
```
int i,j;
signed int k;
signed n;
```

`int`s typically use four bytes (32 bits) and take values from $-(2^{31})$ to $2^{31}-1$ (approximately ± 2 billion). In the example syntax, each of `i`, `j`, `k`, and `n` are defined to be the same data type.

We can use specifiers to get the following integer data types: `unsigned int` or simply `unsigned`, a four-byte integer taking nonnegative values from 0 to $2^{32}-1$; `short int`, `short`, `signed short`, or `signed short int`, all meaning the same thing: a two-byte integer taking values from $-(2^{15})$ to $2^{15}-1$ (i.e., $-32,768$ to $32,767$); `unsigned short int` or `unsigned short`, a two-byte integer taking nonnegative values from 0 to $2^{16}-1$ (i.e., 0 to $65,535$); `long int`, `long`, `signed long`, or `signed long int`, often consisting of eight bytes and representing values from $-(2^{63})$ to $2^{63}-1$; and `unsigned long int` or `unsigned long`, an eight-byte integer taking nonnegative values from 0 to $2^{64}-1$. A `long long int` data type may also be available.

```
float
```
Example definition:
```
float x;
```

This syntax defines the variable `x` to be a four-byte "single-precision" floating point number.

```
double
```
Example definition:
```
double x;
```

This syntax defines the variable `x` to be an eight-byte "double-precision" floating point number. The data type `long double` (quadruple precision) may also be available, using 16 bytes (128 bits). These types allow the representation of larger numbers, to more decimal places, than single-precision `float`s.

The sizes of the data types, both on a typical x86_64 computer with gcc and on the PIC32, are summarized in Table A.1. Note the differences; C lets the compiler determine these details. The C99 standard introduces data types such as `int32_t` (32-bit signed integer) and `unit8_t` (8-bit unsigned integer) which are guaranteed to be the specified size across platforms and compilers.

Table A.1: Data type sizes on two different machines

Type	# bytes on x86_64	# bytes on PIC32
char	1	1
short int	2	2
int	4	4
long int	8	4
long long int	8	8
float	4	4
double	8	4
long double	16	8

Using the data types

If your program requires floating point calculations, you can choose between `float`, `double`, and `long double` data types. The advantages of smaller types are that they use less memory and computations with them (e.g., multiplies, square roots, etc.) may be faster. The advantage of the larger types is the greater precision in the representation (e.g., smaller roundoff error).

If your program needs integer calculations, you should use integer rather than floating point data types due to the higher speed of integer math and the ability to represent a larger range of integers for the same number of bytes.[7] You can decide whether to use `signed` or `unsigned char`s, or {`signed`/`unsigned`} {`short`/`long`} `int`s. The considerations are memory usage, possibly the time of the computations, and whether the type can represent a sufficient range of integer values.[8] For example, if you decide to use `unsigned char`s for integer math to save memory, and you add two of them with values 100 and 240 and assign to a third `unsigned char`, you will get a result of 84 due to *integer overflow*. This example is illustrated in the program `overflow.c` in Section A.4.

Representations of data types

A simple representation for integers is the *sign and magnitude* representation. In this representation, the msb represents the sign of the number (0 = positive, 1 = negative), and the remaining bits represent the magnitude of the number. The sign and magnitude method represents zero twice (positive and negative zero) and is not often used.

A more common representation for integers is called *two's complement*. This method also uses the msb as a sign bit, but it only has a single representation of zero. The two's complement representation of an 8-bit `char` is given below:

binary	`signed char`, **base-10**	`unsigned char`, **base-10**
00000000	0	0
00000001	1	1
00000010	2	2
00000011	3	3
⋮		
01111111	127	127
10000000	−128	128
10000001	−127	129
⋮		
11111111	−1	255

As the binary representation is incremented, the two's complement (signed) interpretation of the binary number also increments, until it "wraps around" to the most negative value when the msb becomes 1 and all other bits are 0. The signed value then resumes incrementing until

[7] Just as a four-byte `float` can represent fractional values that a four-byte `int` cannot, a four-byte `int` can represent more integers than a four-byte `float` can. See the type conversion example program `typecast.c` in Section A.4 for an example.

[8] Computations with smaller data types are not always faster than with larger data types. It depends on the architecture.

it reaches -1 when all bits are 1. To negate a number using two's complement arithmetic, invert all of the bits and add one. For example, 1 (0b00000001) becomes -1 (0b11111111). What happens when you perform the negation procedure on zero?

Another representation choice is *endianness*. The *little-endian* representation of an `int` stores the least significant byte at the lowest address (`ADDRESS`) and the most significant byte at highest (`ADDRESS+3`) (remember, little-lowest-least), while the *big-endian* convention is the opposite, storing the most significant byte at the lowest address (`ADDRESS`) and the least significant byte at the highest address (`ADDRESS+3`).[9] The convention used depends on the processor. For definiteness in this appendix, we always assume little-endian representation, which is used by x86_64 (most likely your computer's architecture) and the PIC32.

`float`s, `double`s, and `long double`s are commonly represented in the IEEE 754 floating point format

$$\text{value} = (-1)^s \times b \times 2^c, \tag{A.1}$$

where one bit is used to represent the sign ($s = 0$ for positive, $s = 1$ for negative); $m = 23/52/112$ bits are used to represent the significand b (also known as the mantissa) in the range 1 to $2 - 2^{-m}$; and $n = 8/11/15$ bits are used to represent the exponent c in the range $-(2^{n-1}) + 2$ to $2^{n-1} - 1$, where n and m depend on whether the type uses 4/8/16 bytes. Certain exponent and significand combinations are reserved for representing special cases like positive and negative infinity and "not a number" (`NaN`).

Specifically for a four-byte `float`, the 32 bits of the IEEE 754 representation are

$$\underbrace{s}_{\text{sign bit}} \;\; \underbrace{e_7\, e_6\, e_5\, e_4\, e_3\, e_2\, e_1\, e_0}_{\text{8 bits of exponent } c} \;\; \underbrace{f_{23} f_{22} \ldots f_2 f_1}_{\text{23 bits of significand } b} \quad .$$

The exponent c in (A.1) is equal to $e - 127$, where e is the unsigned integer value of the eight bits of e, ranging from 0 to $2^8 - 1 = 255$. (The values $e = 0$ and $e = 255$ are reserved to represent special cases, like ±infinity and "not a number.") The significand b in (A.1) is given by

$$b = 1 + \sum_{i=1}^{23} f_i \quad 2^{-i},$$

so b ranges from 1 to $2 - 2^{-23}$.

[9] These phrases come from *Gulliver's Travels* by Jonathan Swift, where Lilliputians fanatically divide themselves according to which end of a soft-boiled egg they crack open.

See Exercise 14 to experiment with two's complement and IEEE 754 floating point representations.

Rarely do you need to worry about the specific bit-level representation of the different data types: endianness, two's complement, IEEE 754, etc. You tell the compiler to store values and retrieve values, and it handles implementing the representations.

A.3.2 Memory, Addresses, and Pointers

Consider the following C syntax:

```
int i;
int *ip;
```

These definitions may appear to define the variables `i` and `*ip` of type `int`; however, the character `*` cannot be part of a variable name. The variable name is actually `ip`, and the special character `*` means that `ip` is a **pointer** to something of type `int`. Pointers store the address of another variable; in other words, they "point" to another variable. We often use the words "address" and "pointer" interchangeably.

When the compiler sees the definition `int i`, it allocates four bytes of memory to hold the integer `i`. When the compiler sees the definition `int *ip`, it creates the variable `ip` and allocates to it whatever amount of memory is needed to hold an address, a platform-dependent quantity.[10] The compiler also remembers the data type that `ip` points to, `int` in this case, so when you use `ip` in a context that requires a pointer to a different data type, the compiler may generate a warning or an error. Technically, the type of `ip` is "pointer to type `int`."

Important! Defining a pointer only allocates memory to hold the pointer. It does **not** allocate memory for a pointee variable to be pointed at. Also, simply defining a pointer does not initialize it to point to anything valid. Do not use pointers without explicitly initializing them!

When we want the address of a variable, we apply the **address** (or **reference**) **operator** to the variable, which returns a pointer to the variable (its address). In C, the address operator is written `&`. Thus the following command assigns the address of `i` to the pointer `ip`:

```
ip = &i;  // ip now holds the address of i
```

[10] When computers switched from the 32-bit x86 to the 64-bit x86_64 architecture, code that relied on pointers being 32 bits long were in trouble; x86_64 uses 64-bit long pointers!

The address operator always returns the lowest address of a multi-byte type. For example, if the four-byte `int i` occupies addresses 0x0004 to 0x0007 in memory, `&i` will return 0x0004.[11]

If we have a pointer (an address) and we want the contents at that address, we apply the **dereference operator** to the pointer. In C, the dereference operator is written `*`. Thus the following command stores the value at the address pointed to by `ip` in `i`:

```
i = *ip;  // i now holds the contents at the address ip
```

However, you should never dereference a pointer until it has been initialized to point to something using a statement such as `ip = &i`.

As an analogy, consider the pages of a book. A page number can be considered a pointer, while the text on the page can be considered the contents of a variable. So the notation `&text` would return the page number (pointer or address) of the text, while `*page_number` would return the text on that page (but only after `page_number` is initialized to point at a page of text).

Even though we are focusing on the concept of pointers, and not C syntax, let us look at some sample C code, remembering that everything after `//` on the same line is a comment:

```
int i,j;      // define i, j as type int
int *ip;      // define ip as type "pointer to type int"
ip = &i;      // set ip to the address of i (& "references" i)
i = 100;      // put the value 100 in the location allocated by the compiler for i
j = *ip;      // set j to the contents of the address ip (* dereferences ip),
              // i.e., 100
j = j+2;      // add 2 to j, making j equal to 102
i = *(&j);    // & references j to get the address, then * gets contents; i is set
              // to 102
*(&j) = 200; // content of the address of j (j itself) is set to 200; i is unchanged
```

The use of pointers can be powerful, but also dangerous. For example, you may accidentally try to access an illegal memory location. The compiler is unlikely to recognize this during compilation, and you may end up with a “segmentation fault” when you execute the code.[12] This kind of bug can be difficult to find, and dealing with it is a C rite of passage. More on pointers in Section A.4.8.

[11] The address my actually be a “virtual” address rather than a physical location in memory. The computer automatically translates the value of `&i` to an actual physical address, when needed.
[12] A good name for a program like this is `coredumper.c`.

A.3.3 Compiling

The process loosely referred to as "compilation" actually consists of four steps:

1. **Preprocessing.** The preprocessor takes the `program.c` source code and produces an equivalent `.c` source code, performing operations such as removing comments. Section A.4.3 discusses the preprocessor in more detail.
2. **Compiling.** The compiler turns the preprocessed code into *assembly* code for the specific processor. The C code becomes a set of instructions that directly correspond to actions that the processor can perform. The compiler can be configured with several options that impact the assembly code generated. For example, the compiler can generate assembly code that increases execution time to reduce the amount of memory needed to store the code. Assembly code generated by a compiler can be inspected with a standard text editor. Coding directly in assembly is still a popular, if painful (or fun), way to program microcontrollers.
3. **Assembling.** The assembler converts the assembly instructions into processor-dependent machine-level binary *object* code. This code cannot be examined using a text editor.[13] Object code is called *relocatable*, in that the exact memory addresses for the data and program statements are not specified.
4. **Linking.** The linker takes one or more object code files and produces a single executable file. For example, if your code includes pre-compiled libraries, such as the C standard library that allows you to print to the screen (described in Sections A.4.3 and A.4.14), this code is included in the final executable. The data and program statements in the various object code files are assigned to specific memory locations.

In our `HelloWorld.c` program, this entire process is initiated by the single command line statement

```
> gcc HelloWorld.c -o HelloWorld
```

If our `HelloWorld.c` program used any mathematical functions in Section A.4.7, the compilation would be initiated by

```
> gcc HelloWorld.c -o HelloWorld -lm
```

where the `-lm` flag tells the linker to link the math library, which may not be linked by default like other libraries are.

If you want to see the intermediate results of the preprocessing, compiling, and assembling steps, Exercise 42 provides an example.

For more complex projects requiring compilation of several files into a single executable or specifying various options to the compiler, it is common to create a `Makefile` that specifies

[13] Well, you can view it in a text editor, but it will be incomprehensible.

how the compilation should be performed, and to then use the command `make` to actually execute the commands to create the executable. Details on the use of `Makefile`s is beyond the scope of this appendix; however, we use one extensively when programming the PIC32. Section A.4.15 gives a simple example of compiling multiple C files into single executable using a `Makefile`.

A.4 C Syntax

So far we have seen only glimpses of C syntax. Let us begin our study of C syntax with a few simple programs. We then jump to a more complex program, `invest.c`, that demonstrates many of the major elements of C structure and syntax. If you can understand `invest.c` and can create programs using similar elements, you are well on your way to mastering C. We defer the more detailed descriptions of the syntax until after introducing `invest.c`.

Printing to screen

Because it is the simplest way to see the results of a program, as well as a useful tool for debugging, let us start with the function `printf` for printing to the screen.[14] We have already seen it in `HelloWorld.c`. Here's a slightly more interesting example. Let us call this program file `printout.c`.

```
#include <stdio.h>

int main(void) {

  int i;
  float f;
  double d;
  char c;

  i = 32;
  f = 4.278;
  d = 4.278;
  c = 'k'; // or, by ASCII table, c = 107;

  printf("Formatted output:\n");
  printf(" i = %4d  c = '%c'\n",i,c);
  printf(" f = %19.17f\n",f);
  printf(" d = %19.17f\n",d);
  return 0;
}
```

[14] Programs called debuggers (such as gdb) also help you debug, allowing you to step through your program line by line as it runs.

The first line of the program

```
#include <stdio.h>
```

tells the preprocessor that the program will use functions from the "standard input and output" library, one of many code libraries provided in standard C installations that extend the power of the language. The `stdio.h` function used in `printout.c` is `printf`, covered in more detail in Section A.4.14.

The next line

```
int main(void) {
```

starts the block of code that defines the `main` function. The `main` code block is closed by the final closing brace `}`. Each C program has exactly one `main` function, and program execution begins there. The type of `main` is `int`, meaning that the function should end by returning a value of type `int`. In our case, it returns a `0`, which indicates to the operating system that the program has terminated successfully.

The next four lines define and allocate memory for four variables with four different types. The following lines assign values to those variables. The `printf` lines will be discussed after we look at the output.

Now that you have created `printout.c`, you can create the executable file `printout` and run it from the command line. Make sure you are in the directory containing `printout.c`, then type the following:

```
> gcc printout.c -o printout
> printout
```

(Again, you may have to use `./printout` to tell your computer to look in the current directory.) Here is the output:

```
Formatted output:
  i =   32  c = 'k'
  f = 4.27799987792968750
  d = 4.27799999999999958
```

The main purpose of this program is to demonstrate formatted output from the code

```
  printf("Formatted output:\n");
  printf(" i = %4d  c = '%c'\n",i,c);
  printf(" f = %19.17f\n",f);
  printf(" d = %19.17f\n",d);
```

Inside the function call to `printf`, everything inside the double quotation marks is printed to the screen, but some character sequences have special meaning. The `\n` sequence creates a newline. The `%` is a special character, indicating that some data will be printed, and for each `%`

in the double quotes, there must be a variable or other expression in the comma-separated list at the end of the `printf` call. The `%4d` means that an `int` type variable is expected, and it will be displayed right-justified using four spaces. (If the number has more than four digits, it will take as much space as is needed.) The `%c` means that a `char` is expected. The `%19.17f` means that a `float` or `double` will be printed right-justified over 19 spaces with 17 spaces after the decimal point. If you did not care how many decimal places were displayed, you could have simply used `%f` and let the C implementation default make the choice for you.[15] More details on `printf` can be found in Section A.4.14.

The output of the program also shows that neither the `float f` nor the `double d` can represent 4.278 exactly, though the double-precision representation comes closer.

Data sizes

Since we have focused on data types, our next program measures how much memory is used by different data types. Create a file called `datasizes.c` that looks like the following:

```
#include <stdio.h>

int main(void) {
  char a;
  char *bp;
  short c;
  int d;
  long e;
  float f;
  double g;
  long double h;
  long double *ip;

  printf("Size of char:                 %2ld bytes\n",sizeof(a)); // "% 2 ell d"
  printf("Size of char pointer:         %2ld bytes\n",sizeof(bp));
  printf("Size of short int:            %2ld bytes\n",sizeof(c));
  printf("Size of int:                  %2ld bytes\n",sizeof(d));
  printf("Size of long int:             %2ld bytes\n",sizeof(e));
  printf("Size of float:                %2ld bytes\n",sizeof(f));
  printf("Size of double:               %2ld bytes\n",sizeof(g));
  printf("Size of long double:          %2ld bytes\n",sizeof(h));
  printf("Size of long double pointer:  %2ld bytes\n",sizeof(ip));
  return 0;
}
```

The first lines in the `main` function define nine variables, telling the compiler to allocate space for these variables. Two of these variables are pointers. The `sizeof()` operator returns the number of bytes allocated in memory for its argument. You can use `sizeof()` on either a variable or a type (i.e., `sizeof(int)`); here we use it exclusively on variables.

[15] `printf` does not distinguish between doubles and floats, so use `%f` for both.

Here is the output of the program:

```
Size of char:                   1 bytes
Size of char pointer:           8 bytes
Size of short int:              2 bytes
Size of int:                    4 bytes
Size of long int:               8 bytes
Size of float:                  4 bytes
Size of double:                 8 bytes
Size of long double:           16 bytes
Size of long double pointer:    8 bytes
```

We see that, on an x86_64 computer with `gcc`, `int`s and `float`s use four bytes, `short int`s two bytes, `long int`s and `double`s eight bytes, and `long double`s 16 bytes. Regardless of whether it points to a `char` or a `long double`, a pointer (address) uses eight bytes, meaning we can address a maximum of $(2^8)^8 = 256^8$ bytes of memory. Considering that corresponds to almost 18 quintillion bytes, or 18 billion gigabytes, we should have enough available addresses (at least for the time-being)!

Overflow

Now let us try the program `overflow.c`, which demonstrates the issue of integer overflow mentioned in Section A.3.1.

```
#include <stdio.h>

int main(void) {
  char i = 100, j = 240, sum;
  unsigned char iu = 100, ju = 240, sumu;
  signed char is = 100, js = 240, sums;

  sum = i+j;
  sumu = iu+ju;
  sums = is+js;

  printf("char:           %d + %d = %3d or ASCII %c\n",i,j,sum,sum);
  printf("unsigned char:  %d + %d = %3d or ASCII %c\n",iu,ju,sumu,sumu);
  printf("signed char:    %d + %d = %3d or ASCII %c\n",is,js,sums,sums);
  return 0;
}
```

In this program we initialize the values of some of the variables when they are defined. You might also notice that we are assigning a `signed char` a value of 240, even though the range for that data type is −128 to 127. So something fishy is happening. The program outputs:

```
char:           100 + -16 =  84 or ASCII T
unsigned char:  100 + 240 =  84 or ASCII T
signed char:    100 + -16 =  84 or ASCII T
```

Notice that, with our C compiler, `char`s are the same as `signed char`s. Even though we assigned the value of 240 to `js` and `j`, they contain the value −16 because the binary representation of 240 has a 1 in the 2^7 column. For the two's complement representation of a `signed char`, this column indicates whether the value is positive or negative. Finally, we notice that the `unsigned char ju` is successfully assigned the value 240 since its range is 0 to 255, but the addition of `iu` and `ju` leads to an overflow. The correct sum, 340, has a 1 in the 2^8 (or 256) column, but this column is not included in the 8 bits of the `unsigned char`. Therefore we see only the remainder of the number, 84. The number 84 is assigned the character T in the standard ASCII table.

Type conversion

Continuing our focus on data types, we try another simple program that illustrates what happens when you mix data types in mathematical expressions. This program uses a helper function in addition to the `main` function. We name this program `typecast.c`.

```
#include <stdio.h>

void printRatio(int numer, int denom) {  // printRatio is a helper function
  double ratio;

  ratio = numer/denom;
  printf("Ratio, %1d/%1d:                         %5.2f\n",numer,denom,ratio);
  ratio = numer/((double) denom);
  printf("Ratio, %1d/((double) %1d):              %5.2f\n",numer,denom,ratio);
  ratio = ((double) numer)/((double) denom);
  printf("Ratio, ((double) %1d)/((double) %1d):  %5.2f\n",numer,denom,ratio);
}

int main(void) {
  int num = 5, den = 2;

  printRatio(num,den);                   // call the helper function
  return(0);
}
```

The helper function `printRatio` "returns" type `void` since it does not return a value. It takes two `int`s as arguments and calculates their ratio in three different ways. In the first, the two `int`s are divided and the result is assigned to a `double`. In the second, the integer `denom` is **typecast** or **cast** to `double` before the division occurs, so an `int` is divided by a `double` and the result is assigned to a `double`.[16] In the third, both the numerator and denominator are cast as `double`s before the division, so two `double`s are divided and the result is assigned to a `double`.

[16] The typecasting does not change the variable `denom` itself; it simply creates a temporary `double` version of `denom` which is lost as soon as the division is complete.

The main function defines two ints, num and den, and passes their values to printRatio, where those values are copied to numer and denom, respectively. The variables num and den are only available to main, and the variables numer and denom are only available to printRatio, since they are defined inside those functions.

Execution of any C program always begins with the main function, regardless of where it appears in the file.

After compiling and running, we get the output

```
Ratio, 5/2:                        2.00
Ratio, 5/((double) 2):             2.50
Ratio, ((double) 5)/((double) 2):  2.50
```

The first answer is "wrong," while the other two answers are correct. Why?

The first division, numer/denom, is an *integer* division. When the compiler sees that there are ints on either side of the divide sign, it assumes you want integer math and produces a result that is an int by simply truncating any remainder (rounding toward zero). This value, 2, is then converted to the floating point number 2.0 so it can be assigned to the double-precision floating point variable ratio. On the other hand, the expression numer/((double) denom), by virtue of the parentheses, first produces a double version of denom before performing the division. The compiler recognizes that you are dividing two different data types, so it temporarily **coerces** the int to a double so it can perform a floating point division. This is equivalent to the third and final division, except that the typecast of the numerator to double is explicit in the code for the third division.

Thus we have two kinds of type conversions:

- **Implicit** type conversion, or **coercion**. This occurs, for example, when a type has to be converted to carry out a variable assignment or to allow a mathematical operation. For example, dividing an int by a double will cause the compiler to treat the int as a double before carrying out the division.
- **Explicit** type conversion. An explicit type conversion is coded using a casting operator, e.g., (double) <expression> or (char) <expression>, where <expression> may be a variable or mathematical expression.

Certain type conversions may result in a change of value. For example, assigning the value of a float to an int results in truncation of the fractional portion; assigning a double to a float may result in roundoff error; and assigning an int to a char may result in overflow. Here's a less obvious example:

```
float f;
int i = 16777217;
f = i;            // f now has the value 16,777,216.0, not 16,777,217!
```

It turns out that $16,777,217 = 2^{24} + 1$ is the smallest positive integer that cannot be represented by a 32-bit `float`. On the other hand, a 32-bit `int` can represent all integers in the range -2^{31} to $2^{31} - 1$.

