

**ENGINEERING ANALYSIS 3
SYSTEM DYNAMICS
Section 20**

Quiz 3, December 1 2023

Name: (1pt)_____. **Please also put your name on the back of the last page!**

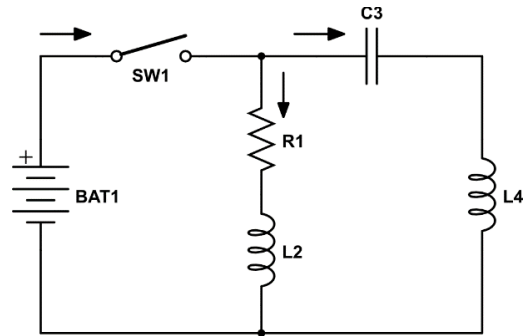
No electronic devices (phones, tablets, laptops, watches, etc.) are allowed during quizzes. No notes or scratch paper.

Do not ask for clarification of the questions; if you think that there is an ambiguity, clearly state your assumption and continue to answer the question.

There are 3 problems. Show all work, and **CLEARLY MARK YOUR ANSWERS**. Good luck!

Problem	Points
1	
2	
3	
Total	

1. (34 points) Consider the RLC circuit to the right, where the battery (BAT1) supplies a constant voltage (V_B). Use the current directions indicated to help write the governing equations. Refer to the voltage/current through each element by using subscripts with element names (e.g., V_{R1} and i_{R1} refer to the voltage across and current through resistor R1, respectively).



Answer the following questions.

- 1-1. List the state variable(s) – for capacitors, use voltage as the state variable.

- 1-2. Write constitutive laws for each element.

- 1-3. Write the appropriate Kirchhoff voltage law (KVL) equation(s) using the sign convention indicated in the figure. Note: all voltages should appear in at least one equation.

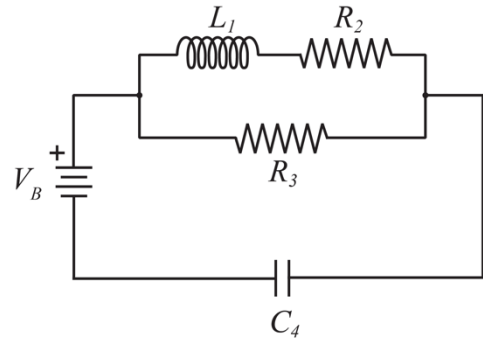
- 1-4. Write the appropriate Kirchhoff current law (KCL) equation(s) using the sign convention indicated in the figure. Note: all currents should appear in at least one equation.

[QUIZ CONTINUES ON NEXT PAGE]

2. (20 points) RLC circuit to the right, derive two couple first-order differential equations, where state variables are the current of the inductor i_{L1} and the voltage of the capacitor V_{C4} .

Given the corresponding KCL, KVL, and CL equations as following:

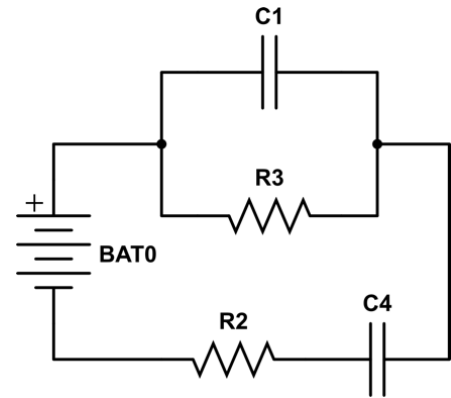
<u>KVL</u>	<u>KCL</u>	<u>CL</u>
$V_B = V_{L1} + V_{R2} + V_{C4}$	$i_{C4} = i_{L1} + i_{R3}$	$V_{L1} = i'_{L1} L_1$
$V_{L1} + V_{R2} = V_{R3}$	$i_{L1} = i_{R2}$	$V_{R2} = i_{R2} R_2$
or $V_B = V_{R3} + V_{C4}$		$V_{R3} = i_{R3} R_3$
		$V_{C4} = Q_{C4} / C_4$



[QUIZ CONTINUES ON NEXT PAGE]

3. (28 points) Consider the RLC circuit to the right, where the battery (BAT0) supplies a constant voltage (V_B).

3-1. Annotate the circuit, labeling currents and voltage polarities. List the currents to be specified here. Refer to the current through each element by using subscripts with element names, e.g., i_{R2} .



3-2. Write down the complete set of node equations (KCL).

3-3. Write down the complete set of loop equations (KVL).

