Chapter 8

Practice Exercises on Dynamics of Open Chains

8.1 Practice Exercises

Practice exercise 8.1 Figure 8.1 illustrates an RP robot moving in a vertical plane. The mass of link 1 is $\mathfrak{m}_1$ and the center of mass is a distance L_1 from joint 1. The scalar inertia of link 1 about an axis through the center of mass and out of the plane is $\mathcal{I}_1$. The mass of link 2 is $\mathfrak{m}_2$, the center of mass is a distance θ_2 from joint 1, and the scalar inertia of link 2 about its center of mass is $\mathcal{I}_2$. Gravity g acts downward on the page.

(a) Let the location of the center of mass of link i be (x_i, y_i). Find (x_i, y_i) for $i = 1, 2$, and their time derivatives, in terms of θ and $\dot{\theta}$.

(b) Write the potential energy of each of the two links, $\mathcal{P}_1$ and $\mathcal{P}_2$, using the

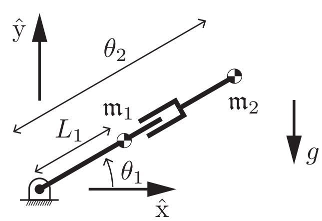

Figure 8.1: An RP robot operating in a vertical plane.

joint variables θ.

(c) Write the kinetic energy of each of the two links, $\mathcal{K}_1$ and $\mathcal{K}_2$.

(d) What is the Lagrangian in terms of $\mathcal{K}_1$, $\mathcal{K}_2$, $\mathcal{P}_1$, and $\mathcal{P}_2$?

(e) One of the terms in the Lagrangian can be expressed as

$$\frac{1}{2}\mathfrak{m}_2\theta_2^2\dot{\theta}_1^2.$$

If this were the complete Lagrangian, what would the equations of motion be? Derive these by hand (no symbolic math software assistance). Indicate which of the terms in your equations are a function of $\ddot{\theta}$, which are Coriolis terms, which are centripetal terms, and which are gravity terms, if any.

(f) Now derive the equations of motion (either by hand or using symbolic math software for assistance) for the full Lagrangian and put them in the form

$$\tau = M(\theta)\ddot{\theta} + c(\theta, \dot{\theta}) + g(\theta).$$

Identify which of the terms in $c(\theta, \dot{\theta})$ are Coriolis and which are centripetal. Explain as if to someone who is unfamiliar with dynamics why these terms contribute to the joint forces and torques.

(g) Consider the configuration-dependent mass matrix $M(\theta)$ from your previous answer. When the robot is at rest (and ignoring gravity), the mass matrix can be visualized as the ellipse of joint forces/torques that are required to generate the unit circle of joint accelerations in $\ddot{\theta}$ space. As θ_2 increases, how does this ellipse change? Describe it in text.

(h) Now visualize the configuration-dependent end-effector mass matrix $\Lambda(\theta)$. For a unit circle of accelerations at the tip of the robot $(\ddot{x}_2, \ddot{y}_2)$, consider the ellipse of linear forces that are required to be applied at the tip of the robot to realize these accelerations. How does the orientation of this ellipse change as θ_1 changes? How does the shape change as θ_2 increases from zero to infinity when $\theta_1 = 0$? If you have access to symbolic computation software (e.g., Mathematica), you can use the Jacobian $J(\theta)$ satisfying

$$\begin{bmatrix} \dot{x}_2 \\ \dot{y}_2 \end{bmatrix} = J(\theta)\dot{\theta}$$

to calculate $\Lambda = J^{-\mathrm{T}}MJ^{-1}$ for the case $\theta_1 = 0$. If you do not have access to symbolic computation software, you can plug in numerical values for $\mathcal{I}_1$, $\mathcal{I}_2$, $\mathfrak{m}_1$, $\mathfrak{m}_2$, and L_1 (make them all equal to 1, for example) to say something about the shape change of the ellipse as θ_2 goes from zero to infinity.