Some type conversions, called **promotions**, never result in a change of value because the new type can represent all possible values of the original type. Examples include converting a `char` to an `int` or a `float` to a `long double`.

As with pointers, typecasts are dangerous and should be used sparingly. Knowing where type coercion occurs, however, can be crucial. In the example below, the first line performs integer division and then converts the result to a double, whereas the second line performs floating point division.

```
double f = 3/2;      // yields 1.0!
double g = 3.0/2.0;  // yields 1.5
```

We will see more on use of parentheses (Section A.4.1), the scope of variables (Section A.4.5), and defining and calling helper functions (Section A.4.6).

Advanced: Pointers can be used in conjunction with typecasts to view the same data in different ways. For example, the declaration `unsigned short s = 0xAB12` stores `0xAB12` in memory as two consecutive bytes: `0x12 0xAB` (remember, the LSB is in the lowest address on a little-endian processor). Performing (`*&s`) dereferences the address `&s` and treats the memory location as an `unsigned short` because `&s` has type `unsigned short *`; thus, the expression yields `0xAB12`. Performing `*(unsigned char *)&s` yields `0x12` because the typecast converts the pointer `&s` into a pointer to an `unsigned char *`. Dereferencing such a pointer yields an `unsigned char`, which is only one byte long. Pointers always refer to the lowest address of the variable, so the result is `0x12` not `0xAB`. On a big-endian system, however, the result would be `0xAB`.

A more complete example: `invest.c`

Until now we have been dipping our toes in the C pool. Now let us dive in headfirst.

Our next program is called `invest.c`, which takes an initial investment amount, an expected annual return rate, and a number of years, and returns the growth of the investment over the years. After performing one set of calculations, it prompts the user for another scenario, and continues this way until the data entered is invalid. The data is invalid if, for example, the initial investment is negative or the number of years to track is outside the allowed range.

The real purpose of `invest.c`, however, is to demonstrate the syntax and several useful features of C.

Here's an example of compiling and running the program. The only data entered by the user are the three numbers corresponding to the initial investment, the growth rate, and the number of years.

```
> gcc invest.c -o invest
> invest
Enter investment, growth rate, number of yrs (up to 100): 100.00 1.05 5
Valid input?  1

RESULTS:

Year   0:      100.00
Year   1:      105.00
Year   2:      110.25
Year   3:      115.76
Year   4:      121.55
Year   5:      127.63

Enter investment, growth rate, number of yrs (up to 100): 100.00 1.05 200
Valid input?  0
Invalid input; exiting.
>
```

Before we look at the full `invest.c` program, let us review two principles that should be adhered to when writing a longer program: modularity and readability.

- *Modularity.* You should break your program into a set of functions that perform specific, well-defined tasks, with a small number of inputs and outputs. As a rule of thumb, no function should be longer than about 20 lines. (Experienced programmers often break this rule of thumb, but if you are a novice and are regularly breaking this rule, you are likely not thinking modularly.) Almost all variables you define should be "local" to (i.e., only recognizable by) their particular function. Global variables, which can be accessed by all functions, should be minimized or avoided altogether, since they break modularity, allowing one function to affect the operation of another without the information passing through the well-defined "pipes" (input arguments to a function or its returned results). If you find yourself typing the same (or similar) code more than once, that's a good sign you should determine how to write a single function and just call that function from multiple places. Modularity makes it much easier to develop large programs and track down the inevitable bugs.
- *Readability.* You should use comments to help other programmers, and even yourself, understand the purpose of the code you have written. Variable and function names should be chosen to indicate their purpose. Be consistent in how you name variables and functions. Any "magic number" (constant) used in your code should be given a name and defined at the beginning of the program, so if you ever want to change this number, you can just change it at one place in the program instead of every place it is used. Global variables and constants should be written in a way that easily distinguishes them from more common local variables; for example, you could WRITE CONSTANTS IN UPPERCASE and Capitalize Globals. You should use whitespace (blank lines, spaces, tabbing, etc.) consistently to make it easy to read the program. Use a fixed-width font

(e.g., `Courier`) so that the spacing/tabbing is consistent. Modularity (above) also improves readability.

The program `invest.c` demonstrates readable modular code using the structure and syntax of a typical C program. In the program's comments, you will see references of the form `==SecA.4.3==` that indicate where you can find more information in the review of syntax that follows the program.

```
/******************************************************************************
 * PROGRAM COMMENTS (PURPOSE, HISTORY)
 ******************************************************************************/

/*
 * invest.c
 *
 * This program takes an initial investment amount, an expected annual
 * return rate, and the number of years, and calculates the growth of
 * the investment.  The main point of this program, though, is to
 * demonstrate some C syntax.
 *
 * References to further reading are indicated by ==SecA.B.C==
 *
 */

/******************************************************************************
 * PREPROCESSOR COMMANDS   ==SecA.4.3==
 ******************************************************************************/

#include <stdio.h>      // input/output library
#define MAX_YEARS 100   // constant indicating max number of years to track

/******************************************************************************
 * DATA TYPE DEFINITIONS (HERE, A STRUCT)  ==SecA.4.4==
 ******************************************************************************/

typedef struct {
  double inv0;                    // initial investment
  double growth;                  // growth rate, where 1.0 = zero growth
  int years;                      // number of years to track
  double invarray[MAX_YEARS+1];   // investment array   ==SecA.4.9==
} Investment;                     // the new data type is called Investment

/******************************************************************************
 * GLOBAL VARIABLES   ==SecA.4.2, A.4.5==
 ******************************************************************************/

// no global variables in this program

/******************************************************************************
 * HELPER FUNCTION PROTOTYPES  ==SecA.4.2==
 ******************************************************************************/

int getUserInput(Investment *invp);     // invp is a pointer to type ...
```

```
void calculateGrowth(Investment *invp); // ... Investment ==SecA.4.6, A.4.8==
void sendOutput(double *arr, int years);

/******************************************************************************
 * MAIN FUNCTION   ==SecA.4.2==
 ******************************************************************************/

int main(void) {

  Investment inv;                // variable definition, ==SecA.4.5==

  while(getUserInput(&inv)) {    // while loop ==SecA.4.13==
    inv.invarray[0] = inv.inv0;  // struct access ==SecA.4.4==
    calculateGrowth(&inv);       // & referencing (pointers) ==SecA.4.6, A.4.8==
    sendOutput(inv.invarray,     // passing a pointer to an array ==SecA.4.9==
               inv.years);       // passing a value, not a pointer ==SecA.4.6==
  }
  return 0;                      // return value of main ==SecA.4.6==
} // ***** END main *****

/******************************************************************************
 * HELPER FUNCTIONS   ==SecA.4.2==
 ******************************************************************************/

/* calculateGrowth
 *
 * This optimistically-named function fills the array with the investment
 * value over the years, given the parameters in *invp.
 */
void calculateGrowth(Investment *invp) {

  int i;

  // for loop ==SecA.4.13==
  for (i = 1; i <= invp->years; i= i + 1) {   // relational operators ==SecA.4.10==
                                              // struct access ==SecA.4.4==
    invp->invarray[i] = invp->growth * invp->invarray[i-1];
  }
} // ***** END calculateGrowth *****


/* getUserInput
 *
 * This reads the user’s input into the struct pointed at by invp,
 * and returns TRUE (1) if the input is valid, FALSE (0) if not.
 */
int getUserInput(Investment *invp) {

  int valid;       // int used as a boolean ==SecA.4.10==

  // I/O functions in stdio.h ==SecA.4.14==
  printf("Enter investment, growth rate, number of yrs (up to %d): ",MAX_YEARS);
  scanf("%lf %lf %d", &(invp->inv0), &(invp->growth), &(invp->years));

  // logical operators ==SecA.4.11==
  valid = (invp->inv0 > 0) && (invp->growth > 0) &&
    (invp->years > 0) && (invp->years <= MAX_YEARS);
  printf("Valid input?  %d\n",valid);

  // if-else ==SecA.4.12==
  if (!valid) { // ! is logical NOT ==SecA.4.11==
    printf("Invalid input; exiting.\n");
  }
  return(valid);
} // ***** END getUserInput *****


/* sendOutput
 *
 * This function takes the array of investment values (a pointer to the first
 * element, which is a double) and the number of years (an int).  We could
 * have just passed a pointer to the entire investment record, but we decided
 * to demonstrate some different syntax.
 */
void sendOutput(double *arr, int yrs) {

  int i;
  char outstring[100];      // defining a string ==SecA.4.9==

  printf("\nRESULTS:\n\n");
  for (i=0; i<=yrs; i++) {  // ++, +=, math in ==SecA.4.7==
    sprintf(outstring,"Year %3d:  %10.2f\n",i,arr[i]);
    printf("%s",outstring);
  }
  printf("\n");
} // ***** END sendOutput *****
```

A.4.1 Basic Syntax

Comments

Everything after a `/*` and before the next `*/` is a comment. Comments are removed during the preprocessing step of compilation. They help make the purpose of the program, function, loop, or statement clear to yourself or other programmers.[17] Keep the comments neat and concise for program readability. Some programmers use extra asterisks or other characters to make the comments stand out (see the examples in `invest.c`), but all that matters is that `/*` starts the comment and the next `*/` ends it.

If your comment is short, you can use `//` instead. Everything after `//` and before the next carriage return will be ignored. The `//` style comments originate from C++ but most modern C compilers support them.

[17]Reading your own code after several months away from it is often like reading someone else's code!

Semicolons

A code statement must be completed by a semicolon. Some exceptions to this rule include preprocessor commands (see `PREPROCESSOR COMMANDS` in the program and Section A.4.3) and statements that end with blocks of code enclosed by braces `{ }`. A single code statement may extend over multiple lines of the program listing until it is terminated by a semicolon (see, for example, the assignment to `valid` in the function `getUserInput`).

Braces and blocks of code

Blocks of code are enclosed in braces `{ }`. Examples include entire functions (see the definition of the `main` function and the helper functions), blocks of code executed inside of a `while` loop (in the `main` function) or `for` loop (in the `calculateGrowth` and `sendOutput` functions), as well as other examples. In `invest.c`, braces are placed as shown here

```
while (<expression>) {
  /* block of code */
}
```

but this style is equivalent

```
while (<expression>)
  {
    /* block of code */
  }
```

as is this

```
while (<expression>) { /* block of code */ }
```

Which brings us to...

Whitespace

Whitespace, such as spaces, tabs, and carriage returns, is only required where it is needed to recognize keywords and other syntax. The whole program `invest.c` could be written on a single line, for example. Indentations and line breaks should be used consistently, however, to make the program readable. Insert line breaks after each semicolon. Statements within the same code block should be left-justified with each other and statements in a code block nested within another code block should be indented with respect to the parent code block. Text editors should use a fixed-width font so that alignment is clear. Most editors provide fixed-width fonts and automatic indentation to enhance readability.

Parentheses

C has rules defining the order in which operations in an expression are evaluated, much like standard math rules that say $3 + 5 * 2$ evaluates to $3 + (10) = 13$, not $(8) * 2 = 16$. If uncertain about the default order of operations, use parentheses `( )` to enclose sub-expressions. For example, $3 + (40/(4 * (3 + 2)))$ evaluates to $3 + (40/(4 * 5)) = 3 + (40/20) = 3 + 2 = 5$, whereas $3 + 40/4 * 3 + 2$ evaluates to $3 + 30 + 2 = 35$.

A.4.2 Program Structure

`invest.c` demonstrates a typical structure for a program written in one `.c` file. When you write larger programs, you may wish to divide your program into multiple files, to increase modularity. Section A.4.15 discusses C programs that consist of multiple source code files.

Let us consider the seven major sections of the program in order of appearance. `PROGRAM COMMENTS` describe the purpose of the program. `PREPROCESSOR COMMANDS` define constants and “header” files that should be included, giving the program access to library functions that extend the power of the C language. This section is described in more detail in Section A.4.3. In some programs, it may be helpful to define a new data type, as shown in `DATA TYPE DEFINITIONS`. In `invest.c`, several variables are packaged together in a single record or `struct` data type, as described in Section A.4.4. Any `GLOBAL VARIABLES` are then defined. These are variables that are available for use by all functions in the program. Because of this special status, the names of global variables could be Capitalized or otherwise written in a way to remind the programmer that they are not local variables (Section A.4.5). Generally, global variables should be avoided because they violate modularity.

The next section of the program contains the `HELPER FUNCTION PROTOTYPES` of the various helper functions. A prototype of a function declares the name, argument types, and return types of a function that will be defined later. Prototypes are used to allow code to call functions that have not yet been fully defined or are defined elsewhere (perhaps in another source file). For example, the function `printRatio` has a return type of `void`, meaning that it does not return a value. It takes two arguments, each of type `int`. The function `getUserInput` returns an `int` and takes a single argument: a pointer to a variable of type `Investment`, a data type defined a few lines above the `getUserInput` prototype.

The next section of the program, `MAIN FUNCTION`, is where the `main` function is defined. Every program has exactly one `main` function, where the program starts execution. The `main` function returns an `int`. By convention, it returns 0 if it executes successfully, and otherwise returns a nonzero value. In `invest.c`, `main` takes no arguments, hence the `void` in the argument list. Some programs accept arguments on the command line; these can be passed as arguments to `main`. For example, we could have written `invest.c` to run with a command such as this:

```
> invest 100.0 1.05 5
```

To allow this, `main` would have been defined with the following syntax:

```
int main(int argc, char *argv[]) {
```

Then when the program is invoked as above, the integer `argc` would be set to four, the number of whitespace-separated strings on the command line, and `argv` would point to a array of four strings, where the string `argv[0]` is 'invest', `argv[1]` is '100.0', etc. You can learn more about arrays and strings in Section A.4.9.

Finally, the last section of the program is the definition of the `HELPER FUNCTIONS` whose prototypes were given earlier. It is not strictly necessary that the helper functions have prototypes, but if not, every function should be defined before it is used by any other function. For example, none of the helper functions uses another helper function, so they could have all been defined before the `main` function, in any order, and their function prototypes eliminated. The names of the variables in a function prototype and in the actual definition of the function need not be the same; for example, the prototype of `sendOutput` uses variables named `arr` and `years`, whereas the actual function definition uses `arr` and `yrs`. What matters is that the prototype and actual function definition have the same number of arguments, of the same types, and in the same order. In fact, in the arguments of the function prototypes, you can leave out variable names altogether, and just keep the comma separated list of argument data types; however, including the names serves as additional documentation and is generally a good practice.

A.4.3 Preprocessor Commands

In the preprocessing stage of compilation, all comments are removed from the program. Additionally, the preprocessor performs actions when encountering the following preprocessor commands, recognizable by the # character:

```
#include <stdio.h>      // input/output header
#define MAX_YEARS 100   // constant indicating max number of years to track
```

Include files

The first preprocessor command in `invest.c` indicates that the program will use standard C input/output functions. The file `stdio.h` is called a **header** file for the library. This file is readable by a text editor and contains constants, function prototypes, and other included headers that are made available to the program. The preprocessor replaces the

`#include <stdio.h>` command with the contents of the header file `stdio.h`.[18] Examples of function prototypes that are included are

```
int printf(const char *Format, ...);
int sprintf(char *Buffer, const char *Format, ...);
int scanf(const char *Format, ...);
```

Each of these three functions is used in `invest.c`. If the program were compiled without including `stdio.h`, the compiler would generate a warning or an error due to the lack of function prototypes. See Section A.4.14 for more information on using the `stdio` input and output functions.

During the linking stage, the object code of `invest.c` is linked with the object code for `printf`, `sprintf`, and `scanf` in your C installation. Libraries like the C standard library provide access to functions beyond the basic C syntax. Other useful libraries (and header files for the C standard library) are briefly described in Section A.4.14.

Constants

The second line defines the constant `MAX_YEARS` to be equal to 100. The preprocessor searches for each instance of `MAX_YEARS` in the program and replaces it with 100. If we later decide that the maximum number of years to track investments should be 200, we can change the definition of this constant in one place, instead of in several places. Since `MAX_YEARS` is constant, not a variable, it can never be assigned another value somewhere else in the program. To indicate that it is not a variable, a common convention is to write constants in UPPERCASE. This is not required by C, however. We should emphasize that the preprocessor performs a text substitution, as if you used the "find and replace" feature of your text editor. So, in this example, the preprocessor literally replaces every occurrence of `MAX_YEARS` with the number 100.

Macros

One more use of the preprocessor is to define simple function-like *macros* that you may use in more than one place in your program. Constants, as described above, are technically a simple macro. Here's an example that converts radians to degrees:

```
#define RAD_TO_DEG(x) ((x) * 57.29578)
```

[18] The preprocesser searches for header files in directories specified by the "include path." If the header file `header.h` sits in the same directory as `invest.c`, we would write `#include "header.h"` instead of `#include <header.h>`.

The preprocessor searches for any instance of `RAD_TO_DEG(x)` in the program, where `x` can be any text, and replaces it with `((x) * 57.29578)`. For example, the initial code

```
angle_deg = RAD_TO_DEG(angle_rad);
```

is replaced by

```
angle_deg = ((angle_rad) * 57.29578);
```

Note the importance of the outer parentheses in the macro definition. If we had instead used the preprocessor command

```
#define RAD_TO_DEG(x) (x) * 57.29578   // don't do this!
```

then the code

```
answer = 1.0 / RAD_TO_DEG(3.14);
```

would be replaced by

```
answer = 1.0 / (3.14) * 57.29578;
```

which is very different from

```
answer = 1.0 / ((3.14) * 57.29578);
```

Moral: if the expression you are defining is anything other than a single constant, enclose it in parentheses, to tell the compiler to evaluate the expression first.

As a second example, the macro

```
#define MAX(A,B)      ((A) > (B) ? (A):(B))
```

returns the maximum of two arguments. The `?` is the *ternary operator* in C, which has the form

```
<test> ? return_value_if_test_is_true : return_value_if_test_is_false
```

The preprocessor replaces

```
maxval = MAX(13+7,val2);
```

with

```
maxval = ((13+7) > (val2) ? (13+7):(val2));
```

Why define a macro instead of just writing a function? One reason is that the macro may execute slightly faster, since no passing of control to another function and no passing of variables is needed. Most of the time, you should use functions.

A.4.4 Typedefs, Structs, and Enums

In simple programs, you will do just fine with the data types `int`, `char`, `float`, `double`, and variations. Sometimes you may find it useful to create an alias for a data type, using the following syntax:

```
typedef <type> newtype;
```

where `<type>` is an existing data type and `newtype` is its alias. Then, you can define a new variable `x` of type `newtype` by

```
newtype x;
```

For example, you could write

```
typedef int days_of_the_month;
days_of_the_month day;
```

You might find it satisfying that your variable `day` (taking values 1 to 31) is of type `days_of_the_month`, but the compiler will still treat it as an `int`. However, if you use this type a lot and later want to change it, using the `typedef` provides one location to make the change rather than needing to go through your whole program: you can think of a `typedef` as a constant but for data types.

In addition to aliasing existing data types, you can also create new types that combine several variables into a single record or `struct`. We gather investment information into a single `struct` in `invest.c`:

```
typedef struct {
  double inv0;                   // initial investment
  double growth;                 // growth rate, where 1.0 = zero growth
  int years;                     // number of years to track
  double invarray[MAX_YEARS+1];  // investment values
} Investment;                    // the new data type is called Investment
```

Notice how the `struct {...}` replaces the data type `int` in our previous `typedef` example. This syntax creates a new data type `Investment` with a record structure, with *fields* named `inv0` and `growth` of type `double`, `years` of type `int`, and `invarray`, an array of `double`s.[19] (Arrays are discussed in Section A.4.9.) With this new type definition, we can define a variable named `inv` of type `Investment`:

```
Investment inv;
```

This definition allocates sufficient memory to hold the two `double`s, the `int`, and the array of `double`s. We can access the contents of the `struct` using the "`.`" operator:

```
int yrs;
yrs = inv.years;
inv.growth = 1.1;
```

An example of this kind of usage is seen in `main`.

Referring to the discussion of pointers in Sections A.3.2 and A.4.8, if we are working with a pointer `invp` that points to `inv`, we can use the "`->`" operator to access the contents of the record `inv`:

```
Investment inv;     // allocate memory for inv, an investment record
Investment *invp;   // invp will point to something of type Investment
int yrs;
invp = &inv;        // invp points to inv
inv.years = 5;      // setting one of the fields of inv
yrs = invp->years;  // inv.years, (*invp).years, and invp->years are all identical
invp->growth = 1.1;
```

Examples of this usage are seen in `calculateGrowth()` and `getUserInput()`. Using the operator `a->b` is equivalent to doing `(*a).b`, dereferencing the `struct` pointer and accessing a specific field.

Another data type you can create is called an `enum`, short for "enumeration." Although `invest.c` does not use an enumeration, they can be useful for describing a type that can take one of a limited set of values. For example, if you wanted a function to use a cardinal direction you could define an enumeration as follows:

```
typedef enum {NORTH, SOUTH, EAST, WEST} Direction;
```

[19] The `typedef` is actually aliasing an anonymous struct with the name `Investment`. You can omit the `typedef`, but then you create a type that must be referred to as `struct Investment` rather than `Investment`. The `typedef` provides a more convenient syntax when you use the type.

Each item in the `enum` gets assigned a constant numerical value. You can explicitly state this value, or use the default compiler-provided values, which start at zero and increment by one for each element. For example, the declaration above is equivalent to

```
typedef enum {NORTH = 0, SOUTH = 1, EAST = 2, WEST = 3} Direction;
```

You can use an `enum` as you would any other data type.

A.4.5 Defining Variables

Variable names

Variable names can consist of uppercase and lowercase letters, numbers, and underscore characters '_'. You should generally use a letter as the first character; `var`, `Var2`, and `Global_Var` are all valid names, but `2var` is not. C is case sensitive, so the variable names `var` and `VAR` are different. A variable name cannot conflict with a reserved keyword in C, like `int` or `for`. Names should be succinct but descriptive. The variable names `i`, `j`, and `k` are often used for integer counters in `for` loops, and pointers often begin with `ptr_`, such as `ptr_var`, or end with `p`, such as `varp`, to remind you that they are pointers. Regardless of how you choose to name your variables, adopting a consistent naming convention throughout a program aids readability.

Scope

The **scope** of a variable refers to where it can be used in the program. A variable may be *global*, i.e., usable by any function, or *local* to a specific function or piece of a function. A global variable is one that is defined in the `GLOBAL VARIABLES` section, outside of and before any function that uses it. Such variables can be referred to or altered in any function.[20] Because of this special status, global variables are often Capitalized. Global variable usage should be minimized for program modularity and readability.

A local variable is one that is defined in a function. Such a variable is only usable inside that function, after the definition.[21] If you choose a local variable name `var` that is also the name of a global variable, inside that function `var` will refer to the local variable, and the global variable will not be available. It is not good practice to choose local variable names to be the

[20] You could also define a variable outside of any function definition but *after* some of the function definitions. This quasi-global variable would be available to all functions defined after the variable is defined, but not those before. This practice is discouraged, as it makes the code harder to read.

[21] Since we recommend that each function be brief, you can define all local variables in that function at the beginning of the function, so we can see in one place what local variables the function uses. Some programmers prefer instead to define variables just before their first use, to minimize their scope. Older C specifications required that all local variables be defined at the beginning of a code block enclosed by braces `{ }`.

same as global variable names, as it makes the program confusing to understand and is often the source of bugs.

