

ME 333 - Mechatronics: HW 7 Solutions

March 7, 2012

Problem 1 Motor Data Sheet

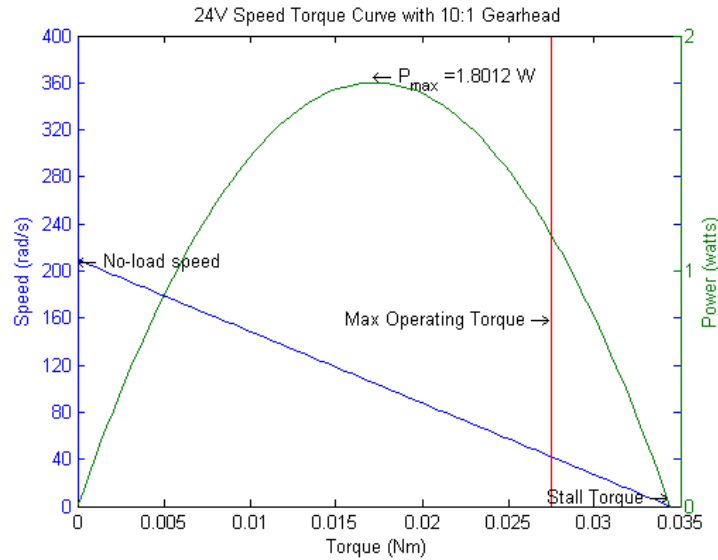
Out of the 19 entries on this table (ignoring friction), only 6 are needed to calculate the rest of the values. The 6 independent values are listed below in **bold**.

Motor Property	Symbol	Value	Units	Derivation
Nominal Voltage	V_{nom}	6	V	From battery pack
Power Rating	P	1.80	W	$P = P_{max}$ because $\tau_c > \frac{\tau_{stall}}{2}$
Max mechanical power	P_{max}	1.80	W	$P_{max} = \frac{\tau_{stall}\omega_0}{4}$
No-load speed	ω_0	209.44	$\frac{rad}{s}$	Measured with encoder
No-load current	I_0	0.186	A	Measured with meter
Max continuous current	I_c	1.04	A	$I_c = \sqrt{\frac{5W}{R}}$
Starting current	I_s	1.3	A	$\frac{V_{nom}}{R}$
Max continuous torque	τ_c	0.0275	Nm	$\tau_c = k_m I_c$
Stall torque	τ_{stall}	0.0344	Nm	$\tau_{stall} = k_m I_s$
Max efficiency	η_{max}	38.7	%	$\eta_{max} = \left(1 - \sqrt{\frac{I_0}{I_s}}\right)^2$
Terminal resistance	R	4.6	Ω	$R = \frac{V_{nom}}{I_s}$
Terminal inductance	L	1.24	mH	Experimentally (See Appendix)
Electrical time constant	T_e	0.27	ms	$T_e = \frac{L}{R}$
Motor constant	k_m	0.02646	$\frac{Nm}{A}$	$k_m = k_e$
Electrical constant	k_e	0.02646	$\frac{Vs}{rad}$	Experimentally (See Appendix)
Speed constant	k_s	37.79	$\frac{rad}{Vs}$	$k_s = \frac{1}{k_e}$
Mechanical time constant	T_m	35	ms	Experimentally (See Appendix)
Rotor inertia	J	53.27	gcm^2	$J = \frac{k_m^2 T_m}{R}$
Short-circuit damping	B	0.000152	$\frac{Nm s}{rad}$	$B = \frac{k_e^2}{R}$
Coulombic Friction	b_0	0.0049	Nm	$b_0 = I_0 k_m$ (Torque with no load)