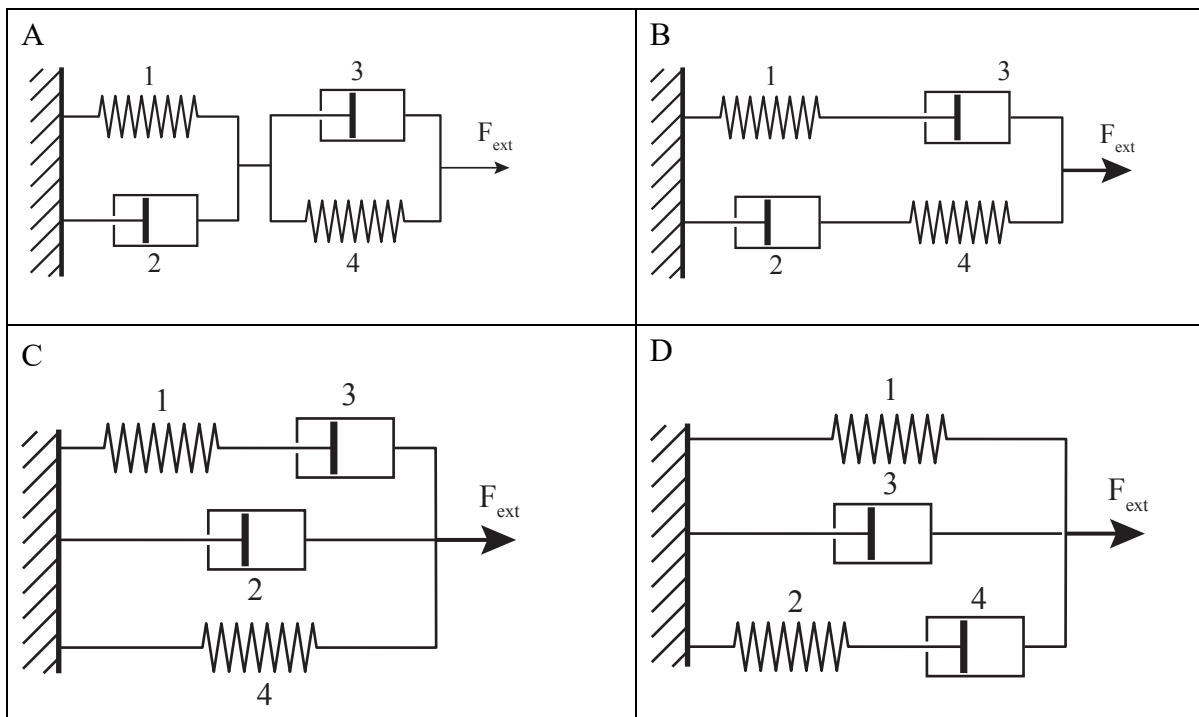
3-4. Map each of the component in the given RLC circuit to the equivalent components in mechanical system.

[QUIZ CONTINUES ON NEXT PAGE]

3-5. Write down the complete set of Geometric Continuity (GC) equations for the equivalent mechanical system.

3-6. Write down the complete set of Force Balance (FB) equations for the equivalent mechanical system.

3-7. The electrical system to the right is analogous to which of the mechanical systems below?



Your Answer:

[END OF QUIZ 3]

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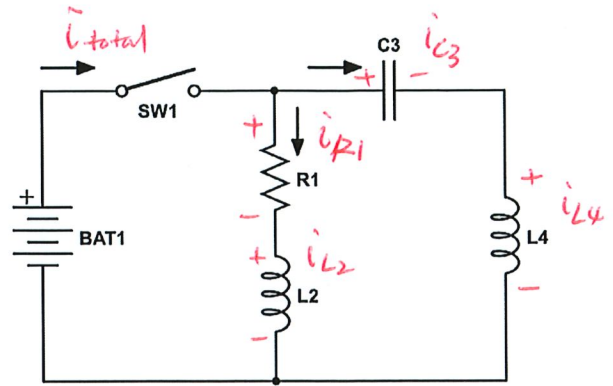
Do not ask for clarification of the questions; if you think that there is an ambiguity, clearly state your assumption and continue to answer the question.

There are 3 problems. Show all work, and **CLEARLY MARK YOUR ANSWERS**. Good luck!

Problem	Points
1	
2	
3	
Total	

(34 pts)

1. Consider the RLC circuit to the right, where the battery supplies a constant voltage (V_B). Use the current directions indicated to help write the governing equations. Before the switch is closed at $t = 0$, there is no current through the inductor, and the capacitor is uncharged. Refer to the voltage/current through each element by using subscripts with element names (e.g., V_{R1} and i_{R1} refer to the voltage across and current through resistor $R1$, respectively).