The parameters to a function are local to that function's definition, as in `sendOutput` at the end of `invest.c`:

```
void sendOutput(double *arr, int yrs) {    // ...
```

The variables `arr` and `yrs` are local to `sendOutput`.

Otherwise, local variables are defined at the beginning of the function code block by syntax similar to that shown in the function `main`.

```
int main(void) {
  Investment inv;    // Investment is a variable type we defined
  // ... rest of the main function ...
```

Since this definition appears within the function, `inv` is local to `main`. Had this definition appeared before any function definition, `inv` would be a global variable.

A global variable can be declared as `static`:

```
static int i;
```

The `static` specifier means that the global variable can only be used from within the given `.c` file; other files cannot use the variable. If you must use a global variable, you should declare it `static` if possible. Preventing other `.c` files in a multi-file C program from accessing the variable helps increase modularity and reduce bugs.

Variables can also have *qualifiers* attached to their types. There are two main qualifiers in C, `const` and `volatile`. The `const` qualifier prevents the "variable" from being modified. The definition

```
const int i = 3;
```

sets `i` to 3, and `i` can never be changed after that. The `volatile` qualifier is used extensively in embedded programming, and indicates that the variable may change outside of the normal flow of the program (i.e., due to interrupts, Chapter 6) that the compiler cannot know about in advance. Therefore, the compiler should not assume anything about the current value of a `volatile` variable. Here is an example usage:

```
volatile int i;
```

We discuss `volatile` more fully in Chapter 6.

Qualifiers can also be applied to pointers. The syntax, however, can be a bit tricky. The first two definitions below make `cp` a `const` pointer, meaning that the content pointed to by `cp` (i.e., `*cp`) is constant: the value of `cp`, however, may change. The third definition makes the pointer `cp` itself constant, but the contents pointed to by `cp` may change. The fourth line makes both the pointer and the data that the pointer references constant.

```
const int * cp; // pointer to const data
int const * cp; // pointer to const data
int * const cp; // pointer to data, value of cp is const
const int * const cp; // pointer and data are const
```

Definition and initialization

When a variable is defined, memory for the variable is allocated. In general, you cannot assume anything about the contents of the variable until you have initialized it. For example, if you want to define a `float x` with value 0.0, the command

```
float x;
```

is insufficient. The memory allocated may have random 0's and 1's already in it, and the allocation of memory does not generally change the current contents of the memory. Instead, you can use

```
float x = 0.0;
```

to initialize the value of `x` when you define it. Equivalently, you could use

```
float x;
x = 0.0;
```

but, when possible, it is better to initialize a variable when you define it, so you do not accidentally use an uninitialized value.

Static local variables

Each time a function is called, its local variables are allocated space in memory. When the function completes, its local variables are discarded, freeing memory. If you want to keep the results of some calculation by the function after the function completes, you could return the results from the function or store them in a global variable. Sometimes, a better alternative is to use the `static` modifier in the local variable definition, as in the following program:

```
#include <stdio.h>
```

```
void myFunc(void) {
  static char ch='d';   // this local variable is static, allocated and initialized
                        // only once during the entire program
  printf("ch value is %d, ASCII character %c\n",ch,ch);
  ch = ch+1;
}

int main(void) {
  myFunc();
  myFunc();
  myFunc();
  return 0;
}
```

The `static` modifier in the definition of `ch` in `myFunc` means that `ch` is only allocated, and initialized to `'d'`, the first time `myFunc` is called during the execution of the program. This allocation persists after the function returns, and the value of `ch` is remembered. The output of this program is

```
ch value is 100, ASCII character d
ch value is 101, ASCII character e
ch value is 102, ASCII character f
```

Numerical values

Just as you can assign an integer a base-10 value using commands like `ch=100`, you can assign a number written in hexadecimal notation by putting "`0x`" at the beginning of the digit sequence, e.g.,

```
unsigned char ch = 0x64;  // ch now has the base-10 value 100
```

This form may be convenient when you want to directly control bit values. This is often useful in microcontroller applications. Some C compilers, including the PIC32 C compiler, allow specifying bits directly using the following syntax:

```
unsigned char ch = 0b1100100;  // ch now has the base-10 value 100
```

A.4.6 Defining and Calling Functions

A function definition consists of the function's return type, function name, argument list, and body (a block of code). Allowable function names follow the same rules as variable names. The function name should make the purpose of the function clear; for example, `getUserInput` (which gets input from the user) in `invest.c`.

If the function does not return a value, it has return type `void`, as with `calculateGrowth`. If it does return a value, such as `getUserInput` which returns an `int`, the function should end with the command

```
return val;
```

where `val` is a variable with the same type as the function's return type. The `main` function returns an `int` and should return 0 upon successful completion.

The function definition

```
void sendOutput(double *arr, int yrs) {  // ...
```

indicates that `sendOutput` returns nothing and takes two arguments, a pointer to type `double` and an `int`. When the function is called with the statement

```
sendOutput(inv.invarray, inv.years);
```

the `invarray` and `years` fields of the `inv` structure in `main` are copied to `sendOutput`, which now has its own local copies of these variables, stored in `arr` and `yrs`. The difference between the two is that `yrs` is just data, while `arr` is a pointer, holding the address of the first element of `invarray`, i.e., `&(inv.invarray[0])`. (Arrays will be discussed in more detail in Section A.4.9.) Since `sendOutput` now has the memory address of the beginning of this array, *it can directly access, and potentially change, the original array seen by* `main`. On the other hand, `sendOutput` cannot, by itself change the value of `inv.years` in `main`, since it only has a copy of that value, not the actual memory address of `main`'s `inv.years`. `sendOutput` takes advantage of its direct access to the `inv.invarray` to print out all the values stored there, eliminating the need to copy all the values of the array from `main` to `sendOutput`. To prevent the function from changing the contents of `arr` we could have (and probably should have) declared the parameter `const`, as in `double const * arr`.

The function `calculateGrowth`, which is called with a pointer to `main`'s `inv` data structure, takes advantage of its direct access to the `invarray` field to change the values stored there.

When a function is passed a pointer argument, it is called a *pass by reference*; the argument is a reference (address, or pointer) to data. When a function is passed non-pointer data, it is called a *pass by value*; data is copied but not an address.

If a function takes no arguments and returns no value, we can define it as `void myFunc(void)`. The function is called using

```
myFunc();
```

A.4.7 Math

Standard *binary* math operators (operators on two operands) include `+`, `-`, `*`, and `/`. These operators take two operands and return a result, as in

```
ratio = a/b;
```

If the operands are the same type, then the CPU carries out a division (or add, subtract, multiply) specific for that type and produces a result of the same type. In particular, if the operands are integers, the result will be an integer, even for division (fractions are rounded toward zero). If one operand is an integer type and the other is a floating point type, the integer type will generally be coerced to a floating point to allow the operation (see the `typecast.c` program description of Section A.4).

The modulo operator `%` takes two integers and returns the remainder of their division, i.e.,

```
int i;
i = 16 % 7;  // i is now equal to 2
```

C also provides `+=`, `-=`, `*=`, `/=`, `%=` to simplify some expressions, as shown below:

```
x = x * 2;  // these two lines
x *= 2;     // are equivalent

y = y + 7;  // these two lines
y += 7;     // are equivalent
```

Since adding one to an integer or subtracting one from an integer are common operations in loops, these have a further simplification. For an integer `i`, we can write

```
++i;  // adds 1 to i, equivalent to i = i+1;
--i;  // equivalent to i = i-1;
```

In fact we also have the syntax `i++` and `i-`. If the `++` or `-` come in front of the variable, the variable is modified before it is used in the rest of the expression. If they come after, the variable is modified after the expression has been evaluated. So

```
int i = 5, j;
j = (++i)*2;   // after this line, i is 6 and j is 12
```

but

```
int i = 5,j;
j = (i++)*2;   // after this line, i is 6 and j is 10
```

But your code would be much more readable if you just wrote `i++` before or after the `j = i*2` line.

If your program includes the C math library with the preprocessor command `#include <math.h>`, you have access to a much larger set of mathematical operations, some of which are listed here:

```
int     abs      (int x);              // integer absolute value
double  fabs     (double x);           // floating point absolute value
double  cos      (double x);           // all trig functions work in radians,
                                          not degrees
double  sin      (double x);
double  tan      (double x);
double  acos     (double x);           // inverse cosine
double  asin     (double x);
double  atan     (double x);
double  atan2    (double y, double x);  // two-argument arctangent
double  exp      (double x);           // base e exponential
double  log      (double x);           // natural logarithm
double  log2     (double x);           // base 2 logarithm
double  log10    (double x);           // base 10 logarithm
double  pow      (double x, double y);  // raise x to the power of y
double  sqrt     (double x);           // square root of x
```

These functions also have versions for `float`s. The names of those functions are identical, except with an ’`f`’ appended to the end, e.g., `cosf`.

When compiling programs using `math.h`, you may need to include the linker flag `-lm`, e.g.,

```
gcc myprog.c -o myprog -lm
```

to tell the linker to link with the math library.

A.4.8 Pointers

It’s a good idea to review the introduction to pointers in Section A.3.2 and the discussion of call by reference in Section A.4.6. In summary, the operator `&` references a variable, returning a pointer to (the address of) that variable, and the operator `*` dereferences a pointer, returning the contents of the address.

These statements define a variable `x` of type `float` and a pointer `ptr` to a variable of type `float`:

```
float x;
float *ptr;
```

At this point, the assignment

```
*ptr = 10.3;
```

would result in undefined behavior, because the pointer `ptr` does not currently point to anything. The following code would be valid:

```
ptr = &x;          // assign ptr to the address of x; x is the "pointee" of ptr
*ptr = 10.3;       // set the contents at address ptr to 10.3; now x is equal to 10.3
*(&x) = 4 + *ptr; // the * and & on the left cancel each other; x is set to 14.3
```

Since `ptr` is an address, it is an integer (technically the type is "pointer to type float"), and we can add and subtract integers from it. For example, say that `ptr` contains the value n, and then we execute the statement

```
ptr = ptr + 1;    // equivalent to ptr++;
```

If we now examined `ptr`, we would find that it has the value $n + 4$. Why? Because the compiler knows that `ptr` points to the type `float`, so when we add 1 to `ptr`, the assumption is that we want to increment by one `float` in memory, not one byte. Since a `float` occupies four bytes, the address `ptr` must increase by 4 to point to the next `float`. The ability to increment a pointer in this way can be useful when dealing with arrays, next.

A.4.9 Arrays and Strings

One-dimensional arrays

An array of five `float`s can be defined by

```
float arr[5];
```

We could also initialize the array at the time we define it:

```
float arr[5] = {0.0, 10.0, 20.0, 30.0, 40.0};
```

Each of these definitions allocates five `float`s in memory, accessed by `arr[0]` (initialized to 0.0 above) through `arr[4]` (initialized to 40.0). The elements are stored consecutively, as per Figure A.2. The assignment

```
arr[5] = 3.2;
```

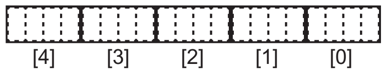


Figure A.2
A float array with five elements, as stored in memory. The dashed lines separate bytes and the solid lines separate floats. Each float has four bytes.

is a mistake, since only `arr[0..4]` have been allocated. This statement will compile successfully because compilers do not check for indexing arrays out of bounds. The best result at this point would be for your program to crash, to alert you to the fact that you are overwriting memory that may be allocated for another purpose. More insidiously, the program could seem to run just fine, but with difficult-to-debug erratic behavior.[22] Bottom line: never access arrays out of bounds!

In the expression `arr[i]`, `i` is an integer called the *index*, and `arr[i]` is of type `float`. The variable `arr` by itself points to the first element of the array, and is equivalent to `&(arr[0])`. The address `&(arr[i])` is at `arr` plus `i*4` bytes, since the elements of the array are stored consecutively, and a `float` uses four bytes. Both `arr[i]` and `*(arr+i)` are correct syntax to access the `i`th element of the array. Since the compiler knows that `arr` is a pointer to the four-byte type `float`, the address represented by `(arr+i)` is `i*4` bytes higher than the address `arr`.

Consider the following code:

```
float arr[5] = {0.0, 10.0, 20.0, 30.0, 40.0};
float *ptr;
ptr = arr + 3;
// arr[0] contains 0.0 and ptr[0] = arr[3] = 30.0
// arr[0] is equivalent to *arr; ptr[0] is equivalent to *ptr and *(arr+3);
// ptr is equivalent to &(arr[3])
```

If we would like to pass the array `arr` to a function that initializes each element of the array, we could call

```
arrayInit(arr,5);
```

or

```
arrayInit(&(arr[0]),5);
```

The function definition for `arrayInit` might look like

```
void arrayInit(float *vals, int length) {

  int i;

  for (i=0; i<length; i++) vals[i] = i*10.0;
  // equivalently, we could substitute the line below for the line above
  // for (i=0; i<length; i++) {*vals = i*10.0; vals++;}
}
```

[22] The mistake could potentially be exploited as a security flaw.

The pointer `vals` in `arrayInit` is set to point to the same location as `arr` in the calling function. Therefore `vals[i]` refers to the same memory contents that `arr[i]` does.

Note that `arr` does not carry any information on the length of the array, so we must send the length separately to `arrayInit`.

Strings

A string is an array of `char`s. The definition

```
char s[100];
```

allocates memory for 100 `char`s, `s[0]` to `s[99]`. We could initialize the array with

```
char s[100] = "cat"; // note the double quotes
```

This places a ’`c`’ (integer value 99) in `s[0]`, an ’`a`’ (integer value 97) in `s[1]`, a ’`t`’ (integer value 116) in `s[2]`, and a value of 0 in `s[3]`, corresponding to the NULL (’`\0`’) character and indicating the end of the string. (You could also do this, less elegantly, by initializing just those four elements using braces as we did with the `float` array above.)

You notice that we allocated more memory than was needed to hold “cat.” Perhaps we will append something to the string in future, so we might want to allocate that extra space just in case. But if not, we could have initialized the string using

```
char s[] = "cat";
```

and the compiler would only assign the minimum memory needed (four bytes in this case, three for each character and one for the NULL character).

The function `sendOutput` in `invest.c` shows an example of constructing a string using `sprintf`, a function provided by `stdio.h`. Other functions for manipulating strings are provided in `string.h`. Both of these headers are described briefly in Section A.4.14.

Multi-dimensional arrays

The definition

```
int mat[2][3];
```

allocates memory for 6 `int`s, `mat[0][0]` to `mat[1][2]`, which can be thought of as a two-dimensional array, or matrix. These occupy a contiguous region of memory, with `mat[0][0]` at the lowest memory location, followed by `mat[0][1]`, `mat[0][2]`, `mat[1][0]`, `mat[1][1]`, and `mat[1][2]`. This matrix can be initialized using nested braces,

```
int mat[2][3] = {{0, 1, 2}, {3, 4, 5}};
```

Higher-dimensional arrays can be created by simply adding more indexes. In memory, a "row" of the rightmost index is completed before incrementing the next index to the left.

Static vs. dynamic memory allocation

A command of the form `float arr[5]` is called *static memory allocation*, meaning that the size of the array is known at compile time. Another option is *dynamic memory allocation*, where the size of the array can be chosen at run time.[23] With the C standard library header `stdlib.h` included using the preprocessor command `#include <stdlib.h>`, the syntax

```
float *arr; // arr is a pointer to float, but no memory has been allocated for the array
int i=5;
arr = (float *) malloc(i * sizeof(float));  // allocate the memory
```

allocates `arr[0..4]`, and

```
free(arr);
```

releases the memory when it is no longer needed. It is crucial to remember to `free` memory allocated with `malloc` when you are finished with it, so you do not run out of memory if you repeatedly allocate memory.[24] If `malloc` cannot allocate the requested memory, perhaps because the computer is out of memory, it returns a NULL pointer (i.e., `arr` will have value 0). You must **always** check that the result of `malloc` is valid before continuing with your program.

A.4.10 Relational Operators and TRUE/FALSE Expressions

`==`	equal
`!=`	not equal
`>, >=`	greater than, greater than or equal to
`<, <=`	less than, less than or equal to

Relational operators operate on two values and evaluate to 0 or 1. A 0 indicates that the expression is FALSE and a 1 indicates that the expression is TRUE. For example, the expression `(3>=2)` is TRUE, so it evaluates to 1, while `(3<2)` evaluates to 0, or FALSE.

[23] Dynamic memory is allocated from the *heap*, a portion of memory set aside for dynamic allocation (and therefore is not available for statically allocated variables and program code). You may have to adjust linker options setting the size of the heap. See Chapter 5.3.

[24] `malloc` tracks the size of the block associated with the address `arr`, so you do not need to tell `free` how much memory to release.

The most common mistake with relational operators is using = to test for equality instead of ==. For example, using the if conditional syntax (Section A.4.12), the test

```
int i = 2;
if (i = 3) {   // error:  this is an assignment, not a test!!
  printf("Test is true.");
}
```

always evaluates to TRUE, because the expression (i=3) assigns the value of 3 to i, and the expression evaluates to 3. Any nonzero value is treated as logical TRUE. If the condition is written (i==3), it will operate as intended, evaluating to 0 (FALSE).

Be aware of potential pitfalls in checking equality of floating point numbers. Consider the following program:

```
#include <stdio.h>
#define VALUE 3.1
int main(void) {
  float  x = VALUE;
  double y = VALUE;
  if (x == VALUE) {
    printf("x is equal to %f.\n",VALUE);
  } else {
    printf("x is not equal to %f!\n",VALUE);
  }
  if (y == VALUE) {
    printf("y is equal to %f.\n",VALUE);
  } else {
    printf("y is not equal to %f!\n",VALUE);
  }
  return 0;
}
```

You might be surprised to see that your program says that x is not equal while y is! In fact, neither x nor y are exactly 3.1 due to roundoff error in the floating point representation. However, by default, the constant 3.1 is treated as a double, so the double y carries the identical (wrong) value. If you want a constant to be treated explicitly as a float, you can write it as 3.1F, and if you want it to be treated as a long double, you can write it as 3.1L.

A.4.11 Logical and Bitwise Operators

~	bitwise NOT
&	bitwise AND
|	bitwise OR
^	bitwise XOR
>>	shift bits to the right (shifting in 0's from the left)
<<	shift bits to the left (shifting in 0's from the right)

Bitwise operators act directly on the bits of the operand(s), as in the following example:

```
unsigned char a=0xC, b=0x6, c;  // in binary, a is 0b00001100 and b is 0b00000110
c = ~a;     // NOT; c is 0xF3 or 0b11110011
c = a & b;  // AND; c is 0x04 or 0b00000100
c = a | b;  // OR;  c is 0x0E or 0b00001110
c = a ^ b;  // XOR; c is 0x0A or 0b00001010
c = a >> 3; // SHIFT RT 3; c is 0x01 or 0b00000001, one 1 is shifted off the
               right end
c = a << 3; // SHIFT LT 3; c is 0x60 or 0b01100000, 1's shifted to more
               significant digits
```

Much like the math operators, we also have the assignment expressions `&=`, `|=`, `^=`, `>>=`, and `<<=`, so `a &= b` is equivalent to `a = a&b`.

A.4.12 Conditional Statements

`if-else`

The basic `if-else` construct takes this form:

```
if (<expression>) {
  // execute this code block if <expression> is TRUE, then exit
}
else {
  // execute this code block if <expression> is FALSE
}
```

If the code block is a single statement, the braces are not necessary; however, good practice dictates that you should always use braces, in case you want to add additional statements later. The `else` and the block after it can be eliminated if no action needs to be taken when `<expression>` is FALSE.

`if-else` statements can be made into arbitrarily long chains:

```
if (<expression1>) {
  // execute this code block if <expression1> is TRUE, then exit this if-else chain
}
else if (<expression2>) {
  // execute this code block if <expression2> is TRUE, then exit this if-else chain
}
else {
  // execute this code block if both expressions above are FALSE
}
```

An example `if` statement is in `getUserInput`.

switch

If you would like to check if the value of a single expression is one of several possibilities, a `switch` may be simpler, clearer, and faster than a chain of `if-else` statements. Here is an example:

```
char ch;
// ... omitting code that sets the value of ch ...
switch (ch) {
  case 'a':      // execute these statements if ch has value 'a'
    <statement>;
    <statement>;
    break;       // exit the switch statement
  case 'b':
    // ... some statements
    break;
  case 'c':
    // ... some statements
    break;
  default:       // execute this code if none of the previous cases applied
    // ... some statements
}
```

Notice the `break;` statement after each `case`. These statements are required to prevent the code from “falling through” to the next case, which is usually undesirable.

A.4.13 Loops

`for` loop

A `for` loop has the following syntax:

```
for (<initialization>; <test>; <update>) {
  // code block
  }
```

If the code block consists of only one statement, the surrounding braces can be eliminated, but it is a good idea to use them anyway.

The sequence is as follows: at the beginning of the loop, the `<initialization>` statement is executed. Then the `<test>` is evaluated. If it is TRUE, the code block is executed, the `<update>` is performed, and we return to the `<test>`. If it is FALSE, the `for` loop exits.

The following `for` loop is in `calculateGrowth`:

```
for (i = 1; i <= invp->years; i = i + 1) {
  invp->invarray[i] = invp->growth*invp->invarray[i-1];
}
```

The `<initialization>` step sets `i = 1`. The `<test>` is TRUE if `i` is less than or equal to the number of years we will calculate growth in the investment. If it is TRUE, the value of the investment in year `i` is calculated from the value in year `i-1` and the growth rate. The `<update>` adds 1 to `i`. In this example, the code block is executed for `i` values of 1 to `invp->years`.

It is possible to perform more than one statement in the `<initialization>` and `<update>` steps by separating the statements by commas. For example, we could write

```
for (i = 1, j = 10; i <= 10; i++, j--)  { /* code */ };
```

if we want `i` to count up and `j` to count down.

`while` loop

A `while` loop has the following syntax:

```
while (<test>) {
  // code block
}
```

First, the `<test>` is evaluated, and if it is FALSE, the `while` loop exits. If the test is TRUE, the code block is executed and we return to the `<test>`.

In `main` of `invest.c`, the `while` loop executes until the function `getUserInput` returns 0, i.e., FALSE. `getUserInput` collects the user's input and returns an `int` that is 0 if the user's input is invalid and 1 if it is valid.

`do-while` loop

This is similar to a `while` loop, except the `<test>` is executed at the end of the code block, guaranteeing that the loop is executed at least once.

```
do {
  // code block
} while (<test>);
```

`break` and `continue`

If anywhere in the loop's code block the command `break` is encountered, the program will exit the loop. If the command `continue` is encountered, the rest of the commands in the code block will be skipped, and control will return to the `<update>` in a `for` loop or the `<test>` in a `while` or `do-while` loop. Examples:

```
while (<test1>) {
  if (<test2>) {
    break;  // jump out of the while loop
  }
  // ...
}
```

```
while (<test1>) {
  if (<test2>) {
    continue;  // skip the rest of the loop and go back to <test1>
  }
  x = x+3;
}
```

Use `break` and `continue` judiciously; they can make your code difficult to read. If you find yourself relying on numerous `break` and `continue` statements in a single loop, you may want to rethink your approach.