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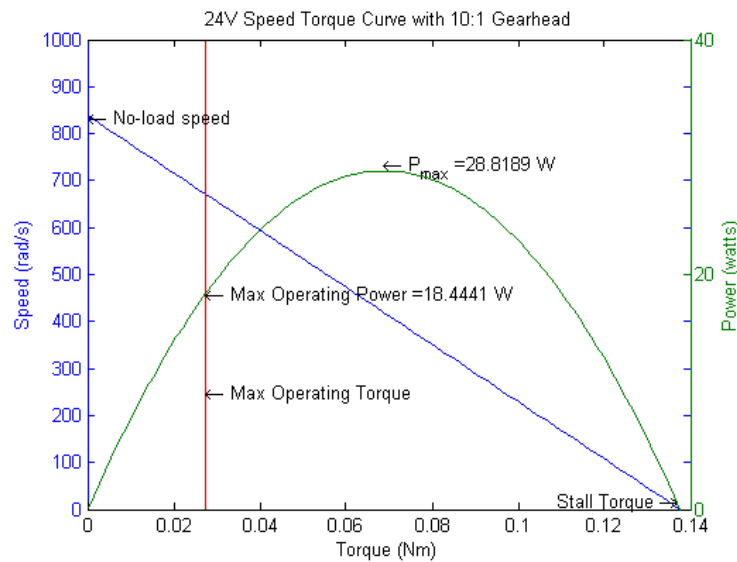
Problem 2 Speed-Torque Curves

- (a) Draw the speed-torque curve for your motor assuming a nominal voltage of 6 V. Indicate the stall torque and no-load speed. Indicate the continuous operating regime. What is the power rating P for this motor? What is the max mechanical power P_{max} ?



Since the maximum power output for the motor at this voltage lies within the continuous operating regime, the power rating for this motor is 1.80 W.

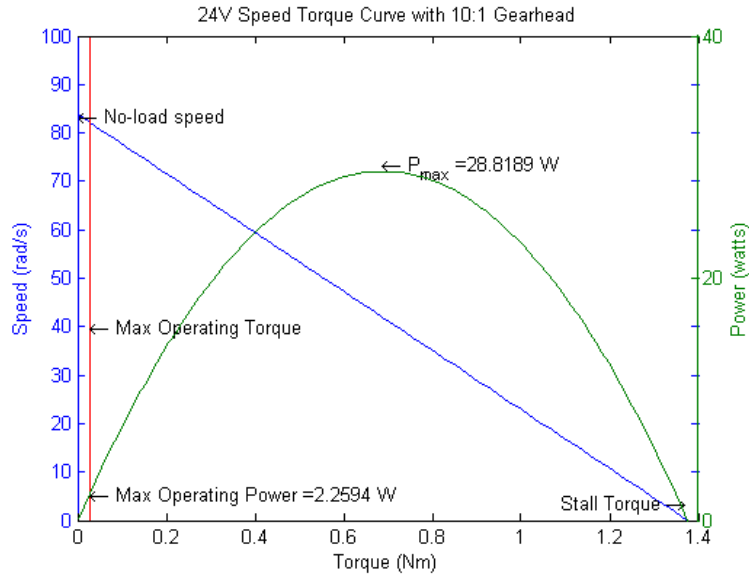
- (b) Draw the speed-torque curve for your motor assuming a nominal voltage of 24 V. Indicate the stall torque and no-load speed. If the motor coils can dissipate a maximum of 5 W continuously before overheating, indicate the continuous operating regime. What is the power rating P for this motor? What is the max mechanical power P_{max} ?



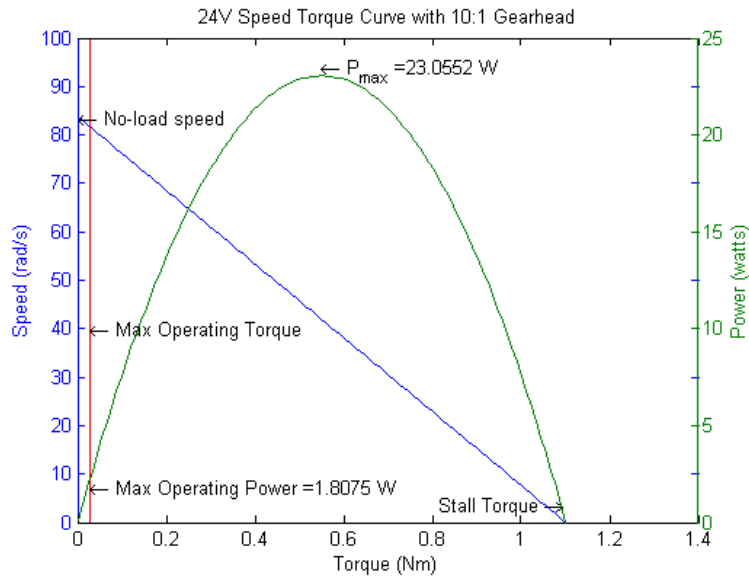
Because the max mechanical power lies outside the continuous operating regime, the power rating for this motor is 18.4 W.