Answer the following questions. Show your work for finding the state equations below.

- 1-1. List the state variable(s) – for capacitors, use voltage as the state variable. 6 pts (2 pts each)

$$V_{C3}, i_{L2}, i_{L4}$$

- 1-2. Write constitutive laws for each element.

8 pts (2 pts each)

$$V_{R1} = i_{R1} \cdot R_1; \quad V_{C3} = \frac{Q_3}{C_3}$$
$$V_{L2} = i_{L2}' \cdot L_2; \quad i_{L4} = i_{L4}' \cdot L_4$$

- 1-3. Write the appropriate Kirchhoff voltage law (KVL) equation(s) using the sign convention indicated in the figure. Note: all voltages should appear in at least one equation. 8 pts (4 pts EA)

$$V_{R1} + V_{L2} = V_B, \quad V_{C3} + V_{L4} = V_B$$
$$(\text{or } V_{R1} + V_{L2} - V_{C3} - V_{L4} = 0)$$

- 1-4. Write the appropriate Kirchhoff current law (KCL) equation(s) using the sign convention indicated in the figure. Note: all currents should appear in at least one equation. 2 pts (4 pts EA)

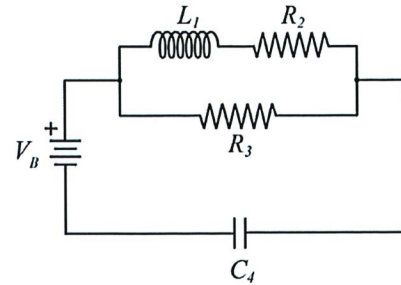
$$i_{\text{total}} = i_{C3} + i_{R1}$$
$$i_{C3} = i_{L4}$$
$$i_{R1} = i_{L2}$$

[QUIZ CONTINUES ON NEXT PAGE]

20 pts

2. RLC circuit to the right, derive two couple first-order differential equations, where state variables are the current of the inductor i_{L1} and the voltage of the capacitor V_{C4} .

The corresponding KCL, KVL, and CL equations as following:



<u>KVL</u>	<u>KCL</u>	<u>CLs</u>
$V_B = V_{L1} + V_{R2} + V_{C4}$ ①	$i_{C4} = i_{L1} + i_{R3}$ ③	$V_{L1} = i'_{L1} L_1$ ⑤
$V_{L1} + V_{R2} = V_{R3}$ ②	$i_{L1} = i_{R2}$ ④	$V_{R2} = i_{R2} R_2$ ⑥
$V_B = V_{R3} + V_{C4}$		$V_{R3} = i_{R3} R_3$ ⑦
		$V_{C4} = Q_{C4} / C_4$ ⑧

$$\begin{aligned} i'_{L1} &= \frac{1}{L_1} \cdot V_{L1} = \frac{1}{L_1} (V_B - V_{R2} - V_{C4}) = \frac{1}{L_1} (V_B - i_{R2} R_2 - V_{C4}) \\ &= \frac{1}{L_1} (V_B - i_{L1} R_2 - V_{C4}) \end{aligned}$$

$$\begin{aligned} V'_{C4} &= \frac{d}{dt} \left(\frac{Q_{C4}}{C_4} \right) = \frac{i_{C4}}{C_4} = \frac{i_{L1} + i_{R3}}{C_4} = \frac{1}{C_4} \left(i_{L1} + \frac{V_{R3}}{R_3} \right) \\ &= \frac{1}{C_4} \left(i_{L1} + \frac{V_B - V_{C4}}{R_3} \right) \end{aligned}$$

$$\begin{aligned} i'_{L1} &= \frac{1}{L_1} (V_B - i_{L1} R_2 - V_{C4}) \\ V'_{C4} &= \frac{1}{C_4} \left(i_{L1} + \frac{V_B - V_{C4}}{R_3} \right) \end{aligned}$$

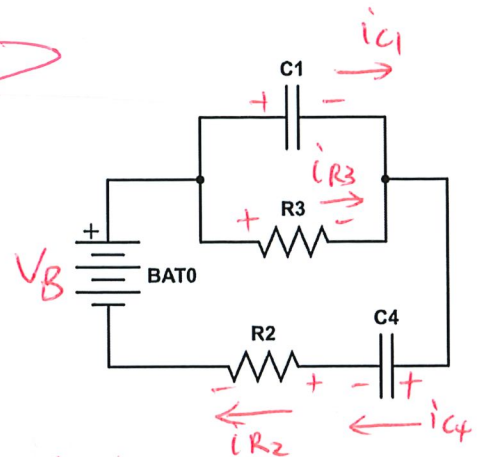
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28pts

3. Consider the RLC circuit to the right.

3-1. Annotate the circuit, labeling currents and voltage polarities. (List the current)

no point



3-2. Write down the complete set of node equations (KCL).