A.4.14 The C Standard Library

C comes with a standard library, aspects of which you can use by including the appropriate `.h` header file. The header file provides data types, function prototypes and macros required for part of the library.[25] We have already seen examples of standard header files such as `stdio.h`, which contains input/output functions; `math.h` in Section A.4.7; and `stdlib.h` in Section A.4.9. Third-party libraries can also be included by including their header files and linking with them.

It is well beyond our scope to provide details on the C standard library. Here we highlight a few particularly useful functions in `stdio.h`, `string.h`, and `stdlib.h`.

Input and output: `stdio.h`

```
int printf(const char *Format, ...);
```

The function `printf` is used to print to the "standard output," which, for a PC, is typically the screen. It takes a formatting string `Format` and a variable number of extra arguments, determined by the formatting string, as indicated by the ... notation. The keyword `const` means that `printf` cannot change the string `Format`.

An example comes from our program `printout.c`:

```
int i;
float f;
double d;
char c;
```

[25] Reminder: if you include `<math.h>`, you should also compile your program with the `-lm` flag, so the math library is linked during the linking stage. The math library is logically part of the C standard library, it just happens to be in a different file.

```
    i = 32;
    f = 4.278;
    d = 4.278;
    c = 'k'; // or, by ASCII table, c = 107;

    printf("Formatted output:\n");
    printf(" i = %4d  c = '%c'\n",i,c);
    printf(" f = %19.17f\n",f);
    printf(" d = %19.17f\n",d);
```

which produces the output

```
  Formatted output:
  i =   32  c = 'k'
  f = 4.27799987792968750
  d = 4.27799999999999958
```

The formatting strings consist of plain text, the special character `\n` that prints a newline, and directives of the form `%4d` and `%19.17f`. Each directive indicates that `printf` will be looking for a corresponding variable in the argument list to insert into the output. A non-exhaustive list of directives is given here:

`%d`	Print an `int`. Corresponding argument should be an `int`.
`%u`	Print an `unsigned int`. Corresponding argument should be an integer data type.
`%ld`	Print a `long int`.
`%f`	Print a `double` or a `float`. Corresponding argument should be a float or a double.
`%c`	Print a character according to the ASCII table. Argument should be `char`.
`%s`	Print a string. Argument should be a pointer to a `char` (first element of a string), terminated with a NULL character (`'\0'`).
`%x`	Print an `unsigned int` as a hexadecimal number.

The directive `%d` can be written instead as `%4d`, for example, meaning that four spaces are allocated to write the integer, which will be right-justified in that space with unused spaces blank. The directive `%f` can be written instead as `%6.3f`, indicating that six spaces are reserved to write out the variable, with one of those spaces being the decimal point and three of the spaces after the decimal point.

```
int sprintf(char *str, const char *Format, ...);
```

Instead of printing to the screen, `sprintf` prints to the string `str`. An example of this is in `sendOutput`. The string `str` must have enough memory allocated to fit the results.

```
int scanf(const char *Format, ...);
```

The function `scanf` is a formatted read from the "standard input," which is typically the keyboard. Arguments to `scanf` consist of a formatting string and pointers to variables where the input should be stored. Typically the formatting string consists of directives like `%d`, `%f`, etc., separated by whitespace. The directives are similar to those for `printf`, except they do not accept spacing modifiers (like the 5 in `%5d`).

One notable difference between formatting strings is that, unlike `printf`, `scanf` does distinguish between `float`s and `double`s. To read a `double` with `scanf` use `%lf` rather than `%f`.

For each directive, `scanf` expects a pointer to a variable of that type in the argument list. A very common mistake is the following:

```
int i;
scanf("%d",i);  // WRONG!  We need a pointer to the variable.
scanf("%d",&i); // RIGHT.
```

The pointer allows `scanf` to put the input into the right place in memory.

`getUserInput` uses the statement

```
scanf("%lf %lf %d", &(invp->inv0), &(invp->growth), &(invp->years));
```

to read in two `double`s and an `int` and place them into the appropriate spots in the investment data structure. `scanf` ignores the whitespace (tabs, newlines, spaces, etc.) between the inputs.

```
int sscanf(char *str, const char *Format, ...);
```

Instead of scanning from the keyboard, `scanf` scans the string pointed to by `str`.

```
FILE* fopen(const char *Path, const char *Mode);
int fclose(FILE *Stream);
int fscanf(FILE *Stream, const char *Format, ...);
int fprintf(FILE *Stream, const char *Format, ...);
```

These commands are for reading from and writing to files. Say you have a file named `inputfile`, sitting in the same directory as the program, with information your program needs. The following code would read from it and then write to the file `outputfile`.

```
int i;
double x;
FILE *input, *output;
input = fopen("inputfile","r");    // "r" means you will read from this file
output = fopen("outputfile","w");  // "w" means you will write to this file
fscanf(input,"%d %lf",&i,&x);
fprintf(output,"I read in an integer %d and a double %lf.\n",i,x);
fclose(input);                     // these streams should be closed ...
fclose(output);                    // ... at the end of the program
```

Normally, you would check the return value of `fopen`. If it returns NULL, than it failed to open the file.

```
int fputc(int character, FILE *stream);
int fputs(const char *str, FILE *stream);
int fgetc(FILE *stream);
char* fgets(char *str, int num, FILE *stream);
int puts(const char *str);
```

These commands write (put) a character or string to a file, get a character or string from a file, or write a string to the screen. The variable `stdin` is a `FILE *` that corresponds to keyboard input, and `stdout` is a `FILE *` that corresponds to screen output.

String manipulation: `string.h`

```
char* strcpy(char *destination, const char *source);
```

Given two strings, `char destination[100],source[100]`, we cannot simply copy one to the other using the assignment `destination = source`. Instead we use `strcpy(destination,source)`, which copies the string `source` (until reaching the string terminator character, integer value 0) to `destination`. The string `destination` must have enough memory allocated to hold the source string.

```
char* strcat(char *destination, const char *source);
```

Appends the string in `source` to the end of the string `destination`, which must be large enough to hold the concatenated string.

```
int strcmp(const char *s1, const char *s2);
```

Returns 0 if the two strings are identical, a positive integer if the first unequal character in `s1` is greater than `s2`, and a negative integer if the first unequal character in `s1` is less than `s2`.

```
size_t strlen(const char *s);
```

The type `size_t` is an unsigned integer type. `strlen` returns the length of the string `s`, where the end of the string is indicated by the NULL character ('`\0`').

```
void* memset(void *s, int c, size_t len);
```

`memset` writes `len` bytes of the value `c` (converted to an `unsigned char`) starting at the beginning of the string `s`. So

```
char s[10];
memset(s,'c',5);
```

would fill the first five characters of the string `s` with the character `'c'` (or integer value 99). This can be a convenient way to initialize a string.

General purpose functions in `stdlib.h`

```
void* malloc(size_t objectSize)
```

`malloc` is used for dynamic memory allocation. An example use is in Section A.4.9.

```
void free(void *objptr)
```

`free` is used to release memory allocated by `malloc`. An example use is in Section A.4.9.

```
int rand()
```

It is sometimes useful to generate random numbers, particularly for games. The code

```
int i;
i = rand();
```

places in `i` a pseudo-random number between 0 and `RAND_MAX`, a constant which is defined in `stdlib.h` (2,147,483,647 in our gcc installation). To convert this to an integer between 1 and 10, you could follow with

```
i = 1 + (int) ((10.0*i)/(RAND_MAX+1.0));
```

One drawback of the code above is that calling `rand` multiple times will lead to the same sequence of random numbers every time the program runs. The usual solution is to "seed" the random number algorithm with a different number each time, and this different number is often taken from a system clock. The `srand` function is used to seed `rand`, as in the example below:

```
#include <stdio.h>   // allows use of printf()
#include <stdlib.h>  // allows use of rand() and srand()
#include <time.h>    // allows use of time()

int main(void) {
  int i;
```

```
    srand(time(NULL)); // seed the random number generator with the current time
    for (i=0; i<10; i++) printf("Random number:  %d\n",rand());
    return 0;
  }
```

If we take out the line with `srand`, this program produces the same ten "random" numbers every time we run it. Note that this program includes the `time.h` header to allow the use of the `time` function.

```
void exit(int status)
```

When `exit` is invoked, the program exits with the exit code `status`. `stdlib.h` defines `EXIT_SUCCESS` with value 0 and `EXIT_FAILURE` with value −1, so that a typical call to `exit` might look like

```
  exit(EXIT_SUCCESS);
```

A.4.15 Multiple File Programs and Libraries

So far our programs have used the C Standard Library, which provides several functions, including `printf` and `scanf`. Accessing these functions is possible because:

1. The preprocessor command `#include <stdio.h>` inserts the header file `stdio.h` into the current compilation unit, providing function prototypes for library functions.
2. The linker links the pre-compiled library object code in the C Standard Library with your program.

Thus, using a library usually requires header files and object code. We could also loosely define a library to consist of a header file and a C file (without a `main` function) containing source code for the library functions.

The purpose of a library is to collect functions that are likely to be useful in multiple programs so you do not have to rewrite the code for each program. The same principles apply when dividing your project into multiple source files. Think about what functions may be generally useful yet related and put them into their own file. Having code in multiple files not only promotes reuse, but it also isolates your project's components making them easier to test and debug. Many libraries evolve from a collection of source files that were designed for one particular project, but prove more generally useful.

Let us look at an example.

A simple example: The rad2volume library

Pretend you want to write several programs that need to calculate the volume of a sphere given its radius. You could copy and paste the formula into all of your programs. If, however, you made some mistake and wanted to fix it, you would then need to find where you used the formula in every program and change it. By placing the formula in a function, in its own file, you only need to make one correction to fix everything.

In this example, you decide to write one helper C file, `rad2volume.c`, with a function `double radius2Volume(double r)` that can be used by other C files. For good measure, you decide to make the constant `MY_PI` available also. To test your new rad2volume library consisting of `rad2volume.c` and `rad2volume.h`, you create a `main.c` file that uses it. The three files are given below.

```
// ***** file:  rad2volume.h *****
#ifndef RAD2VOLUME_H           // "include guard"; don't include twice in one compilation
#define RAD2VOLUME_H           // second line of the "include guard"

#define MY_PI 3.1415926           // constant available to files including rad2Volume.h
double radius2Volume(double r);   // prototype available to files including rad2Volume.h

#endif                            // third line, and end, of "include guard"
```

```
// ***** file:  rad2volume.c *****
#include <math.h>                 // for the function pow
#include "rad2volume.h"           // if the header is in the same directory, use "quotes"

static double cuber(double x) {   // this function is not available externally
  return pow(x,3.0);
}

double radius2Volume(double rad) {  // function definition
  return (4.0/3.0)*MY_PI*cuber(rad);
}
```

```
// ***** file:  main.c *****
#include <stdio.h>
#include "rad2volume.h"

int main(void) {
  double radius = 3.0, volume;
  volume = radius2Volume(radius);
  printf("Pi is approximated as %25.23lf.\n",MY_PI);
  printf("The volume of the sphere is %8.4lf.\n",volume);
  return 0;
}
```

The C file `rad2volume.c` contains two functions, `cuber` and `radius2Volume`. The function `cuber` is only meant for internal, private use by `rad2volume.c`, so there is no prototype in rad2volume.h and it is also declared static so it is not visible to other source files. The function radius2Volume is meant for public use by other C files, so a prototype for radius2Volume is included in the library header file rad2volume.h. The constant MY_PI is also meant for public use, so it is defined in rad2volume.h. Now radius2Volume and MY_PI are available to any file that includes rad2volume.h. In this case, they are available to main.c and rad2volume.c. Typically, it is good practice for the implementation file to #include its own header; this prevents problems relating to a mismatch between function prototypes and function definitions.

Each of main.c and rad2volume.c is compiled independently to create the object code files main.o and rad2volume.o. The linker combines these files into the final executable. main.c compiles successfully because it has a prototype for rad2Volume from including rad2volume.h, and it expects that, during the linking stage, rad2Volume will be present. If no object code passed to the linker defines rad2Volume then a linker error occurs.

Note the three lines making up the *include guard* in rad2volume.h. During preprocessing of a C file, if rad2volume.h is included, the macro RAD2VOLUME_H is defined. If the same C file tries to include rad2volume.h again, the include guard will recognize that RAD2VOLUME_H already exists and therefore skip the prototype and constant definition, down to the #endif. Without include guards, if we wrote a .c file including both header1.h and header2.h, for example, not knowing that header2.h already includes header1.h, header1.h would be included twice, possibly causing errors.

The two C files, rad2volume.c and main.c, can be compiled into object code using the commands

```
gcc -c rad2volume.c -o rad2volume.o
gcc -c main.c -o main.o
```

where the -c flag indicates that the source code should be compiled and assembled, but not linked. The result is the object files rad2volume.o and main.o. The two object files can be linked into a final executable using

```
gcc rad2volume.o main.o -o myprog
```

Instead of typing these three lines, the single command

```
gcc rad2volume.c main.c -o myprog
```

will automatically compile and link the files.

Executing myprog, the output is

```
Pi is approximated as 3.14159260000000006840537.
The volume of the sphere is 113.0973.
```

More general multi-file projects

Generalizing from the previous example, a header file declares constants, macros, new data types, and function prototypes that are needed by the files that `#include` them. A header file can be included by C source files or other header files. Figure A.3 illustrates a project consisting of one C source file with a `main` function and two helper C source files without a `main` function. (Every C program has exactly one `.c` file with a `main` function.) Each of the helper C files has its own header file. This project also has another header file, `general.h`, without an associated C file. This header contains general constant, macro, and data type definitions that are not specific to either helper source file or the `main` C file. The arrows indicate that the pointed-to file `#include`s the pointed-from header file.

Assuming that all the files are in the same directory, the project in Figure A.3 can be built by the following four commands, which create three object files (one for each source file) and link them together into `myprog`:

```
gcc -c main.c -o main.o
gcc -c helper1.c -o helper1.o
gcc -c helper2.c -o helper2.o
gcc main.o helper1.o helper2.o -o myprog
```

The build is illustrated in Figure A.4. Each C file is compiled independently and requires the constants, macros, data types, and function prototypes needed to successfully compile and assemble into an object file. During compilation of a single C file, the compiler neither has nor needs access to the source code for functions in other C files. If `main.c` uses a function in `helper1.c`, for example, it needs only a prototype of the function, provided by `helper1.h`. The prototype tells the compiler the return type of the function and what arguments it takes, allowing the compiler to check if the code in `main.c` uses the function properly. Calls to the function from `main.o` are linked to the actual function in `helper1.o` at the linker stage.

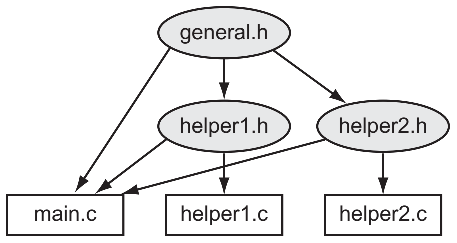


Figure A.3
An example project consisting of three C files and three header files. Arrows point from header files to files that include them.

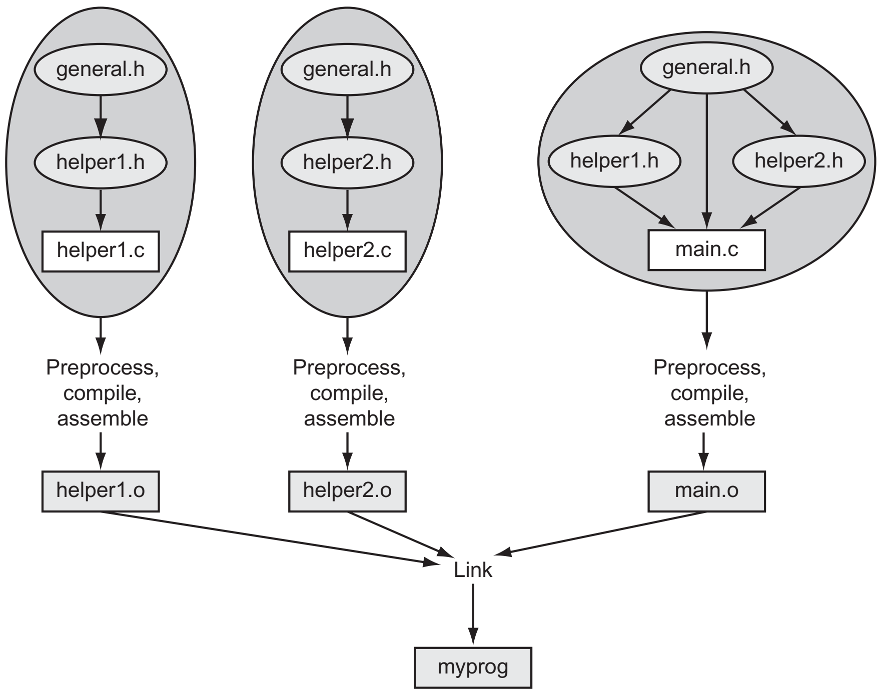


Figure A.4
The building of the project in Figure A.3.

Another benefit of splitting your project comes during compilation. If a `.c` file and the header files it includes do not change, then the `.c` file need not be compiled every time you build your project; the existing `.o` file may be used during the linking stage.

According to Figures A.3 and A.4, `main.c` has the following preprocessor commands:

```
#include "general.h"
#include "helper1.h"
#include "helper2.h"
```

The preprocessor replaces these commands with copies of the files `general.h`, `helper1.h`, and `helper2.h`. But when it includes `helper1.h`, it finds that `helper1.h` tries to include a second copy of `general.h` (see Figure A.3; `helper1.h` has a `#include "general.h"` command). Since `general.h` has already been copied in, it should not be copied again; otherwise we would have multiple copies of the same function prototypes, constant definitions, etc. The include guards prevent this duplication from happening.

In summary, the `general.h`, `helper1.h`, and `helper2.h` header files contain definitions that are made public to files including them. We might see the following items in the `helper1.h` header file, for example:

- an include guard
- other include files
- constants and macros defined with `#define` made public (and which may also be used by `helper1.c`)
- new data types (which may also be used by `helper1.c`)
- function prototypes of those functions in `helper1.c` which are meant to be used by other files

If a variable, function prototype, or constant is private to one C file, you should define it with the `static` keyword in the C file and not include it in the header file.

A header file like `helper1.h` could also have the declaration

```
extern int Helper1_Global_Var;   // no space is allocated by this declaration
```

where `helper1.c` has the global variable definition

```
int Helper1_Global_Var;          // space is allocated by this definition
```

Then any file including `helper1.h` would have access to the global variable `Helper1_Global_Var` allocated by `helper1.c`. Global variables defined in `helper1.c` with the `static` keyword are private to `helper1.c` and cannot be accessed by other files. If you have to use global variables (which should be avoided generally), declare them `static` whenever possible. Only in rare circumstances should you need to use `extern`.

Makefiles

When you are ready to build your executable, you can type the `gcc` commands at the command line, as we have seen previously. A `Makefile` simplifies the process, particularly for multi-file projects, by specifying the dependencies and commands needed to build the project. A `Makefile` for our rad2volume example is shown below, where everything after a `#` is a comment.

```
# ***** file:  Makefile *****
# Comment:  This is the simplest of Makefiles!

# Here is a template:
# [target]:  [dependencies]
# [tab] [command to execute]

# The thing to the left of the colon in the first line is what is created,
# and the thing(s) to the right of the colon are what it depends on.  The second
# line is the action to create the target.  If the things it depends on
# haven't changed since the target was last created, no need to do the action.
# Note:  The tab spacing in the second line is important!  You can't just use
# individual spaces.

# "make myprog" or "make" links two object codes to create the executable
myprog:  main.o rad2volume.o
        gcc main.o rad2volume.o -o myprog

# "make main.o" produces main.o object code; depends on main.c and rad2volume.h
main.o:  main.c rad2volume.h
        gcc -c main.c -o main.o

# "make rad2volume.o" produces rad2volume.o; depends on one .c and one h file
rad2volume.o:  rad2volume.c rad2volume.h
        gcc -c rad2volume -o rad2volume.o

# "make clean" throws away any object files to ensure make from scratch
clean:
        rm *.o
```

With this `Makefile` in the same directory as your other files, you should be able to type the command `make [target]`,[26] where `[target]` is `myprog`, `main.o`, `rad2volume.o`, or `clean`. If the `target` depends on other files, `make` will make sure those are up to date first, and if not, it will call the commands needed to create them. For example, `make myprog` triggers a check of `main.o`, which triggers a check of `main.c` and `rad2volume.h`. If either of those have changed since the last time `main.o` was made, then `main.c` is compiled and assembled to create a new `main.o` before the linking step.

The command `make` with no target specified will make the first target (which is `myprog` in this case).

Ensure that your `Makefile` is saved without any extensions (e.g., `.txt`) and that the commands are preceded by a tab (not spaces).

There are many more sophisticated uses of `Makefile`s which you can learn about from other sources.

A.5 Exercises

1. Install C, create the `HelloWorld.c` program, and compile and run it.
2. Explain what a pointer variable is, and how it is different from a non-pointer variable.
3. Explain the difference between interpreted and compiled code.

[26] In some C installations `make` is named differently, like `nmake` for Visual Studio or `mingw32-make`. If you can find no version of `make`, you may not have selected the `make` tools installation option when you performed the C installation.

For more information on thet

4. Write the following hexadecimal (base-16) numbers in eight-digit binary (base-2) and three-digit decimal (base-10). Also, for each of the eight-digit binary representations, give the value of the most significant bit. (a) 0x1E. (b) 0x32. (c) 0xFE. (d) 0xC4.
5. What is 333_{10} in binary and 1011110111_2 in hexadecimal? What is the maximum value, in decimal, that a 12-bit number can hold?
6. Assume that each byte of memory can be addressed by a 16-bit address, and every 16-bit address has a corresponding byte in memory. How many total bits of memory do you have?
7. (Consult the ASCII table.) Let `ch` be of type `char`. (a) The assignment `ch = 'k'` can be written equivalently using a number on the right side. What is that number? (b) The number for `'5'`? (c) For `'='`? (d) For `'?'`?
8. What is the range of values for an `unsigned char`, `short`, and `double` data type?
9. How many bits are used to store a `char`, `short`, `int`, `float`, and `double`?
10. Explain the difference between `unsigned` and `signed` integers.
11. (a) For integer math, give the pros and cons of using `char`s vs. `int`s. (b) For floating point math, give the pros and cons of using `float`s vs. `double`s. (c) For integer math, give the pros and cons of using `char`s vs. `float`s.
12. The following `signed short int`s, written in decimal, are stored in two bytes of memory using the two's complement representation. For each, give the four-digit hexadecimal representation of those two bytes. (a) 13. (b) 27. (c) −10. (d) −17.
13. The smallest positive integer that cannot be exactly represented by a four-byte IEEE 754 `float` is $2^{24} + 1$, or 16,777,217. Explain why.
14. Give the four bytes, in hex, that represent the following decimal values: (a) 20 as an `unsigned int`. (b) −20 as a two's complement `signed int`. (c) 1.5 as an IEEE 754 `float`. (d) 0 as an IEEE 754 `float`.

