ME 449 Robotic Manipulation
Spring 2014
Problem Set 1
Due Monday April 14 at beginning of class

1. Use the planar version of Grübler's formula to determine the degrees of freedom of the mechanisms shown in Figure 1.

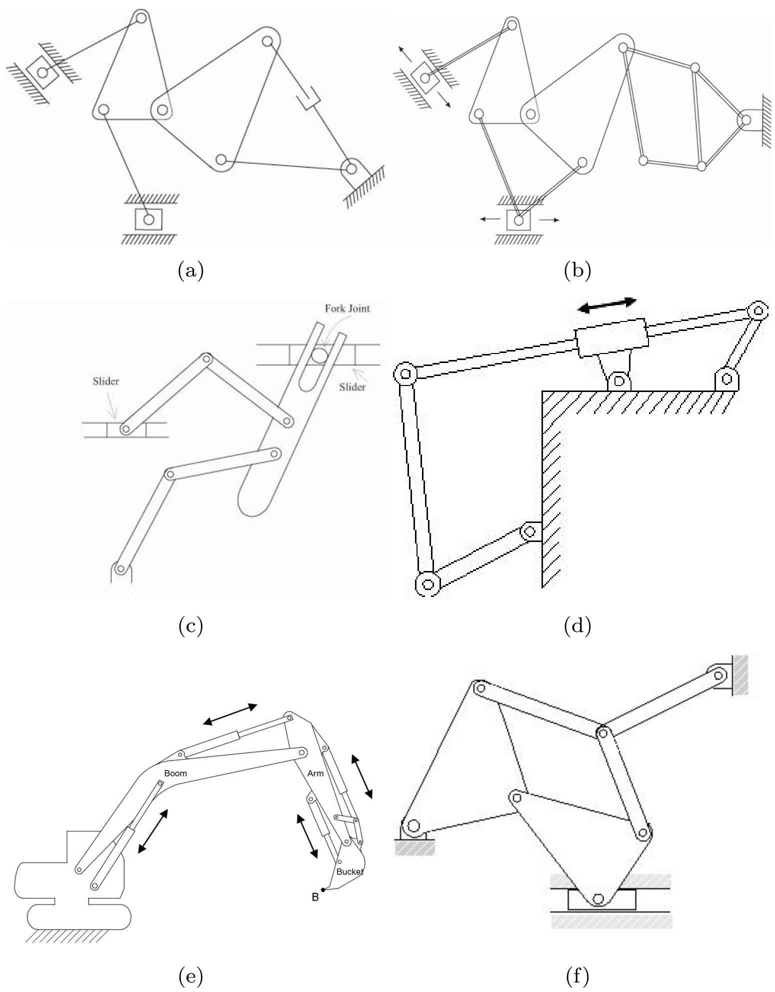


Figure 1: A collection of planar mechanisms.

2. (a) Use an appropriate version of Grübler's formula to determine the dof of the $6 \times RUS$ platform of Figure 2.

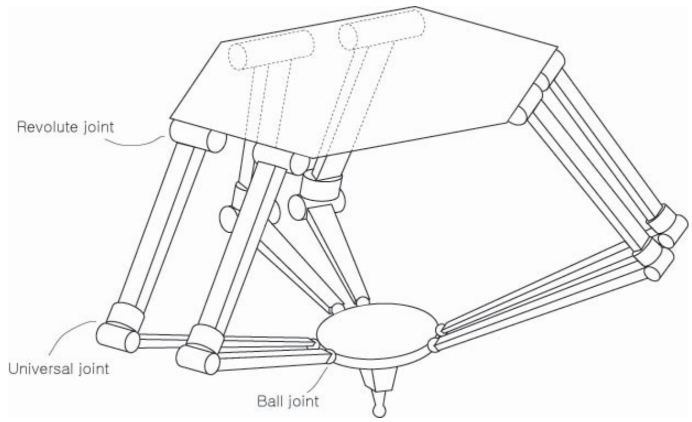


Figure 2: $6 \times RUS$ platform.

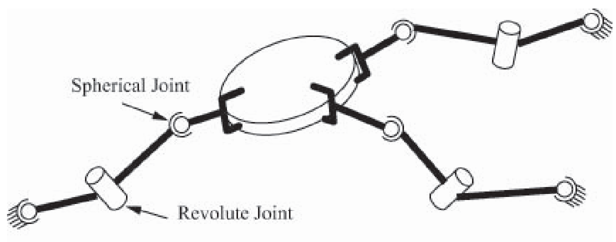


Figure 3: Three cooperating open chain arms grasping a common object.