- (c) Often DC motors spin at speeds that are too high, and torques that are too low, to be useful.

If we put a $G = 10$, or 10:1, gearhead on the output of our motor, however, the speed of the motor is reduced by a factor of 10 ($\omega_{out} = \omega_{in}/G$) and the torque is increased by a factor of 10 ($T_{out} = GT_{in}$). Draw the speed-torque curve for the 24 V motor (the previous curve you drew) with a 10:1 gearhead and indicate the no-load speed and stall torque.



- (d) Gearheads are not 100% efficient; some power is lost due to friction and impact between gear teeth. Now assume our 10:1 gearhead from the previous example is $\eta = 80\%$ efficient. The relation $\omega_{out} = \omega_{in}/G$ must be preserved (its enforced by the teeth), so we will use the relation $T_{out} = \eta GT_{in}$, giving $P_{out} = T_{out}\omega_{out} = \eta T_{in}\omega_{in} = \eta P_{in}$. Draw the speed-torque curve for the 24 V motor with an 80% efficient 10:1 gearhead.

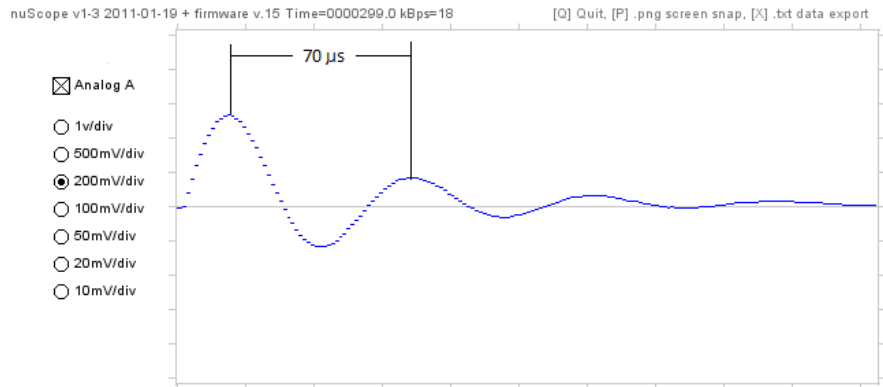


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3 Appendix

Terminal Inductance Measurement

The terminal inductance is measured by observing the electrical frequency of an RLC circuit, where the motor is used as the resistance and inductance. Observing oscillations during a step response gives a good estimation of the natural frequency. As shown in the figure below, one period is roughly $70\mu s$, corresponding to a frequency of about 14.3 kHz. The natural frequency of oscillation for an RLC circuit is given by the equation $1/\sqrt{LC}$. With our known capacitor of 100 nF, the inductance works out to 1.24 mH.



Electrical Constant Measurement

The electrical constant is measured by observing the generated V_{emf} on the terminals in terms of the rotor velocity. By spinning the motor, measuring the speed and back-emf, the electrical constant $k_e = V_{emf}/\omega$ can be calculated. I measured a back-emf of 4.75 V at a rotor velocity of 179.5 rad/s. This corresponds to an electrical constant of 0.02646 Vs/rad.

Mechanical Time Constant Measurement

The mechanical time constant is measured by observing how long it takes the motor to get to 63% of its final velocity at a fixed voltage input. I measured the rotor velocity over time, and captured the data with a PIC chip. Since the encoder is very low resolution, I filtered the data to better approximate the time constant. As shown in the figure below, at about 35 milliseconds after starting, the motor reached 63% of its final speed at the test voltage.