    To verify your answers, use the program `typereps.c` below. This program allows the user to enter four bytes as eight hex characters, then prints the value of those four bytes when they are interpreted as an `unsigned int`, a two's complement `signed int`, an IEEE 754 `float`, or four consecutive `char`s. To do this, the program creates a new data type `four_types_t` consisting of four bytes, or 32 bits. These same 32 bits are interpreted as either an `unsigned int`, `int`, `float`, or four `char`s depending on whether we reference the bits using the fields `u`, `i`, `f`, or `char0`–`char3` in the `union`.

```
#include <stdio.h>

typedef union {    // a new data type consisting of four bytes
  unsigned int u; // the 32 bits interpreted as an unsigned int
  int i;          // the same 32 bits interpreted as a two's complement int
  float f;        // the same 32 bits interpreted as an IEEE 754 single prec float
  struct {
    char char0:8; // bits  0 -  7 interpreted as char, called char0
    char char1:8; // bits  8 - 15 interpreted as char, called char1
    char char2:8; // bits 16 - 23 interpreted as char, called char2
```

```
      char char3:8; // bits 24 - 31 interpreted as char, called char3
    };
  } four_types_t;  // the new type is called four_types_t

  int main(void) {
    four_types_t val;

    while (1) {     // exit the infinite loop using ctrl-c or similar
      printf("Enter four bytes as eight hex characters 0-f, e.g., abcd0123:  ");
      scanf("%x",&val.u);
      printf("\nThe 32 bits in hex:                        %x\n",val.u);
      printf("The 32 bits as an unsigned int, in decimal:  %u\n",val.u);
      printf("The 32 bits as a signed int, in decimal:     %d\n",val.i);
      printf("The 32 bits as a float:                      %.20f\n",val.f);
      printf("The 32 bits as 4 chars:                      %c %c %c %c\n\n",
             val.char3, val.char2, val.char1, val.char0);
    }
    return 0;
  }
```

Below is a sample output. Note that only the ASCII values 32-126 have a visible printed representation, so the printouts as `char`s are meaningless in the first two examples.

```
Enter four bytes as eight hex characters 0-f, e.g., abcd0123: c0000000

The 32 bits in hex:                         c0000000
The 32 bits as an unsigned int, in decimal: 3221225472
The 32 bits as a signed int, in decimal:    -1073741824
The 32 bits as a float:                     -2.00000000000000000000
The 32 bits as 4 chars:                     ?

Enter four bytes as eight hex characters 0-f, e.g., abcd0123: ff800000

The 32 bits in hex:                         ff800000
The 32 bits as an unsigned int, in decimal: 4286578688
The 32 bits as a signed int, in decimal:    -8388608
The 32 bits as a float:                     -inf
The 32 bits as 4 chars:                     ? ?

Enter four bytes as eight hex characters 0-f, e.g., abcd0123: 48494a4b

The 32 bits in hex:                         48494a4b
The 32 bits as an unsigned int, in decimal: 1212762699
The 32 bits as a signed int, in decimal:    1212762699
The 32 bits as a float:                     206121.17187500000000000000
The 32 bits as 4 chars:                     H I J K
```

You can easily modify the code to allow the user to enter the four bytes as a `float` or `int` to examine their hex representations.

15. Write a program that prints out the sign, exponent, and significand bits of the IEEE 754 representation of a `float` entered by the user.
16. Technically the data type of a pointer to a `double` is "pointer to type `double`." Of the common integer and floating point data types discussed in this chapter, which is the most similar to this pointer type? Assume pointers occupy eight bytes.

17. To keep things simple, let us assume we have a microcontroller with only $2^8 = 256$ bytes of RAM, so each address is given by a single byte. Now consider the following code defining four global variables:

```
unsigned int i, j, *kp, *np;
```

Let us assume that the linker places `i` in addresses 0xB0..0xB3, `j` in 0xB4..0xB7, `kp` in 0xB8, and `np` in 0xB9. The code continues as follows:

```
             // (a) the initial conditions, all memory contents unknown
kp = &i;     // (b)
j = *kp;     // (c)
i = 0xAE;    // (d)
np = kp;     // (e)
*np = 0x12;  // (f)
j = *kp;     // (g)
```

For each of the comments (a)-(g) above, give the contents (in hexadecimal) at the address ranges 0xB0..0xB3 (the `unsigned int i`), 0xB4..0xB7 (the `unsigned int j`), 0xB8 (the pointer `kp`), and 0xB9 (the pointer `np`), at that point in the program, after executing the line containing the comment. The contents of all memory addresses are initially unknown or random, so your answer to (a) is "unknown" for all memory locations. If it matters, assume little-endian representation.

18. Invoking the `gcc` compiler with a command like `gcc myprog.c -o myprog` actually initiates four steps. What are the four steps called, and what is the output of each step?
19. What is `main`'s return type, and what is the meaning of its return value?
20. Give the `printf` statement that will print out a `double d` with eight digits to the right of the decimal point and four spaces to the left.
21. Consider three `unsigned char`s, `i`, `j`, and `k`, with values 60, 80, and 200, respectively. Let `sum` also be an `unsigned char`. For each of the following, give the value of `sum` after performing the addition. (a) `sum = i+j;` (b) `sum = i+k;` (c) `sum = j+k;`
22. For the variables defined as

```
int a=2, b=3, c;
float d=1.0, e=3.5, f;
```

give the values of the following expressions. (a) `f = a/b;` (b) `f = ((float) a)/b;` (c) `f = (float) (a/b);` (d) `c = e/d;` (e) `c = (int) (e/d);` (f) `f = ((int) e)/d;`

23. In each snippet of code in (a)-(d), there is an arithmetic error in the final assignment of `ans`. What is the final value of `ans` in each case?

a.
```
char c = 17;
float ans = (1 / 2) * c;
```
b.
```
unsigned int ans = -4294967295;
```
c.
```
double d = pow(2, 16);
short ans = (short) d;
```
d.
```
double ans = ((double) -15 * 7) / (16 / 17) + 2.0;
```

24. Truncation is not always bad. Say you wanted to store a list of percentages rounded down to the nearest percent, but you were tight on memory and cleverly used an array of `chars` to store the values. For example, pretend you already had the following snippet of code:

```
char percent(int a, int b) {
  // assume a <= b
  char c;
  c = ???;
  return c;
}
```

    You cannot simply write `c = a / b`. If $\frac{a}{b} = 0.77426$ or $\frac{a}{b} = 0.778$, then the correct return value is `c` = 77. Finish the function definition by writing a one-line statement to replace `c = ???`.
25. Explain why global variables work against modularity.
26. What are the seven sections of a typical C program?
27. You have written a large program with many functions. Your program compiles without errors, but when you run the program with input for which you know the correct output, you discover that your program returns the wrong result. What do you do next? Describe your systematic strategy for debugging.
28. Erase all the comments in `invest.c`, recompile, and run the program to make sure it still functions correctly. You should be able to recognize what is a comment and what is not. Turn in your modified `invest.c` code.
29. The following problems refer to the program `invest.c`. For all problems, you should modify the original code (or the code without comments from the previous problem) and run it to make sure you get the expected behavior. For each problem, turn in the modified portion of the code only.
    a. *Using* `if`, `break` *and* `exit`. Include the header file `stdlib.h` so we have access to the `exit` function (Section A.4.14). Change the `while` loop in `main` to be an infinite loop by inserting an expression `<expr>` in `while(<expr>)` that always evaluates to 1 (TRUE). (What is the simplest expression that evaluates to 1?) Now the first command inside the while loop gets the user's input. `if` the input is not valid, `exit` the program; otherwise continue. Next, change the `exit` command to a `break` command, and see the different behavior.
    b. *Accessing fields of a* `struct`. Alter `main` and `getUserInput` to set `inv.invarray[0]` in `getUserInput`, not `main`.
    c. *Using* `printf`. In `main`, before `sendOutput`, echo the user's input to the screen. For example, the program could print out `You entered 100.00, 1.05, and 5.`
    d. *Altering a string.* After the `sprintf` command of `sendOutput`, try setting an element of `outstring` to 0 before the `printf` command. For example, try setting the third element of `outstring` to 0. What happens to the output when you run the program? Now try setting it to `'0'` instead and see the behavior.

e. *Relational operators.* In `calculateGrowth`, eliminate the use of `<=` in favor of an equivalent expression that uses `!=`.
f. *Math.* In `calculateGrowth`, replace `i=i+1` with an equivalent statement using `+=`.
g. *Data types.* Change the fields `inv0`, `growth`, and `invarray[]` to be `float` instead of `double` in the definition of the `Investment` data type. Make sure you make the correct changes everywhere else in the program.
h. *Pointers.* Change `sendOutput` so that the second argument is of type `int *`, i.e., a pointer to an integer, instead of an integer. Make sure you make the correct changes everywhere else in the program.
i. *Conditional statements.* Use an `else` statement in `getUserInput` to print `Input is valid` if the input is valid.
j. *Loops.* Change the `for` loop in `sendOutput` to an equivalent `while` loop.
k. *Logical operators.* Change the assignment of `valid` to an equivalent statement using `||` and `!`, and no `&&`.

30. Consider this array definition and initialization:

```
int x[4] = {4, 3, 2, 1};
```

For each of the following, give the value or write "error/unknown" if the compiler will generate an error or the value is unknown. (a) `x[1]` (b) `*x` (c) `*(x+2)` (d) `(*x)+2` (e) `*x[3]` (f) `x[4]` (g) `*(&(x[1]) + 1)`

31. For the (strange) code below, what is the final value of `i`? Explain why.

```
int i,k=6;
i = 3*(5>1) + (k=2) + (k==6);
```

32. As the code below is executed, give the value of `c` in hex at the seven break points indicated, (a)-(g).

```
unsigned char a=0x0D, b=0x03, c;
c = ~a;      // (a)
c = a & b;   // (b)
c = a | b;   // (c)
c = a ^ b;   // (d)
c = a >> 3;  // (e)
c = a << 3;  // (f)
c &= b;      // (g)
```

33. In your C installation, or by searching on the web, find a listing of the header file `stdio.h`. Find the function prototype for one function provided by the library, but not mentioned in this appendix, and describe what that function does.
34. Write a program to generate the ASCII table for values 33 to 127. The output should be two columns: the left side with the number and the right side with the corresponding character. Turn in your code and the output of the program.
35. We will write a simple *bubble sort* program to sort a string of text in ascending order according to the ASCII table values of the characters. A bubble sort works as follows.

Given an array of n elements with indexes 0 to $n - 1$, we start by comparing elements 0 and 1. If element 0 is greater than element 1, we swap them. If not, leave them where they are. Then we move on to elements 1 and 2 and do the same thing, etc., until finally we compare elements $n - 2$ and $n - 1$. After this, the largest value in the array has "bubbled" to the last position. We now go back and do the whole thing again, but this time only comparing elements 0 up to $n - 2$. The next time, elements 0 to $n - 3$, etc., until the last time through we only compare elements 0 and 1.
Although this simple program `bubble.c` could be written in one function (`main`), we are going to break it into some helper functions to get used to using them. The function `getString` will get the input from the user; the function `printResult` will print the sorted result; the function `greaterThan` will check if one element is greater than another; and the function `swap` will swap two elements in the array. With these choices, we start with an outline of the program that looks like this.

```
#include <stdio.h>
#include <string.h>
#define MAXLENGTH 100       // max length of string input

void getString(char *str);  // helper prototypes
void printResult(char *str);
int greaterThan(char ch1, char ch2);
void swap(char *str, int index1, int index2);

int main(void) {
  int len;                  // length of the entered string
  char str[MAXLENGTH];      // input should be no longer than MAXLENGTH
  // here, any other variables you need

  getString(str);
  len = strlen(str);        // get length of the string, from string.h
  // put nested loops here to put the string in sorted order
  printResult(str);
  return(0);
}

// helper functions go here
```

Here's an example of the program running. Everything after the first colon is entered by the user. Blank spaces are written using an underscore character, since `scanf` assumes that the string ends at the first whitespace.

```
Enter the string you would like to sort:  This_is_a_cool_program!
Here is the sorted string:  !T____aacghiilmoooprrss
```

Complete the following steps in order. Do not move to the next step until the current step is successful.

a. Write the helper function `getString` to ask the user for a string and place it in the array passed to `getString`. You can use `scanf` to read in the string. Write a simple call in `main` to verify that `getString` works as you expect before moving on.

b. Write the helper function printResult and verify that it works correctly.
c. Write the helper function greaterThan and verify that it works correctly.
d. Write the helper function swap and verify that it works correctly.
e. Now define the other variables you need in main and write the nested loops to perform the sort. Verify that the whole program works as it should.

Turn in your final documented code and an example of the output of the program.

36. A more useful sorting program would take a series of names (e.g., Doe_John) and scores associated with them (e.g., 98) and then list the names and scores in two columns in descending order. Modify your bubble sort program to do this. The user enters a name string and a number at each prompt. The user indicates that there are no more names by entering 0 0.

    Your program should define a constant MAXRECORDS which contains the maximum number of records allowable. You should define an array, MAXRECORDS long, of struct variables, where each struct has two fields: the name string and the score. Write your program modularly so that there is at least a sort function and a readInput function of type int that returns the number of records entered.

    Turn in your code and example output.

37. Modify the previous program to read the data in from a file using fscanf and write the results out to another file using fprintf. Turn in your code and example output.

38. Consider the following lines of code:

```
int i, tmp, *ptr, arr[7] = {10, 20, 30, 40, 50, 60, 70};

ptr = &arr[6];
for(i = 0; i < 4; i++) {
  tmp = arr[i];
  arr[i] = *ptr;
  *ptr = tmp;
  ptr-;
}
```

a. How many elements does the array arr have?
b. How would you access the middle element of arr and assign its value to the variable tmp? Do this two ways, once indexing into the array using [] and the other with the dereferencing operator and some pointer arithmetic. Your answer should only be in terms of the variables arr and tmp.
c. What are the contents of the array arr before and after the loop?

39. The following questions pertain to the code below. For your responses, you only need to write down the changes you would make using valid C code. You should verify that your modifications actually compile and run correctly. Do not submit a full C program for this question. Only write the changes you would make using legitimate C syntax.

```
#include <stdio.h>
#define MAX 10

void MyFcn(int max);
```

```
int main(void) {
  MyFcn(5);
  return(0);
}

void MyFcn(int max) {
  int i;
  double arr[MAX];

  if(max > MAX) {
printf("The range requested is too large.  Max is %d.\n", MAX);
return;
  }
  for(i = 0; i < max; i++) {
    arr[i] = 0.5 * i;
    printf("The value of i is %d and %d/2 is %f.\n", i, i, arr[i]);
  }
}
```

a. `while` loops and `for` loops are essentially the same thing. How would you write an equivalent `while` loop that replicates the behavior of the `for` loop?
b. How would you modify the `main` function so that it reads in an integer value from the keyboard and then passes the result to `MyFcn`? (This replaces the statement `MyFcn(5);`.) If you need to use extra variables, make sure to define them before you use them in your snippet of code.
c. Change `main` so that if the input value from the keyboard is between −`MAX` and `MAX`, you call `MyFcn` with the absolute value of the input. If the input is outside this range, then you simply call `MyFcn` with the value `MAX`. How would you make these changes using conditional statements?
d. In C, you will often find yourself writing nested loops (a loop inside a loop) to accomplish a task. Modify the `for` loop to use nested loops to set the ith element in the array `arr` to half the sum of the first $i-1$ integers, i.e., `arr[i]` $= \frac{1}{2}\sum_{j=0}^{i-1} j$. (You can easily find a formula for this that does not require the inner loop, but you should use nested loops for this problem.) The same loops should print the value of each `arr[i]` to 2 decimal places using the `%f` formatting directive.

40. If there are n people in a room, what is the chance that two of them have the same birthday? If $n = 1$, the chance is zero, of course. If $n > 366$, the chance is 100%. Under the assumption that births are distributed uniformly over the days of the year, write a program that calculates the chances for values of $n = 2$ to 100. What is the lowest value n^* such that the chance is greater than 50%? (The surprising result is sometimes called the "birthday paradox.") If the distribution of births on days of the year is not uniform, will n^* increase or decrease? Turn in your answer to the questions as well as your C code and the output.
41. In this problem you will write a C program that solves a "puzzler" that was presented on NPR's CarTalk radio program. In a direct quote of their radio transcript, found here

http://www.cartalk.com/content/hall-lights?question, the problem is described as follows:

***RAY**: This puzzler is from my "ceiling light" series. Imagine, if you will, that you have a long, long corridor that stretches out as far as the eye can see. In that corridor, attached to the ceiling are lights that are operated with a pull cord.*

There are gazillions of them, as far as the eye can see. Let us say there are 20,000 lights in a row.

They're all off. Somebody comes along and pulls on each of the chains, turning on each one of the lights. Another person comes right behind, and pulls the chain on every second light.

***TOM**: Thereby turning off lights 2, 4, 6, 8 and so on.*

***RAY**: Right. Now, a third person comes along and pulls the cord on every third light. That is, lights number 3, 6, 9, 12, 15, etc. Another person comes along and pulls the cord on lights number 4, 8, 12, 16 and so on. Of course, each person is turning on some lights and turning other lights off.*

If there are 20,000 lights, at some point someone is going to come skipping along and pull every 20,000th chain.

When that happens, some lights will be on, and some will be off. Can you predict which lights will be on?

You will write a C program that asks the user the number of lights *n* and then prints out which of the lights are on, and the total number of lights on, after the last (*n*th) person goes by. Here's an example of what the output might look like if the user enters 200:

```
How many lights are there? 200

You said 200 lights.
Here are the results:
  Light number   1 is on.
  Light number   4 is on.
  ...
  Light number 196 is on.
  There are 14 total lights on!
```

Your program lights.c should follow the template outlined below. Turn in your code and example output.

```
/***************************************************************************
 * lights.c
 *
 * This program solves the light puzzler.  It uses one main function
 * and two helper functions:  one that calculates which lights are on,
 * and one that prints the results.
 *
 ***************************************************************************/

#include <stdio.h>
#include <stdlib.h>         // allows the use of the "exit()" function
```

```
#define MAX_LIGHTS 1000000  // maximum number of lights allowed

// here's a prototype for the light toggling function
// here's a prototype for the results printing function

int main(void) {

  // Define any variables you need, including for the lights' states

  // Get the user's input.
  // If it is not valid, say so and use "exit()" (stdlib.h, Sec 1.2.16).
  // If it is valid, echo the entry to the user.

  // Call the function that toggles the lights.
  // Call the function that prints the results.

  return(0);
}

// definition of the light toggling function
// definition of the results printing function
```

42. We have been preprocessing, compiling, assembling, and linking programs with commands like

    ```
    gcc HelloWorld.c -o HelloWorld
    ```

    The `gcc` command recognizes the first argument, `HelloWorld.c`, is a C file based on its `.c` extension. It knows you want to create an output file called `HelloWorld` because of the `-o` option. And since you did not specify any other options, it knows you want that output to be an executable. So it performs all four of the steps to take the C file to an executable. We could have used options to stop after each step if we wanted to see the intermediate files produced. Below is a sequence of commands you could try, starting with your `HelloWorld.c` code. Do not type the "comments" to the right of the commands!

    ```
    > gcc HelloWorld.c -E > HW.i  // stop after preprocessing, dump into file HW.i
    > gcc HW.i -S -o HW.s         // compile HW.i to assembly file HW.s and stop
    > gcc HW.s -c -o HW.o         // assemble HW.s to object code HW.o and stop
    > gcc HW.o -o HW              // link with stdio printf code, make executable HW
    ```

    At the end of this process you have `HW.i`, the C code after preprocessing (`.i` is a standard extension for C code that should not be preprocessed); `HW.s`, the assembly code corresponding to `HelloWorld.c`; `HW.o`, the unreadable object code; and finally the executable code `HW`. The executable is created from linking your `HW.o` object code with object code from the `stdio` (standard input and output) library, specifically object code for `printf`.
    Try this and verify that you see all the intermediate files, and that the final executable works as expected. (An easier way to generate the intermediate files is to use `gcc`

`HelloWorld.c -save-temps -o HelloWorld`, where the `-save-temps` option saves the intermediate files.)

If our program used any math functions, the final linker command would be

```
> gcc HW.o -o HW -lm          // link with stdio and math libraries, make
                                 executable HW
```

The C standard library is linked automatically, but often the math library is not, requiring the extra `-lm` option.

The `HW.i` and `HW.s` files can be inspected with a text editor, but the object code `HW.o` and executable `HW` cannot. We can try the following commands to make viewable versions:

```
> xxd HW.o v1.txt             // can't read obj code; this makes viewable v1.txt
> xxd HW v2.txt               // can't read executable; make viewable v2.txt
```

The utility `xxd` just turns the first file's string of 0's and 1's into a string of hex characters, represented as text-editor-readable ASCII characters 0..9, A..F. It also has an ASCII sidebar: when a byte (two consecutive hex characters) has a value corresponding to a printable ASCII character, that character is printed. You can even see your message "Hello world!" buried there!

Take a quick look at the `HW.i`, `HW.s`, and `v1.txt` and `v2.txt` files. No need to understand these intermediate files any further. If you do not have the `xxd` utility, you could create your own program `hexdump.c` instead:

```
#include <stdio.h>
#define BYTES_PER_LINE 16

int main(void) {
  FILE *inputp, *outputp;                         // ptrs to in and out files
  int c, count = 0;
  char asc[BYTES_PER_LINE+1], infile[100];

  printf("What binary file do you want the hex rep of? ");
  scanf("%s",infile);                             // get name of input file
  inputp = fopen(infile,"r");                     // open file as "read"
  outputp = fopen("hexdump.txt","w");             // output file is "write"

  asc[BYTES_PER_LINE] = 0;                        // last char is end-string
  while ((c=fgetc(inputp)) != EOF) {              // get byte; end of file?
    fprintf(outputp,"%x%x ",(c >> 4),(c & 0xf));  // print hex rep of byte
    if ((c>=32) && (c<=126)) asc[count] = c;      // put printable chars in asc
    else asc[count] = '.';                        // otherwise put a dot
    count++;
    if (count==BYTES_PER_LINE) {                  // if BYTES_PER_LINE reached
      fprintf(outputp,"  %s\n",asc);              // print ASCII rep, newline
      count = 0;
    }
  }
  if (count!=0) {                                 // print last (short) line
    for (c=0; c<BYTES_PER_LINE-count; c++)        // print extra spaces
      fprintf(outputp,"   ");
      asc[count]=0;                               // add end-string char to asc
      fprintf(outputp,"  %s\n",asc);              // print ASCII rep, newline
    }
    fclose(inputp);                               // close files
    fclose(outputp);
    printf("Printed hexdump.txt.\n");
    return(0);
  }
```

Further Reading

Bronson, G. J. (2006). *A first book of ANSI C* (4th ed.). Boston, MA: Course Technology Press.

Kernighan, B. W., & Ritchie, D. M. (1988). *The C programming language* (2nd ed.). Upper Saddle River, NJ: Prentice Hall.

APPENDIX B

Circuits Review

This appendix is meant as a brief refresher on basic analysis of circuits with resistors, capacitors, inductors, diodes, bipolar junction transistors, and operational amplifiers, at the level they are used in this book. In particular, this appendix does not cover the general frequency response of circuits with complex impedance. It also does not cover digital circuit design; in this book, most logical operations are performed by the PIC32.