4 pts (2 pts EA)

$$i_{C1} + i_{R3} = i_{C4}$$

$$i_{R2} = i_{C4}$$

3-3. Write down the complete set of loop equations (KVL).

4 pts (2 pts EA)

$$V_{C1} = V_{R3}, \quad V_B = V_{C1} + V_{R2} + V_{C4}$$

$$\text{or } V_B = V_{R3} + V_{R2} + V_{C4}$$

3-4. Map each of the component in the given RLC circuit to the equivalent components in mechanical system.

4 pts, (1 pt EA)

$$C_1 \rightarrow \text{Spring 1}, \quad C_4 \rightarrow \text{Spring 4}$$

$$R_2 \rightarrow \text{damper 2}$$

$$R_3 \rightarrow \text{damper 3}$$

3-5. Write down the complete set of Geometric Continuity (GC) equations for the equivalent mechanical system.

6 pts (3 pts EA)

$$V_{s1} + V_{d3} = V_{s4}$$

$$V_{d2} = V_{s4}$$

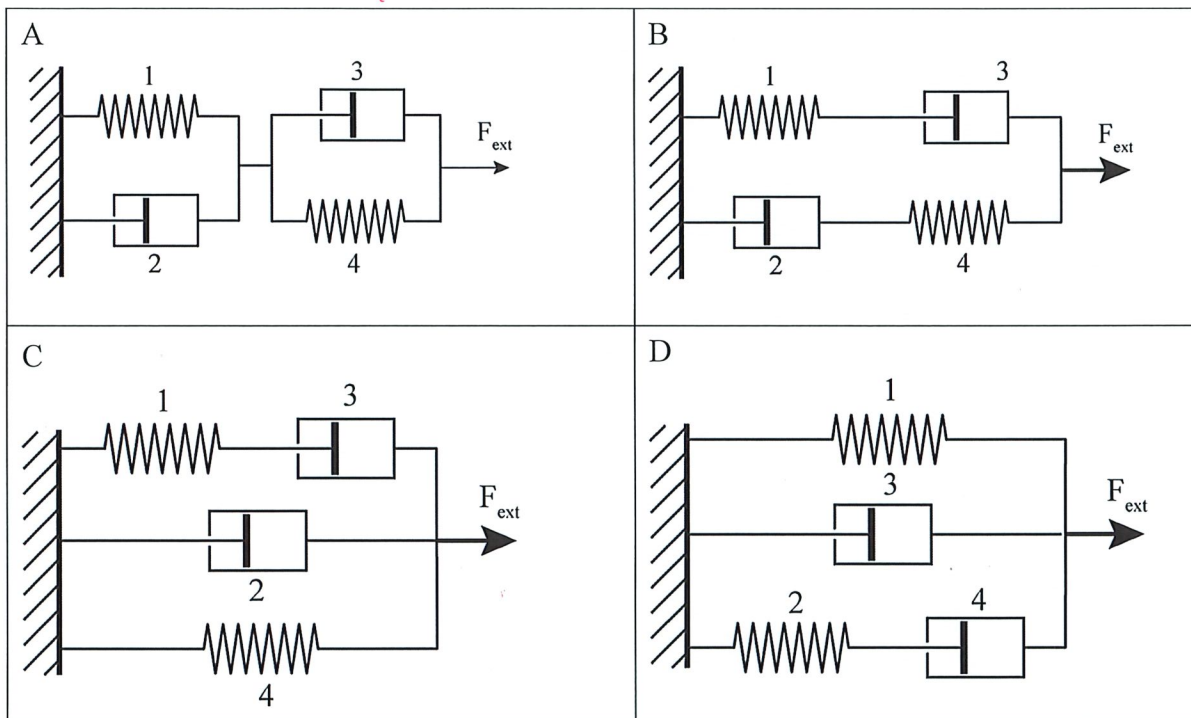
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3-6. Write down the complete set of Force Balance (FB) equations for the equivalent mechanical system. *16 pts, 3 pts EA*

$$F_{s1} = F_{d3} ; F_{ext} = F_{s1} + F_{d2} + F_{s4}$$

or $F_{ext} = F_{d3} + F_{d2} + F_{s4}$

3-7. The electrical system to the right is analogous to which of the mechanical systems below? *(4 pts)*



Your Answer:

C

[END OF QUIZ 3]