B.1 Basics

Primary quantities of interest in circuit analysis and design are **voltage** and **current**.

Voltage is an effort variable, analogous to force in mechanical systems. In fact, voltage is sometimes referred to as *electromotive force*. Just as a force causes a mass to move, a voltage causes electrons (and therefore current) to flow. The unit of voltage is a Volt (V). Voltage is measured across elements (e.g., the voltage, or potential, at the positive terminal of a 9 V battery is 9 V greater than at the negative terminal). By defining the voltage at a particular point in a circuit as 0 V, or ground (GND), it is possible to refer to the voltage at a point, implicitly comparing it to ground.

Current is a flow variable, analogous to velocity in mechanical systems. The unit of current is an Ampere (A), commonly shortened to amp. Current is measured as a flow through circuit elements. Current into a circuit element must equal the current coming out of the element, and therefore current can only flow around a closed loop. It cannot, for example, flow into an element and stop there.

Just as force times velocity is power in mechanical systems, voltage times current is power in electrical systems. The unit of power is the Watt (W), and 1 W = 1 A $\times$ 1 V. For example, Figure B.1 shows a generic circuit element (perhaps a battery, resistor, capacitor, diode, etc.). The voltage V across the element is defined to be positive if the potential is higher at the end of the element labeled $+$; otherwise V is negative. The current I through the element is defined to be positive if it is in the direction of the arrow, from + to $-$; otherwise I is negative.[1] With

[1] Note: The labeling of the ends of the element as $+$ and $-$ does *not* necessarily indicate which end has higher potential. It just indicates the convention chosen to call the voltage positive or negative. Similarly, the arrow does not necessarily indicate which direction the current actually flows.

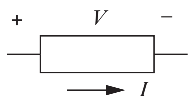


Figure B.1
Defining positive voltage *V* across, and current *I* through, a circuit element.

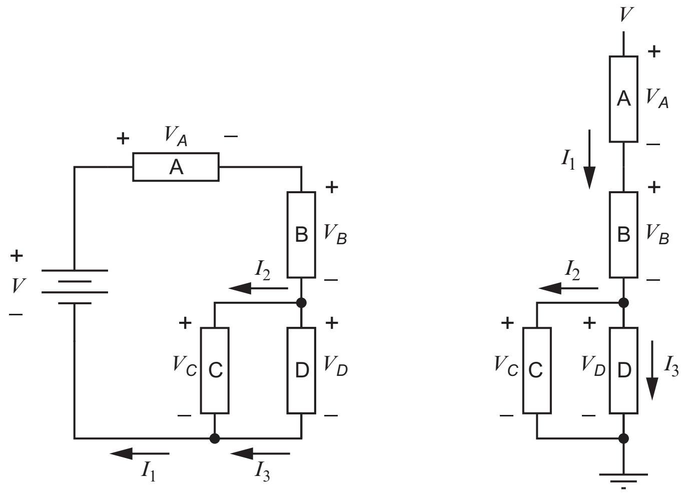


Figure B.2
The same circuit drawn two different ways, with a battery of voltage *V* and four generic elements, A through D.

these conventions, the power consumed by the element is $P = IV$. When $IV > 0$, the element is consuming electrical power, either dissipating it (as with a resistor) or storing it as energy (as with a capacitor or inductor). When $IV < 0$, the element is providing electrical power (e.g., a battery or a discharging capacitor). The unit of energy is the Joule (J), and 1 J = 1 W × 1 s.

Figure B.2 shows a circuit with a battery of voltage V and four generic elements, labeled A through D. The voltages across the elements are V_A through V_D. The current I_1 flows through the battery, A, and B; I_2 flows through C; and I_3 flows through D. The same circuit is drawn in two different ways. In one, a battery is drawn explicitly, allowing a closed loop for current to be clearly visualized. In the other, which is more common in circuit schematics, the closed loop through the battery is left implicit. This circuit also introduces the ground symbol the voltage level defined as zero volts (at the negative terminal of the battery in this case).

To solve for voltages and currents in this circuit, we use *Kirchhoff's current law* (KCL) and *Kirchhoff's voltage law* (KVL). KCL says that current is preserved at any node: current into the node is equal to current out of the node. In Figure B.2, there are two nodes where currents come together, indicated by dots, and each provides the same equation:

$$I_1 = I_2 + I_3.$$

KVL says that the sum of voltages around any closed loop must be zero. As you step around a loop, add voltages from elements where you proceed from the $-$ terminal to the $+$ terminal, and subtract voltages from elements where you proceed from the $+$ terminal to the $-$ terminal. For example, there are three loops in Figure B.2: through the battery, A, B, and C; through the battery, A, B, and D; and through C and D. These yield the following equations, respectively:

$$\begin{aligned} V - V_A - V_B - V_C &= 0 \\ V - V_A - V_B - V_D &= 0 \\ V_C - V_D &= 0. \end{aligned}$$

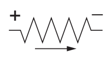
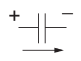
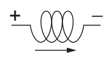
Only two of these equations are independent. For example, the third equation can be used to show that the first two are equivalent.

We now have three independent equations (one from KCL and two from KVL) to solve for seven unknowns in the circuit: the three currents I_1, I_2, and I_3, and the four voltages across the elements, V_A, V_B, V_C, and V_D. To get four more equations, we need the *constitutive laws* of the elements, relating the voltages across the elements to the currents through them. Let us begin with the constitutive laws of the common linear circuit elements: resistors, capacitors, and inductors.

B.2 Linear Elements: Resistors, Capacitors, and Inductors

Resistors, capacitors, and inductors are called *linear* circuit elements because the voltages across the elements are proportional to the current, time integral of the current, or derivative of the current, respectively. The symbols, units, constitutive laws, and information about power and energy are summarized in Table B.1. Resistors only dissipate power, as heat, while

Table B.1: The three linear circuit elements and the constitutive laws relating the current *I* through them and the voltage *V* across them

Element	Schematic	Symbol	Unit	Constitutive Law	Power (W)	Energy Stored (J)
Resistor		R	Ohm (Ω)	$V = IR$	I^2R dissipated	0
Capacitor		C	Farad (F)	$I = C\frac{dV}{dt}$	$CV\frac{dV}{dt}$	$\frac{1}{2}CV^2$
Inductor		L	Henry (H)	$V = L\frac{dI}{dt}$	$LI\frac{dI}{dt}$	$\frac{1}{2}LI^2$

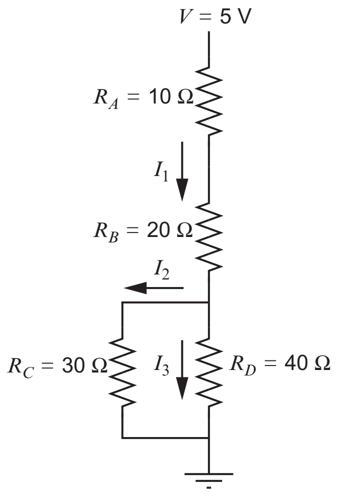


Figure B.3
A resistor network.

capacitors and inductors do not dissipate any power, but either charge (consuming electrical power and storing it as energy) or discharge (providing electrical power).

Figure B.3 shows the circuit of Figure B.2 with the generic elements replaced by resistors. We can solve for the four voltages across the resistors and the three currents by simultaneously solving the seven KCL, KVL, and constitutive law equations:

$$\begin{aligned}
\text{KCL: } I_1 &= I_2 + I_3 \\
\text{KVL: } 0 &= V - V_A - V_B - V_D \\
0 &= V_C - V_D \\
\text{Constitutive laws: } V_A &= I_1 R_A \\
V_B &= I_1 R_B \\
V_C &= I_2 R_C \\
V_D &= I_3 R_D
\end{aligned}$$

Substituting the battery voltage $V = 5$ V and the resistances $R_A = 10\ \Omega$, $R_B = 20\ \Omega$, $R_C = 30\ \Omega$, and $R_D = 40\ \Omega$, the currents and voltages can be solved as

$$\begin{aligned}
&I_1 = 0.106 \text{ A}, \quad I_2 = 0.061 \text{ A}, \quad I_3 = 0.045 \text{ A} \\
&V_A = 1.061 \text{ V}, \quad V_B = 2.121 \text{ V}, \quad V_C = 1.818 \text{ V}, \quad V_D = 1.818\ V.
\end{aligned}$$

Table B.2: Equivalent resistance, capacitance, and inductance of elements in series and parallel

Elements	In Series	In Parallel
Resistors R_1, R_2	$R_1 + R_2$	$R_1R_2/(R_1 + R_2)$
Capacitors C_1, C_2	$C_1C_2/(C_1 + C_2)$	$C_1 + C_2$
Inductors L_1, L_2	$L_1 + L_2$	$L_1L_2/(L_1 + L_2)$

According to our sign convention, where I is defined as positive if it flows from the terminal labeled $+$ to the terminal labeled $-$, the power consumed by the battery is $-I_1V = -(0.106\text{ A})(5\text{ V}) = -0.53$ W. Since the power consumed is negative, the battery is providing power. The power consumed by R_A is $I_1V_A = I_1^2R_A = 0.112$ W. The power consumed by R_B, R_C, and R_D can be calculated as 0.225, 0.112, and 0.081 W, respectively, and the sum of the power dissipated by the resistors is 0.53 W, equal to the power provided by the battery, as we would expect.

If any of the elements were capacitors or inductors, those constitutive laws would relate the current through a capacitor to the rate of change of the voltage, or the voltage across an inductor to the rate of change of current. Instead of simply solving linear equations as above, we must now solve linear differential equations. In this book we do not delve into analysis of linear circuits with arbitrary combinations of resistors, inductors, and capacitors, but focus on circuits with resistors only, as above, as well as circuits with resistors and either a single capacitor or a single inductor (Section B.2.1). Such circuits cover many practical cases of interest in mechatronics.

In Figure B.3, the resistors R_A and R_B are said to be *in series*. The resistors R_C and R_D are said to be *in parallel*. A simple derivation shows that resistors in series act like a single resistor of greater resistance, $R_{\text{series}} = R_A + R_B$, and resistors in parallel act like a single resistor of lesser resistance (since there are now two paths for the current to follow), $R_{\text{parallel}} = R_CR_D/(R_C + R_D)$. Similar relationships can be derived for capacitors and inductors (Table B.2).

The last linear element we will use is the *potentiometer*, or *pot* for short (Figure B.4). A pot is a resistor with three connections: the terminals at either end, like a regular resistor, and a third connection called the *wiper*. The wiper is an electrical contact that can slide from one end of the resistor to the other, creating a variable resistance between the wiper and the end connections. If R_+ is the resistance between the $+$ terminal of the resistor and the wiper, and R_- is the resistance between the $-$ terminal and the wiper, then the sum of R_+ and R_- always equals R, where R is the total resistance of the pot between the two ends. Pots often come packaged in rotary knobs, and turning the knob moves the wiper to allow R_+ and R_- to be varied from approximately 0 to R.

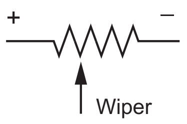


Figure B.4
Symbol for a potentiometer.

B.2.1 Time Response of RC and RL Circuits

Figure B.5(a)–(d) shows four circuits, each consisting of a resistor R and either a capacitor C or an inductor L. For mechatronics, circuits with resistors and a single inductor are important for understanding the behavior of motors, which have significant inductance. Circuits with resistors and a single capacitor are often used for signal filtering.

The circuits in Figure B.5(a)–(d) are powered by a time-varying voltage $V_{\text{in}}(t)$ that periodically switches between $V_{\text{hi}} > 0$ and 0 V relative to ground. Let us focus on the circuit in Figure B.5(a), where $V_{\text{out}} = V_C$ is the voltage across the capacitor.

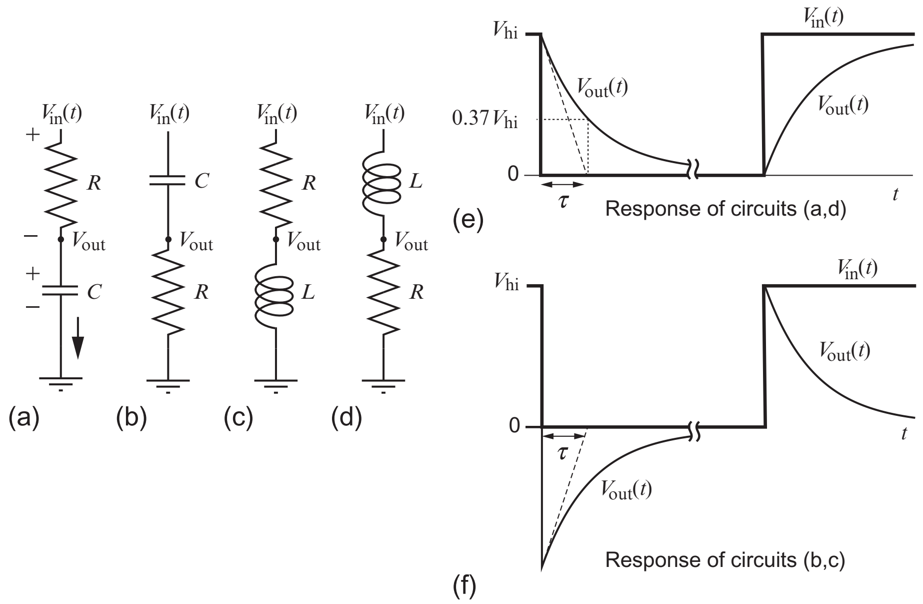


Figure B.5
(a–d) RC circuits and RL circuits. (e) Response of the circuits in (a) and (d) to a changing $V_{\text{in}}(t)$. Note the discontinuity in time in the middle of the plot, to allow the response to reach steady state. (f) Response of the circuits in (b) and (c) to a changing $V_{\text{in}}(t)$.

KVL tells us that the circuit in Figure B.5(a) satisfies

$$V_{\text{in}}(t) = V_R(t) + V_C(t) = I(t)R + \frac{1}{C}\int_0^t I(s)\,\mathrm{d}s.$$

Assume that the input voltage $V_{\text{in}}(t)$ is equal to V_{hi}, and has been for a long time so that any transients have died out. Current flowing through the capacitor has charged it up until, in steady state, the capacitor is fully charged to a voltage $V_C = V_{\text{hi}}$ and an energy $\frac{1}{2}CV_{\text{hi}}^2$. Therefore there is no voltage across the resistor, so $I = 0$. The capacitor is acting like an open circuit.

The key point is that the voltage across the capacitor $V_C(t)$ cannot be discontinuous in time if current is finite. For example, V_C cannot change from 0 to 5 V instantaneously; it takes time for the current to integrate to develop a voltage across the capacitor.

Now consider what happens when $V_{\text{in}}(t)$ instantly changes from V_{hi} to 0 V. Since the voltage across the capacitor cannot change instantaneously, just after the switch occurs, the voltage across the capacitor is still V_{hi}. This means that the voltage across the resistor R is $-V_{\text{hi}}$. Therefore current must be flowing from ground through the capacitor and resistor. By KVL, and the constitutive law of the resistor, we can calculate the current I just after the switch at time $t = 0$ (let us call this time 0^+):

$$V_{\text{in}}(0^+) = 0 = V_R(0^+) + V_C(0^+) = I(0^+)R + V_{\text{hi}} \quad \rightarrow \quad I(0^+) = -\frac{V_{\text{hi}}}{R}.$$

This negative current begins to discharge the energy stored in the capacitor, and therefore the voltage across the capacitor begins to drop. To solve for $\mathrm{d}V_C/\mathrm{d}t$, the rate of change of the voltage across the capacitor, at time 0^+, we use the constitutive law of the capacitor:

$$I(0^+) = -\frac{V_{\text{hi}}}{R} = C\frac{\mathrm{d}V_C}{\mathrm{d}t}(0^+) \quad \rightarrow \quad \frac{\mathrm{d}V_C}{\mathrm{d}t}(0^+) = -\frac{V_{\text{hi}}}{RC}.$$

If the capacitor continued to discharge at this rate, it would fully discharge in RC seconds.

Of course the capacitor does not continue to discharge at this rate; the rate slows as the voltage across the capacitor drops. To fully solve for $V_C(t)$ using KVL and the constitutive law of the capacitor, we solve the first-order linear differential equation

$$\begin{aligned} 0 &= I(t)R + V_C(t) \\ 0 &= C\frac{\mathrm{d}V_C}{\mathrm{d}t}(t)R + V_C(t) \\ \frac{\mathrm{d}V_C}{\mathrm{d}t}(t) &= -\frac{1}{RC}V_C(t) \end{aligned} \tag{B.1}$$

to get

$$V_C(t) = V_0 \mathrm{e}^{-t/RC} \tag{B.2}$$

where the initial voltage V_0 is V_{hi}. This is an exponential decay to zero as $t \to \infty$. The *time constant* of the decay is $\tau = RC$, in seconds. One time constant after the switch, the voltage is $V_C(\tau) = V_0\mathrm{e}^{-1} = 0.37V_0$; the voltage has decayed by 63%. After 3τ, the voltage has decayed to 5% of its initial value.

Note that the time constant $\tau = RC$ is large if R is large, since the large resistance limits the current to charge or discharge the capacitor, or if C is large, because it takes more time for the current to charge or discharge energy in the capacitor, which is $\frac{1}{2}CV_C^2$.

If instead $V_{\mathrm{in}}(t)$ has been at 0 V for a long time and then switches to V_{hi} at $t = 0$, a similar derivation yields

$$V_{\mathrm{out}} = V_C(t) = V_{\mathrm{hi}}(1 - \mathrm{e}^{-t/RC}),$$

a rise from 0 V asymptoting at V_{hi}. The voltage across the capacitor rises to 63% (95%) of V_{hi} after time τ (3τ).

Figure B.5(e) shows a plot of the fall and rise of the voltage $V_{\mathrm{out}}(t) = V_C(t)$ in the circuit in Figure B.5(a) in response to a $V_{\mathrm{in}}(t)$ occasionally switching between V_{hi} and 0.

In Figure B.5(b), the positions of the capacitors and the resistors are reversed, so $V_{\mathrm{out}} = V_R = V_{\mathrm{in}}(t) - V_C(t)$. The response of $V_{\mathrm{out}}(t)$ to the switching $V_{\mathrm{in}}(t)$ is shown in Figure B.5(f). In this case, V_{out} spikes to $-V_{\mathrm{hi}}$ on a falling edge of $V_{\mathrm{in}}(t)$, then decays back to zero, and spikes to V_{hi} on a rising edge of $V_{\mathrm{in}}(t)$, then decays back to zero.

In summary, the output of the circuit in Figure B.5(a) is a smoothed version of $V_{\mathrm{in}}(t)$, where the output gets smoother as RC gets larger, while the output in circuit in Figure B.5(b) responds most strongly at the times of the switches of $V_{\mathrm{in}}(t)$. Smoothing is characteristic of a *low-pass filter*, while strong response to signal changes is characteristic of a *high-pass filter*; see Section B.2.2.

The circuits in Figure B.5(c) and (d) can be analyzed similarly, now using the constitutive law $V_L = L\,\mathrm{d}I/\mathrm{d}t$ for the inductor instead of $I = C\,\mathrm{d}V_C/\mathrm{d}t$ for the capacitor. It is also important to realize that the inductor does not allow current to change discontinuously, as a discontinuous current implies an unbounded voltage $L\,\mathrm{d}I/\mathrm{d}t$ across the inductor. It takes time to charge or discharge the energy in the inductor, $\frac{1}{2}LI^2$, and therefore I cannot change instantaneously.

Based on this analysis, we see that the response of the RL circuit in Figure B.5(c) is that shown in Figure B.5(f), but now with a time constant $\tau = L/R$. Similarly, the response of the RL circuit in Figure B.5(d) is shown in Figure B.5(e), again with a time constant $\tau = L/R$.

Table B.3: Summary of capacitor and inductor behavior

Element	Rule Enforced	Discharged	Charged
Capacitor	Continuous voltage	Wire	Open circuit
Inductor	Continuous current	Open circuit	Wire

Note that the time constant is large if R is small, since the low resistance does not dissipate much power for a given current, or if L is large, since it takes longer to charge or discharge the inductor's energy $\frac{1}{2}LI^2$.

Some rules of analyzing circuits with a capacitor or inductor are summarized in Table B.3. When a capacitor is initially discharged, it lets current flow freely (like a wire), and when it is fully charged, it behaves like an open circuit (no current flows). When an inductor is initially discharged, it behaves like an open circuit (it takes time for current to build up as initially all voltage is claimed by $L\,\mathrm{d}I/\mathrm{d}t$), and when it is fully charged and $dI/dt = 0$, it lets current flow freely with no voltage across it (like a wire).

Application: Switch debouncing

Figure B.6 illustrates a closing mechanical switch, nominally generating a clean falling edge from GND to V. In practice, mechanical switches tend to *bounce*; the two metal contacts impact and bounce before coming into steady contact. The result is a $V_0(t)$ that rapidly switches between V and GND before settling at GND. Switch bounce is a common problem, and programs responding to button presses should not respond to the bounces.

To remedy the signal bounce, a debouncing circuit, as shown in Figure B.6, can be used. First, an RC filter is used to slow down the voltage variations, creating the signal $V_1(t)$. Because the

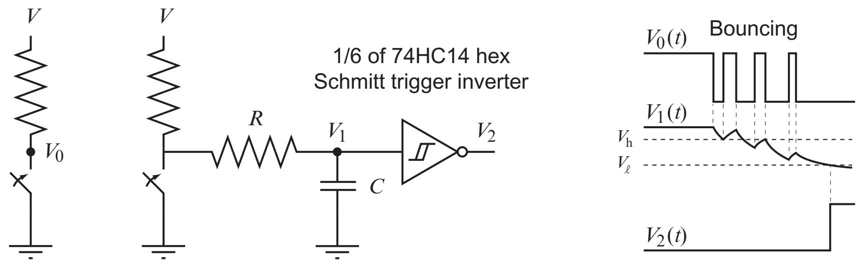


Figure B.6
(Left) Bounces on the closing of a mechanical switch generate the output signal $V_0(t)$ on the right. (Middle) A debouncing circuit. The bouncing signal is RC filtered, creating the signal $V_1(t)$. This signal then passes through a Schmitt trigger inverter, creating a single clean rising edge $V_2(t)$. (Right) The signals $V_0(t)$, $V_1(t)$, and $V_2(t)$. The Schmitt trigger hysteresis voltages V_h and V_ℓ are shown on the signal $V_1(t)$.

bouncing transitions occur quickly, the signal $V_1(t)$ changes little during the bouncing. Once the bouncing has ended, $V_1(t)$ drops steadily to zero, according to the RC time constant.

The signal V_1 is then fed to a digital output Schmitt trigger chip. The purpose of a Schmitt trigger is to implement *hysteresis*: if the input is currently low, the output does not change until the input has risen past a voltage V_h; if the input is high, the output does not change until the input has dropped below a voltage V_ℓ; and $V_\mathrm{h} > V_\ell$. This hysteresis means that small variations of the input signal should not change the output signal. Further, a Schmitt trigger *inverter* makes the digital output opposite the input. The 74HC14 chip has six Schmitt trigger inverters on it.

Because the Schmitt trigger inverter ignores the small voltage variations at $V_1(t)$ during bouncing, it does not change its output until the switch contact is steady. The end result of the debouncing circuit, $V_2(t)$, is a single clean rising edge, after the bounces have terminated.

Since it is unlikely that you will need to press and release a button in less than 10 ms, it is not unreasonable to choose $RC \approx 10$ ms.

There are other debouncing circuits, and debouncing can instead be performed in software. See Exercise 16 of Chapter 6.

B.2.2 Frequency Response of RC and RL Circuits

In the previous section we focused on the time response of RC and RL circuits in response to step changes in voltage. The step response is helpful to understand, as microcontrollers and some sensors output digital signals. We should remember, however, that by Fourier decomposition, any periodic signal of frequency f can be represented by a sum of sinusoids of frequency f, $2f$, $3f$, etc. For example, the 50% duty cycle square wave of amplitude 1 and frequency f in Figure B.7 can be represented by an infinite sum of sinusoids at frequencies f, $3f$, $5f$, etc. Therefore it is useful to understand the behavior of circuits in response to sinusoidal inputs.

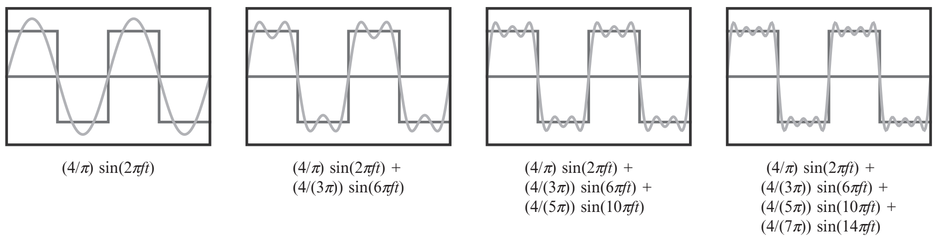


Figure B.7
The lowest four frequency components of a Fourier decomposition of a square wave of amplitude 1 and frequency f.

If the input to a linear system, like an RCL circuit, is a sinusoid of the form $V_{\text{in}}(t) = A\sin(2\pi ft)$, the output is a scaled, phase-shifted sinusoid of the same frequency, $V_{\text{out}}(t) = G(f)A\sin(2\pi ft + \phi(f))$, where the system's gain $G(f)$ and phase $\phi(f)$ are a function of the frequency f of the input (Figure B.8). Collectively the gain $G(f)$ and the phase $\phi(f)$ are called the *frequency response* of the system. For periodic non-sinusoidal input signals like the square wave in Figure B.7, the output is the sum of the individually scaled and shifted sinusoids that constitute the Fourier decomposition of the input.

Each of the RC and RL circuits in Figure B.5 is a linear system, with $V_{\text{in}}(t)$ as input and $V_{\text{out}}(t)$ as output. Without derivation (take a linear systems or circuits course!), the frequency responses of the circuits are plotted in Figure B.9, where Figure B.9(a) is the frequency response $G(f)$ and $\phi(f)$ of the circuits in Figure B.5(a) and (d), and Figure B.9(b) is the frequency response of the circuits in Figure B.5(b) and (c). Note that the frequency and gain

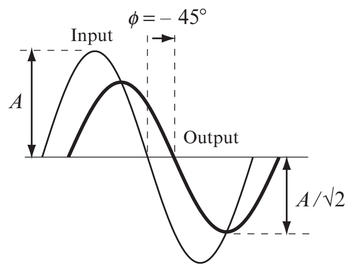


Figure B.8
An example linear system time response to a sinusoidal input of amplitude A. The output is phase shifted by $\phi = -45°$ and scaled by a gain $G = 1/\sqrt{2}$.

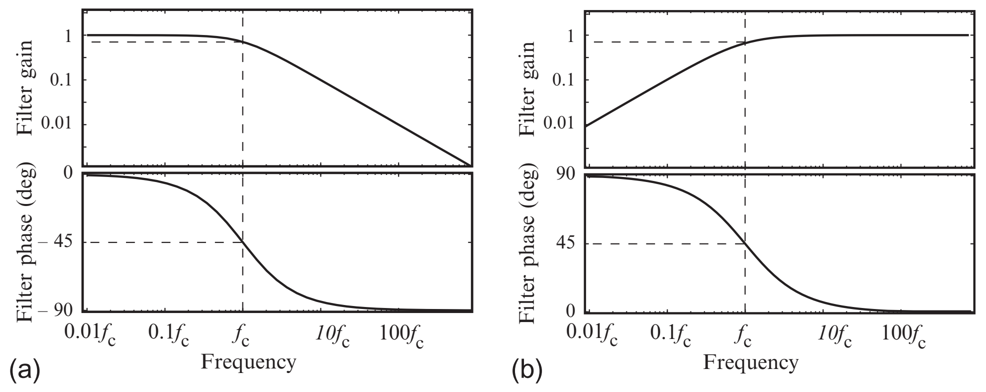


Figure B.9
(a) The frequency response of a first-order low-pass filter, e.g., the circuits in Figure B.5(a) and (d). (b) The frequency response of a first-order high-pass filter, e.g., the circuits in Figure B.5(b) and (c).

are plotted on log scales, to cover larger ranges of values and to more clearly show the essential features of the response.

The frequency response in Figure B.9(a) corresponds to a *low-pass filter* (LPF). This name comes from the fact that low-frequency sinusoids are passed from the input to the output with little change: $G \approx 1$ and $\phi \approx 0$. As the frequency increases, the gain drops and the output signal begins to lag the input signal ($\phi < 0$). At the *cutoff frequency* $f_c = 1/(2\pi\tau)$, where $\tau = RC$ for the RC circuits and $\tau = L/R$ for the RL circuits, the gain has dropped to $G(f_c) = 1/\sqrt{2}$ and the phase has dropped to $\phi(f_c) = -45°$. Beyond the cutoff frequency the gain drops by a factor of 10 for every increase in the frequency by a factor of 10, so $G(10f_c) \approx 0.1$ and $G(100f_c) \approx 0.01$. The phase ϕ continues to drop, asymptoting at $-90°$ at high frequencies.

The frequency response in Figure B.9(b) corresponds to a *high-pass filter* (HPF). This name comes from the fact that high-frequency sinusoids are passed from the input to the output with little change: $G \approx 1$ and $\phi \approx 0$. As the frequency decreases, the gain drops and the output signal begins to lead the input signal ($\phi > 0$). At the cutoff frequency $f_c = 1/(2\pi\tau)$, the gain has dropped to $G(f_c) = 1/\sqrt{2}$ and the phase has risen to $\phi(f_c) = 45°$. Beyond the cutoff frequency the gain drops by a factor of 10 for every decrease in the frequency by a factor of 10, so $G(0.1f_c) \approx 0.1$ and $G(0.01f_c) \approx 0.01$. This means that DC (constant) signals are completely suppressed by the filter. The phase ϕ continues to rise with decreasing frequency, asymptoting at 90° at low frequencies.

Low-pass and high-pass filters are useful for isolating a signal of interest from other signals summed with it. For example, LPFs can be used to smooth and suppress high-frequency noise on a sensor line. HPFs can be used to suppress DC signals and only look for sudden changes in a sensor reading, just as the output depicted in Figure B.5(f) is largest when the input suddenly changes value and drops to zero when the signal is constant.

The LPFs and HPFs illustrated in Figure B.9 are called *first order* because the circuit response is described by a first-order differential equation, e.g., (B.1). First-order filters have relatively slow *rolloff*—in the cutoff frequencies, the filter gain drops by only a factor of 10 for every factor of 10 in frequency. By using more passive elements, it is possible to design second-order filters with a gain rolloff of 100 for every factor of 10 in frequency, which are better at suppressing signals at frequencies we want to eliminate with less effect on signals at frequencies we want to preserve. Higher-order filters, with even steeper rolloff, can be constructed by putting first- and second-order filters in series. LPFs and HPFs can also be combined to create *bandpass* filters, which suppress frequency components below some $f_{\min}$ and above some $f_{\max}$, or *bandstop* or *notch* filters, which suppress frequency components between $f_{\min}$ and $f_{\max}$.

Filters constructed purely with resistors, capacitors, and inductors are called *passive* filters, as these circuit elements do not generate power. More sophisticated *active* filters can be created using operational amplifiers (Section B.4). These circuits have the advantage of having high input impedance (drawing very little current from V_{in}) and low output impedance (capable of supplying a lot of current at V_{out} while maintaining the desired behavior as a function of V_{in}).

The gain (or magnitude) portion of the frequency response is often written in terms of decibels as M_{dB}, where

$$M_{\text{dB}} = 20 \log_{10} G.$$

So a gain of 1 corresponds to 0 dB, a gain of 0.1 corresponds to −20 dB, and a gain of 100 corresponds to 40 dB.

B.3 Nonlinear Elements: Diodes and Transistors

Nonlinear circuit elements are critical for computing and nearly all modern circuits. Two common types of nonlinear elements are diodes and transistors. While there are many kinds of transistors, in this section we focus on bipolar junction transistors (BJTs), notably excluding field effect transistors (FETs), which are extraordinarily useful and come in many different varieties. In keeping with the spirit of this appendix, we do not get into the semiconductor physics of these nonlinear elements, but focus on simplified models that facilitate analysis.

Analyzing circuits with simplified models of nonlinear elements is quite different from circuits with only linear elements. We do not simply write a set of equations and solve them. Instead, a nonlinear element can operate in different regimes (two regimes for a diode: conducting and not conducting; and three regimes for a BJT: off, linear, and saturated), each regime with its own governing equations. In principle, we have to solve a complete set of circuit equations for each possible combination of regimes for the nonlinear elements in the circuit. All but one of these guesses at the operating regimes will be wrong, leading to equations and inequalities without valid solutions.

B.3.1 Diodes

Figure B.10 shows the circuit symbol and the simplified current-voltage behavior of a diode. When the voltage across the diode is less than the *forward bias voltage* $V_d \geq 0$, no current flows through the diode. When current flows, it is only allowed to flow in the direction indicated in Figure B.10, from anode to cathode, and the voltage across the diode is V_d. It is never possible to have a voltage greater than V_d across the diode. A typical forward bias voltage for a diode is around 0.7 V, but other values are also possible.

Figure B.11(a) shows a simple resistor-diode circuit. Assume $V_{\text{in}} = 5$ V and $V_d = 0.7$ V. To solve for V_{out}, we analyze the circuit for the two possible cases of the diode: conducting or not conducting.

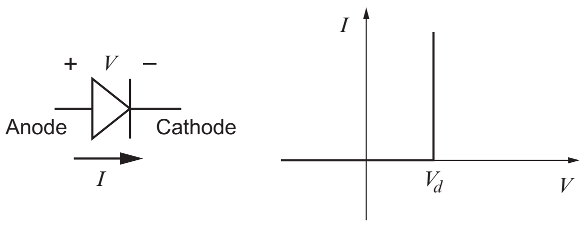


Figure B.10
(Left) The circuit symbol for a diode, indicating positive current and positive voltage. (Right) The simplified current-voltage relationship for a diode with forward bias voltage V_d.

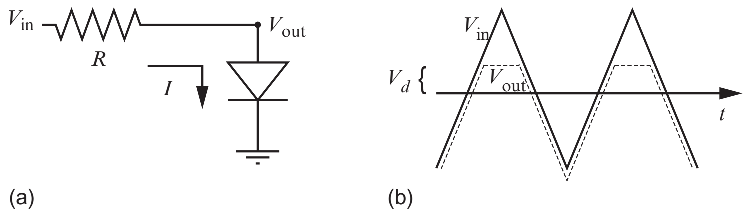


Figure B.11
(a) A resistor-diode circuit. (b) The output voltage V_{out} for a sawtooth V_{in}. The output voltage is equal to the input voltage for $V_{in} < V_d$, but the output voltage is capped at V_d by the diode. Reversing the direction of the diode would cause the output to track the input for $V_{in} > -V_d$, and the voltage would never drop below $-V_d$.

- **Case 1: diode is not conducting.** In this case, we know $I = 0$, so there is no voltage drop across the resistor, so $V_{out} = V_{in} = 5$ V. But we know that the diode can never have more than $V_d = 0.7$ V across it. Therefore this regime is not valid.
- **Case 2: diode is conducting.** Since the diode is conducting in this case, we know that the voltage across the diode is the forward bias voltage $V_d = 0.7$ V. Therefore the current I must be $(5\text{ V} - 0.7\text{ V})/R$ from the constitutive law of the resistor. This current is flowing in the right direction (positive current) and therefore does not violate the current-voltage relationship of the diode, so this is a valid solution.

Figure B.11(b) illustrates V_{out} as V_{in} follows a sawtooth profile, showing that V_{out} can never exceed V_d. The power dissipated by the diode when current I flows is IV_d.

A *light-emitting diode* (LED) is just a diode that emits visible or invisible light when current flows. A typical forward bias voltage for an LED is 1.7 V.

If a large negative voltage is placed across a diode, it may break down, allowing negative current to flow. While this is a failure mode for most diodes, for *Zener* diodes it is the intended use. Zener diodes are designed to have specific (and often relatively small) negative

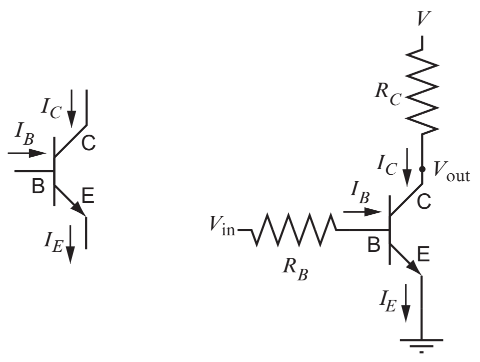


Figure B.12
(Left) The circuit symbol for an NPN transistor, showing the collector, base, and emitter. (Right) A resistor-transistor circuit.

breakdown voltages and to allow current to flow easily while at that breakdown voltage. Zener diodes can be used in voltage regulator applications.

B.3.2 Bipolar Junction Transistors

Bipolar junction transistors come in two flavors: NPN and PNP. We will focus on the NPN BJT, then return to the PNP.

Figure B.12 shows the circuit symbol for an NPN BJT. It has three connections: the collector (C), base (B), and emitter (E). Current flows into the collector and base, and the sum of those currents flows out of the emitter, $I_E = I_C + I_B$. The voltage drop from the base to the emitter is denoted V_{BE} and the voltage drop from the collector to the emitter is written V_{CE}. In normal usage, I_C, I_B, I_E, and V_{CE} are all nonnegative.

The basic function of the NPN BJT is to attempt to generate a collector current that amplifies the base current, $I_C = \beta I_B$, where β is the gain of the transistor (also commonly referred to as h_{FE}). A typical value of β is 100. Depending on the amount of base current flowing, the transistor can be in one of three modes—off, linear, or saturated—and each mode provides three equations governing the transistor voltages and currents:

- **Linear:** $I_B > 0$ and $V_{CE} > V_{CE,sat}$. In this mode, the collector current is $I_C = \beta I_B$, and the transistor is not yet saturated, so if I_B increases, I_C will also increase. Saturation occurs when V_{CE} drops to the collector-emitter saturation voltage $V_{CE,sat}$, which is commonly around 0.2 V or so. In the linear mode, V_{BE} is equal to $V_{BE,on}$, the PN junction diode voltage drop from the base to the emitter. A typical value is $V_{BE,on} = 0.7$ V. **Governing equations:** $I_C = \beta I_B$, $V_{BE} = V_{BE,on}$, and $I_E = I_C + I_B$.

- **Off:** $I_B = 0$. This means that $I_C = 0$ and therefore $I_E = 0$. V_{BE} is less than $V_{BE,on}$. **Governing equations:** $I_B = I_C = I_E = 0$.
- **Saturated:** $I_B > 0$ and $V_{CE} = V_{CE,sat}$. In this mode the collector cannot provide more current even if the base current increases; the transistor is saturated. This is because the voltage between the collector and emitter cannot drop below $V_{CE,sat}$. This means that the relationship $I_C = \beta I_B$ no longer holds. **Governing equations:** $V_{CE} = V_{CE,sat}$, $V_{BE} = V_{BE,on}$, and $I_E = I_C + I_B$.

When $I_E > 0$, the power dissipated by the transistor is $I_B V_{BE,on} + I_C V_{CE,sat}$.

Figure B.12 shows an NPN BJT in a *common emitter* circuit, so called because the emitter is attached to ground ("common"). We can determine the transistor's operating mode as a function of V_{in}:

- **Off:** $V_{\text{in}} \leq V_{BE,on}$. Input voltages in this range do not provide enough voltage to turn on the base-emitter PN junction while also providing a base current $I_B > 0$. Since $I_C = 0$, there is no voltage drop across R_C, and $V_{\text{out}} = V$.
- **Saturated:** $V_{\text{in}} \geq V_{BE,on} + R_B(V - V_{CE,sat})/(\beta R_C)$. When the transistor is saturated, the output voltage is $V_{\text{out}} = V_{CE,sat}$, and the voltage across R_C is $V - V_{CE,sat}$. This means $I_C = (V - V_{CE,sat})/R_C$. At the boundary between the linear and saturated regions, the relationship $I_C = \beta I_B$ is still satisfied, so $I_B = I_C/\beta$. So the minimum V_{in} for saturation is the sum of $V_{BE,on}$ and the voltage drop $I_B R_B$ across the base resistor.
- **Linear:** All V_{in} between the off and saturated regimes. In this regime, $I_B = (V_{\text{in}} - V_{BE,on})/R_B$ and $I_C = \beta I_B$, so

$$V_{\text{out}} = V - I_C R_C = V - \frac{\beta R_C}{R_B}(V_{\text{in}} - V_{BE,on}).$$

To increase the gain of a transistor we can use two transistors, Q1 and Q2, as a *Darlington pair* (Figure B.13(a)). The two collectors are connected and the emitter of Q1 feeds the base of Q2. The two together act like a single transistor, with the base of Q1 as the base of the pair

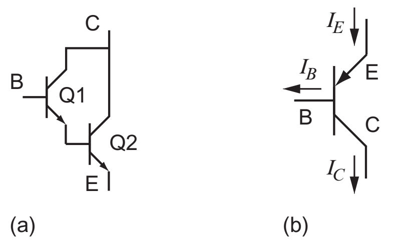


Figure B.13
(a) An NPN Darlington pair. (b) A PNP BJT.

and the emitter of Q2 as the emitter of the pair, but now the overall gain is $\beta_1\beta_2$. $V_{BE,on}$ for the Darlington pair is the sum of the individual base-emitter voltages.

Finally, Figure B.13(b) shows the circuit symbol for a PNP transistor. For a PNP BJT, $I_C = \beta I_B$ as with the NPN, but now I_B and I_C flow out of the transistor and $I_E = I_C + I_B$ flows into it. At saturation, V_E is greater than V_C (typically by about 0.2 V), and when the transistor is on, the voltage drop from V_E to V_B is approximately 0.7 V. The PNP BJT is off if $I_B = 0$; it is saturated if $I_B > 0$ and the voltage drop from V_E to V_C indicates saturation; and otherwise it is in the linear mode.

B.4 Operational Amplifiers

The circuit symbol for an operational amplifier (op amp) is shown in Figure B.14(a). Apart from the power supply inputs, the op amp has two inputs, a noninverting input labeled + and an inverting input labeled −, and one output. Figure B.14(b) shows a particular chip, the 8-pin Texas Instruments TLV272, which has two op amps.

An ideal op amp obeys the following rules:

1. Input impedance is infinite. No current flows in or out of the inputs.
2. Output impedance is zero. The op amp can produce any current necessary to satisfy the following rule.

3(a). If there is no feedback connection between the output and the inputs, then the output satisfies $V_{\text{out}} = G(V_{\text{in}+} - V_{\text{in}-})$, where the gain G is very large, effectively infinite. (The output goes to its maximum positive or negative value if $V_{\text{in}+}$ and $V_{\text{in}-}$ are different.)

3(b). If there is a current path from the output to the inverting input (*negative feedback*), for example through a capacitor or resistor, then the voltage at the two inputs are equal. This is because the large gain G of the op amp attempts to eliminate the voltage difference $V_{\text{in}+} - V_{\text{in}-}$.

Almost all useful op amp circuits have negative feedback, so rule 3(b) applies.

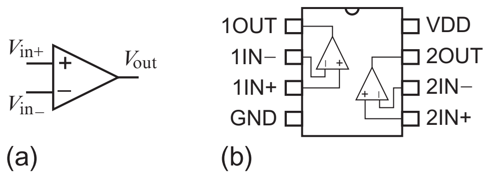


Figure B.14
(a) The op amp circuit symbol. (b) The 8-pin TLV272 integrated circuit, with two op amps.

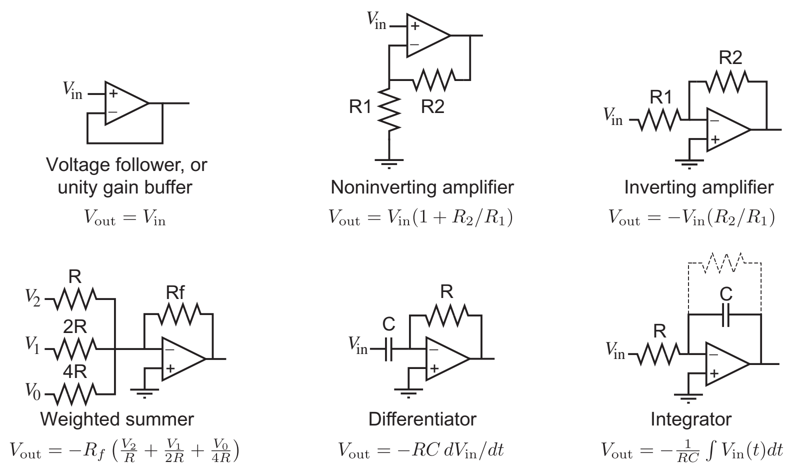

Vin
Voltage follower, or unity gain buffer
$V_{\text{out}} = V_{\text{in}}$
Vin
R1
R2
Noninverting amplifier
$V_{\text{out}} = V_{\text{in}}(1 + R_2/R_1)$
R2
R1
Vin
Inverting amplifier
$V_{\text{out}} = -V_{\text{in}}(R_2/R_1)$
R
V2
Rf
2R
V1
4R
V0
Weighted summer
$V_{\text{out}} = -R_f\left(\frac{V_2}{R} + \frac{V_1}{2R} + \frac{V_0}{4R}\right)$
R
C
Vin
Differentiator
$V_{\text{out}} = -RC\,dV_{\text{in}}/dt$
C
R
Vin
Integrator
$V_{\text{out}} = -\frac{1}{RC}\int V_{\text{in}}(t)dt$

Figure B.15

Common op amp circuits. Of these circuits, note that only the unity gain buffer and the noninverting amplifier present the very high input impedance of the op amp at the V_{in} input; in the other circuits, the input impedance is dominated by external resistors or capacitors. See Section B.5 for a discussion of input impedance.

Figure B.15 shows several useful circuits built with op amps. To analyze these circuits, remember that no current flows in or out of the op amp inputs (current only flows through the external resistors and capacitors), and since there is negative feedback in each of them, the voltages at the two inputs are equal.

For example, to analyze the weighted summer circuit, we recognize that both inputs are at 0 V (ground). Therefore the currents flowing through R, $2R$, and $4R$ are simply V_2/R, $V_1/(2R)$, and $V_0/(4R)$. Since no current flows in or out of the $-$ input, these currents sum to give the current I through the feedback resistor R_f, and the output voltage is simply $V_{\text{out}} = -IR_f$. If the input voltages V_i are binary, this circuit provides an analog voltage representation of the three-digit binary number $V_2V_1V_0$, where V_2 represents the most significant bit (since it provides the most current) and V_0 represents the least significant bit. If instead the three resistors $R, 2R, 4R$ are replaced by variable resistances set by potentiometers, the weighted summer is similar to an audio mixer.

The response of the integrator circuit is obtained by recognizing that the current flowing from V_{in} is $I = -V_{\text{in}}/R$ and that

$$V_{\text{out}} = -V_C = -\frac{1}{C}\int I(t)\,\mathrm{d}t = -\frac{1}{RC}\int V_{\text{in}}(t)\,\mathrm{d}t.$$

If V_{in} is constant at zero, ideally V_{out} should also be zero. In practice, however, the voltage across the capacitor is likely to drift due to nonidealities of the op amp (Section B.4.1), including slight input offset. For this reason, it is good practice to put another resistor in parallel with the capacitor. This resistor serves to slowly bleed off capacitor charge, counteracting voltage drift. This resistance should be much larger than R to prevent significant impact on the circuit's behavior at frequencies of interest.

The voltage follower circuit simply implements $V_{\text{out}} = V_{\text{in}}$, but it is quite useful because it draws essentially no current from the source providing V_{in} (such as a sensor) while being able to provide significant current at the output. For this reason the circuit is sometimes called a unity gain *buffer*: the op amp provides a buffer that prevents the circuit connected to the output of the op amp from affecting the behavior of the circuit connected to the input of the op amp. This allows individual circuits to be designed and tested modularly, then cascaded using buffers in between (Section B.5).

One application of the unity gain buffer is to implement an RC LPF or HPF (Section B.2.2). By cascading a unity gain buffer, then a passive RC LPF or HPF, we get the ideal frequency response of the passive filter but with high input impedance, as opposed to the relatively low input impedance of the passive filter alone. There are many more sophisticated higher-order op amp filter designs that achieve better attenuation at frequencies to be suppressed; consult any text on the design of op amp filters. In particular, for LPFs, HPFs, bandpass, and notch filters, popular filters are Butterworth, Chebyshev, and Bessel filters, each with somewhat different properties. These names refer to the form of the mathematical transfer function from input to output. To implement these transfer functions using op amps, resistors, and capacitors, there are different types of circuit designs; popular choices are the Sallen-Key circuit topology and the multiple feedback circuit topology.

B.4.1 Practical Op Amp Considerations

If you want to purchase an op amp chip, you will find that there are tens of thousands to choose from! How do you choose? Op amp data sheets can be bewildering to read, with many different characteristics, most of them depending on the particular operating condition (the power supply voltage, the load at the output, etc.). Here are a few characteristics to consider, along with the values for the flexible and inexpensive TLV272.

- **Supply voltage range.** An op amp has both a minimum and a maximum allowable voltage across the power supply lines. TLV272: 2.7-16 V.
- **Output voltage swing (rail-to-rail or not).** The maximum outputs of some op amps do not reach all the way to the power supply rails, falling short by 1 V or more. Other op amps are *rail-to-rail*, meaning that the output voltage can come close to the power supply rails. The TLV272 is rail-to-rail, with a maximum output voltage swing to within about 0.1 V of the rails.

- **Input voltage range.** Even if the output goes rail-to-rail, the differential inputs may not be allowed to approach the rails. TLV272: inputs can go rail-to-rail.
- **Output current.** The maximum amount of current that can be provided by a single output. TLV272: up to 100 mA.
- **Unity gain bandwidth.** This is a specification of how fast the op amp output can change. For an input signal at this frequency, the effective gain of the amplifier (which we assumed to be infinite) has dropped to one. TLV272: 3 MHz.
- **Slew rate.** This is another measure of how fast the output can change, in V/s. TLV272: 2.4 V/μs.
- **Input bias current.** There is actually a very small current at the inputs (we assumed it to be zero). This is the typical average of those input currents. TLV272: 1 pA.
- **Input offset current.** This is the typical difference between the two input currents. TLV272: 1 pA.
- **Input offset voltage.** Ideally zero voltage difference at the inputs would cause zero voltage at the open-loop output. In practice, the input levels may have to be slightly different to achieve a zero voltage output. TLV272: 0.5 mV.
- **Common-mode rejection ratio.** The ideal amplifier amplifies only the difference between the voltages at the + and − inputs, but there is actually a small amplification of the common voltage between them. For example, if the voltage V_+ is 5.1 V and the voltage V_- is 5.0 V, the usual amplifier gain acts on the 0.1 V difference while the common-mode gain acts on the average, 5.05 V. The CMRR specifies the ratio of the differential gain to the common-mode gain. TLV272: 80 dB (or 10,000).
- **Number of op amps.** Some chips have more than one op amp. TLV272: two op amps.
- **Packaging.** Op amps come in different types of packaging. DIP packages are easiest to work with for breadboard prototyping. TLV272: available in a variety of packages, including an 8-pin DIP.
- **Price.** Price increases for higher bandwidth and slew rates, higher output current, lower offset voltage, higher common-mode rejection ratio, rail-to-rail operation, etc. TLV272: about one dollar.

B.4.2 Single Supply Design and Virtual Ground

When using an op amp in microcontroller applications, often the only power supply available is a positive voltage rail and ground (no negative rail), and the positive voltage may be small (e.g., 3.3 V). First of all, this likely means that a rail-to-rail op amp should be used, to maximize the output voltage range, and it should be capable of being powered by the microcontroller voltage (e.g., 3.3 V). Secondly, notice that many of the standard op amp circuits (Figure B.15) provide an output voltage that has a sign opposite of the input voltage, which the op amp cannot produce if there is no negative rail.

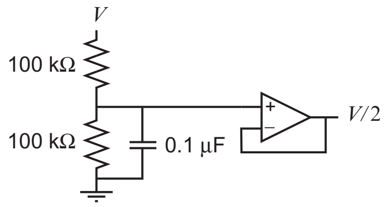


Figure B.16
Creating a virtual ground at $V/2$.

One way to handle this issue is to introduce a *virtual ground* at a voltage halfway between the positive supply rail and ground, effectively creating a bipolar supply about this virtual ground. Figure B.16 illustrates the idea. A unity gain buffer is used in combination with a voltage divider to create a virtual ground at $V/2$. The capacitor helps stabilize the reference voltage to any transients on the power supply line. This virtual ground is then used in place of ground in the inverting circuits in Figure B.15. Inverted voltages are now with respect to $V/2$ instead of 0.

Since the op amp likely sinks or sources less current than a typical power supply, care should be taken to make sure that these limits are never exceeded.

B.4.3 Instrumentation Amps

An *instrumentation amp* is a specialized amplifier designed to precisely amplify the difference in voltage between two inputs, $V_{\text{out}} = G(V_{\text{in}+} - V_{\text{in}-})$. Like an op amp, it has inputs $V_{\text{in}+}$ and $V_{\text{in}-}$, but unlike an op amp, it is not used with a negative feedback path. Instead, an instrumentation amp like the Texas Instruments INA128 allows you to connect a single external resistor R_G to determine the gain G, where

$$G = 1 + \frac{50\ \text{k}\Omega}{R_G}.$$

The INA128 is typically used to implement gains G from 1 ($R_G = \infty$, i.e., no connection at the gain resistor inputs) to 10,000 ($R_G \approx 5\ \Omega$).

Other instrumentation amps allow you to choose from a fixed set of very precise gains, not dependent on an external resistor (with its associated tolerance). These are sometimes called programmable-gain instrumentation amps, and an example is the TI PGA204, which uses two digital inputs to choose gains of 1, 10, 100, or 1000. A related design is the TI INA110, offering gains of 1, 10, 100, 200, or 500.

Instrumentation amps distinguish themselves in their very high common-mode rejection ratio (120 dB for the INA128). Instrumentation amps are typically more expensive than op amps, e.g., in the range of 10 to 20 USD for quantities of one. If you are trying to save money, you can build your own difference-and-gain circuit using multiple op amps, but you will not achieve the same performance as an instrumentation amp.

B.5 Modular Circuit Design: Input and Output Impedance

One way to design a complex circuit is to design subcircuits, each with a specific function, and then cascade them so that the output of one circuit is the input of another. Modular circuit design is similar to modular code design: each subcircuit has a specific function and well-defined inputs and outputs.

Modularity requires that connecting the output of circuit A to the input of circuit B does not change the behavior of either circuit. Modularity is assured if circuit A has low output impedance (it can source or sink a lot of current with little change in the output voltage) and circuit B has high input impedance (it draws little current at the input).[2] For constant (DC) voltages and currents, if a change ΔI in the current drawn from the output of a circuit causes a change of voltage ΔV, then the DC output impedance (or simply the output resistance, since the voltage is DC) is $|\Delta V/\Delta I|$. A circuit's input resistance can be measured similarly. High input impedance means that a change in input voltage gives a very small change in input current.

Input and output impedance are generally frequency dependent. For sinusoidal signals of any frequency $\omega = 2\pi f$, the impedance of a resistor is simply its resistance R. The magnitude of the impedance of an inductor is ωL, meaning that the impedance increases linearly with ω—the impedance is zero at DC and infinite at infinite frequency. The magnitude of the impedance of a capacitor is $1/(\omega C)$, indicating that the impedance magnitude is infinite at DC and zero at infinite frequency.

As a simple DC example, consider the following design problem. We want to provide a user the ability to choose an input voltage between 0 and 3 V by turning a potentiometer knob. So we decide the circuit B will be a 10 kΩ potentiometer with one end at 3 V and the other end at 0 V, with the wiper providing the user's input signal. No 3 V supply is available, however; there is only a 6 V supply. So we decide to design a circuit A, a voltage divider consisting of two resistors of resistance R, to create the 3 V reference. The output of circuit A becomes the input for circuit B (see Figure B.17).

Let us say we choose R = 100 kΩ for the voltage divider. Then the currents in Figure B.17 can be calculated using

[2] It is actually the ratio of input impedance to output impedance that matters. This ratio should ideally be multiple orders of magnitude.

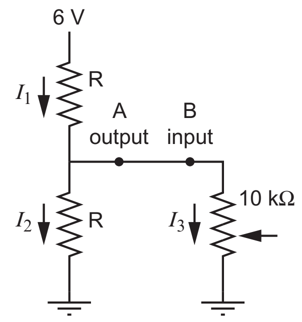


Figure B.17
A voltage divider circuit A feeding a potentiometer circuit B.

$$\begin{aligned} I_1 &= I_2 + I_3 \\ 6\,\text{V} &= I_1\,100\,\text{k}\Omega + I_2\,100\,\text{k}\Omega \\ I_2\,100\,\text{k}\Omega &= I_3\,10\,\text{k}\Omega \end{aligned}$$

to find $I_1 = 55\ \mu\text{A}$, $I_2 = 5\ \mu\text{A}$, and $I_3 = 50\ \mu\text{A}$. This means the voltage divider actually creates a voltage at the output of A of 6 V − (55 μA)(100 kΩ) = 0.5 V instead of 3 V. Circuit B "loads" or "pulls down" the output of circuit A. The output impedance *R* of circuit A is too high relative to the 10 kΩ input impedance of circuit B, defeating circuit modularity. Our attempt to design circuits A and B independently and put them together has failed.

On the other hand, if we choose *R* = 100 Ω for the voltage divider, we find that $I_1 = 30.15$ mA, so $V_A = 2.985$ V, very close to our target of 3 V. The output impedance of circuit A is much lower, so modularity is more closely achieved. This comes at the cost of greater power dissipated by the voltage divider, $V^2/R = (6\ \text{V})^2/200\ \Omega = 180$ mW vs. $(6\ \text{V})^2/200\ \text{k}\Omega = 0.18$ mW.

Op amps, with their high input impedance and low output impedance, are quite useful in achieving circuit modularity. In particular, a unity gain buffer between the output A and input B in Figure B.17 would eliminate any loading of the circuit A by circuit B, allowing us to use higher resistances for *R* and therefore wasting less power.

Further Reading

Hambley, A. R. (2000). *Electronics* (2nd ed.). Upper Saddle River, NJ: Prentice Hall.
Horowitz, P., & Hill, W. (2015). *The art of electronics* (3rd ed.). New York, NY: Cambridge University Press.

Other PIC32 Models

As of this writing, there are nearly 200 different PIC32 models, arranged in six major families: PIC32MX1xx/2xx, PIC32MX3xx/4xx, PIC32MX5xx/6xx/7xx (from which our PIC32MX795F512H was chosen), PIC32MX1xx/2xx/5xx 64-100 pins, PIC32MX330/350/370/430/450/470, and PIC32MZ. The five PIC32MX families all use the MIPS32 M4K processor as the CPU, at speeds of 40-120 MHz, while the MZ family has a different architecture and uses the MIPS32 microAptiv microprocessor as the CPU at up to 200 MHz. "MIPS32" refers to CPU architectures and associated assembly language instructions licensed by Microchip from Imagination Technologies.

The main differences between the families are the CPU architecture (MIPS32 M4K vs. microAptiv), CPU clock speeds, amount of RAM and flash, physical packaging, available peripherals, number of pins, and the extent to which the pins can be mapped to different functions. This appendix provides a brief introduction to the features of the different families.

C.1 The PIC32MX5xx/6xx/7xx Family

Devices in the PIC32MX5xx/6xx/7xx family have names of the form

$$\text{PIC32MX} \quad \underbrace{\text{5, 6, or 7}}_{\text{communication options}} \quad \underbrace{\text{x x}}_{\text{other model options}} \quad \text{F} \quad \underbrace{\text{yyy}}_{\text{flash in KB}} \quad \underbrace{\text{H or L}}_{\text{64 or 100 pins}}$$

The 5xx series has full-speed USB and CAN peripherals, the 6xx series has full-speed USB and Ethernet, and the 7xx series has full-speed USB, Ethernet, and CAN. The xx code can be 34, 64, 75, or 95, corresponding to other model options, but primarily indicating the amount of RAM available (16, 32, 64, and 128 KB, respectively). The yyy code indicates the amount of flash memory, in KB. (All devices in this family also have an additional 12 KB of boot flash.) Devices ending in H have 64 pins and devices ending in L have 100 pins. Therefore the PIC32MX795F512H has full-speed USB, CAN, and Ethernet; 128 KB of RAM; 512 KB of flash; and 64 pins.

The M4K CPU can operate at up to 80 MHz for all devices in the PIC32MX5xx/6xx/7xx family. All devices have full-speed USB, five 16-bit counter/timers with up to two 32-bit counter/timers, five input capture devices, five output compare modules with 16-bit resolution, 16 10-bit ADC inputs, six UARTs, two comparators, DMA, and PMP. Devices with 64 pins have three SPI and four I^2C peripherals; devices with 100 pins have four SPI and five I^2C peripherals. All devices are available only in surface mount packages.

C.2 PIC32MX3xx/4xx Family

PIC32MX3xx/4xx devices are the first to have appeared in the PIC32 line. The M4K CPU can operate at up to 80 MHz for most devices in this family, but a few devices are limited to 40 MHz. Devices in this family have up to 32 KB of RAM and 512 KB of flash and have names of the form

$$\text{PIC32MX}\quad \underbrace{\text{3 or 4}}_{\text{3: no USB; 4: with USB}}\quad \underbrace{\text{xx}}_{\text{20, 40, or 60}}\quad \text{F}\quad \underbrace{\text{yyy}}_{\text{flash in KB}}\quad \underbrace{\text{H or L}}_{\text{64 or 100 pins}}$$

Thus the PIC32MX460F512H has full-speed USB, 32 KB of RAM (with the “60” option), 512 KB of flash, and 64 pins.

Devices in the PIC32MX3xx/4xx family have similar capabilities to those in the PIC32MX5xx/6xx/7xx family, except they do not offer Ethernet or CAN, have fewer UART, SPI, and I^2C peripherals, and have only one 32-bit counter/timer.

C.3 PIC32MX1xx/2xx Family

PIC32MX1xx/2xx devices are more recent than the 3xx/4xx and 5xx/6xx/7xx families. They are smaller devices, coming in 28-, 36-, and 44-pin devices. The 28-pin devices are available in DIP (dual inline package), convenient for breadboarding. The maximum CPU clock speed for a 1xx/2xx device is 40 or 50 MHz, depending on the model. The 1xx/2xx devices do not have a prefetch cache module; they run at full speed pulling instructions from flash.

Devices in this family have up to 64 KB of RAM and 256 KB of flash and have names of the form

$$\text{PIC32MX}\quad \underbrace{\text{1 or 2}}_{\text{1: no USB; 2: with USB}}\quad \underbrace{\text{xx}}_{\text{other model options}}\quad \text{F}\quad \underbrace{\text{yyy}}_{\text{flash in KB}}\quad \underbrace{\text{B, C, or D}}_{\text{28, 36, or 44 pins}}$$

The xx code can be 10, 20, 30, 50, or 70, which correspond to 4, 8, 16, 32, or 64 KB of RAM, respectively. Thus the PIC32MX230F064B has full-speed USB, 16 KB of RAM, 64 KB of flash, and 28 pins.

Devices in this family differ from the 5xx/6xx/7xx family in that they have only 3 KB of boot flash; fewer ADC inputs; fewer UART, SPI, and I^2C peripherals; and no Ethernet or CAN.

The 1xx/2xx devices have three comparator modules instead of two, programmable with up to 32 reference voltages as compared to the 16 of the 5xx/6xx/7xx family. Other interesting new features, which we have not seen in the previous models, include Peripheral Pin Select (PPS), audio interface by SPI, and charge-time measurement for capacitive touch sensing, as discussed below.

PPS allows a wide range of digital-only peripherals to be assigned flexibly to different device pins, a major feature for low-pin-count devices like PIC32MX1xx/2xx devices. While a pin of our PIC32MX795F512H can support several possible peripherals, devices with PPS have much more flexibility in mapping certain peripherals to different pins. A remappable peripheral does not have default I/O pins; SFRs must be configured to assign the peripheral to specific pins before it can be used. Examples of peripherals that can be remapped by PPS include UARTs, SPI modules, counter/timer inputs, input capture, output compare, and interrupt-on-change inputs. Some peripherals cannot be remapped, such as I^{2}C peripherals and ADC inputs, because of special requirements on the I/O circuitry for those peripherals.

The 1xx/2xx devices' SPI modules support audio interface protocols for 16-, 24-, and 32-bit audio data. One example is the Inter-IC Sound (I^{2}S) protocol, which allows the transmission of two channels of digital audio data using the SPI peripheral. The I^{2}S capability allows a 1xx/2xx device to communicate with digital audio equipment as either the master or slave.

Finally, the 1xx/2xx's Charge-Time Measurement Unit (CTMU) provides a current source to interface with an external capacitive touch sensor, such as a capacitive on/off button or even an *x*-*y* touchpad. The CTMU is used with one or more ADC channels to measure the capacitance of one or more analog capacitive sensors.

C.4 PIC32MX1xx/2xx/5xx 64-100 Pin Family

The PIC32MX1xx/2xx/5xx 64-100 pin family expands on the features of the PIC32MX1xx/2xx family, which includes PPS, CTMU, and audio interface protocols. The M4K CPU operates at speeds up to 50 MHz, and the devices have 64 or 100 pins and up to 64 KB of RAM and 512 KB of flash. These devices feature more analog input channels (up to 48), more UART and SPI peripherals, and some models feature USB and CAN. Device names have the form

$$\text{PIC32MX}\quad \underbrace{\text{1, 2, or 5}}_{\text{communication options}}\quad \underbrace{\text{xx}}_{\text{other model options}}\quad \text{F}\quad \underbrace{\text{yyy}}_{\text{flash in KB}}\quad \underbrace{\text{H or L}}_{\text{64 or 100 pins}}$$

The 1xx series has neither CAN nor USB, the 2xx series features full-speed USB but no CAN, and the 5xx series has both full-speed USB and CAN. The code xx can be 20, 30, 50, or 70, corresponding to 8, 16, 32, or 64 KB of RAM, respectively. Thus the PIC32MX570F512H offers both USB and CAN and has 64 KB of RAM, 512 KB of flash, and 64 pins.

C.5 PIC32MX330/350/370/430/450/470 Family

The PIC32MX330/350/370/430/450/470 family differs from the PIC32MX1xx/2xx/5xx 64-100 pin family by offering 12 KB of boot flash, up to 128 KB of RAM, and CPU speeds up to 120 MHz, the fastest in the PIC32MX families. Devices in this family have two SPI channels (compared to the three or four of the PIC32MX1xx/2xx/5xx 64-100 pin family) and do not offer CAN. 4xx devices in this family offer full-speed USB while 3xx devices do not.

C.6 PIC32MZ Family

The most recent addition to the PIC32 line, the PIC32MZ family is the most advanced. Devices in this family have names of the form

$$\text{PIC32MZ} \quad \underbrace{\text{xxxx}}_{\text{flash in KB}} \quad \text{three-letter code} \quad \underbrace{\text{yyy}}_{\text{number of pins}}$$

The number of pins (yyy) is either 064, 100, 124, or 144. The three-letter code indicates whether the PIC32 has CAN modules; an external bus interface (EBI), described below; a floating point unit (FPU) for fast floating point operations; and a Crypto Engine, a hardware module used to accelerate applications requiring encryption, decryption, and authentication. PIC32MZs currently have up to 512 KB of RAM.

All PIC32MZs have a much larger boot flash segment (160 KB), nine 16-bit counter/timers configurable to up to four 32-bit counter/timers, nine output compares with up to 32-bit resolution, nine input captures, six UARTs, up to five I^{2}C modules, up to six SPI modules supporting audio interfaces, PPS for more flexible pin remapping, Ethernet, and high-speed USB, as opposed to the slower full-speed USB of the PIC32MX models. PIC32MZs do not currently have a CTMU.

Two new capabilities on the PIC32MZ are the 50 MHz External Bus Interface (EBI) and the 50 MHz Serial Quad Interface (SQI). SQI is similar to SPI, except it has four data lines and supports single lane, dual lane, and quad lane modes of operation. In single lane mode, it is identical to SPI. EBI allows a high-speed connection to external memory devices, like NOR flash devices or SRAM, allowing you to seamlessly address external memory in your C code.

The PIC32MZ family also has several different peripheral buses, each potentially clocked at different frequencies.

Certainly the biggest difference of the PIC32MZ family from PIC32MX devices is its different microprocessor, the MIPS32 microAptiv core. The microAptiv CPU can be clocked at up to 200 MHz, and it has multiple shadow register sets, a larger number of interrupt sources, and new assembly instructions and hardware to accelerate digital signal processing calculations. Other capabilities of the microAptiv core can be found in Section 50 of the Reference Manual.

C.7 Conclusion

To learn more about a specific PIC32 model, first consult the Data Sheet for the appropriate PIC32 family to learn the specific capabilities of each model. After that, you can consult the sections of the Reference Manual for more information. You will find it helpful to be armed with the knowledge of the features that your PIC32 model has or does not have, since the Reference Manual is currently written to cover all PIC32 models. After that, you can modify the sample code provided in this book for your particular PIC32, or start with sample code provided by Microchip.