(b) What happens to the dof if the universal joints are replaced by spherical joints?

3. Three identical open chain SRS arms are grasping an object as shown in Figure 3.
(a) Find the mobility of this system of cooperating robots.
(b) Suppose there are now a total of n 7-dof open chain arms grasping the object. What is the mobility of this system of robots?
(c) Suppose the spherical wrist joint in each of the n seven degree of freedom open chains is replaced by a universal joint. What is the mobility of the overall system?

4. Find simple formulas for the number of linear degrees of freedom and the number of angular degrees of freedom of a rigid body in n-dimensional space. ($n = 3$ is the usual spatial case, but consider higher-dimensional spaces!)

5. The mobile manipulator of Figure 4 consists of a $6R$ arm and multifingered hand mounted on a mobile base with a single wheel that steers and rolls (other

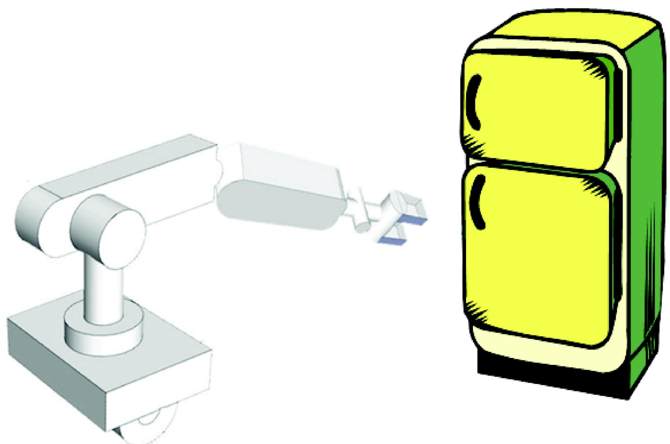


Figure 4: Mobile manipulator.

caster wheels keep the robot from falling over). The wheel rotates without slip, and its axis of rotation is always parallel to the ground.
(a) Ignoring the multifingered hand, describe the configuration space of the mobile manipulator. How many degrees of freedom does it have? What is the dimension of the feasible velocity space for the system?
(b) Now suppose the robot hand rigidly grasps the refrigerator door handle, and opens the door using only its arm. Assume the wheel is locked in place, so that the mobile base does not move. After the door has been opened, what is the number of degrees of freedom of the mechanism formed by the arm and open door?
(c) A second identical mobile manipulator comes along, and after parking its mobile base, also rigidly grasps the refrigerator door handle. What is the number of degrees of freedom of the mechanism formed by the two arms and the open referigerator door?
(d) If the two robots no longer have to be parked, what is the dimension of the C-space and the dimension of the feasible velocity space?

6. Elements of the Special Orthogonal Group $SO(2)$ (planar rotations) commute, while those of $SO(3)$ do not. (Recall that a matrix in $SO(2)$ consists of the 2×2 upper-left submatrix of a matrix in $SO(3)$.) Prove these statements.

7. In terms of the $\hat{x}$-$\hat{y}$-$\hat{z}$ coordinates of a fixed space frame {s}, the frame {a} has an $\hat{x}_a$-axis pointing in the direction $(0, 0, 1)$ and a $\hat{y}_a$-axis pointing in the direction $(-1, 0, 0)$, and the frame {b} has an $\hat{x}$-axis pointing in the direction $(1, 0, 0)$ and a $\hat{y}_b$-axis pointing in the direction $(0, 0, -1)$.

(i) Give your best hand drawing of the three frames. Draw them at different locations so they are easy to see.

(ii) Write the rotation matrices R_{sa} and R_{sb}.

(iii) Given R_{sb}, how do you calculate R_{sb}^{-1} without using a matrix inverse? Write R_{sb}^{-1} and verify its correctness with your drawing.

(iv) Given R_{sa} and R_{sb}, how do you calculate R_{ab} (again no matrix inverses)? Compute the answer and verify its correctness with your drawing.

(v) R_{sb} is obtained by rotating the frame {s} about its $\hat{x}$-axis by $-90°$. Let $R = R_{sb}$. Calculate $R_1 = R_{sa}R$. Does R_1 correspond to rotating R_{sb} by $-90°$ about the world-fixed $\hat{x}$-axis or the body-fixed $\hat{x}_b$-axis? Now calculate $R_2 = RR_{sa}$. Does R_2 correspond to rotating R_{sb} by $-90°$ about the world-fixed $\hat{x}$-axis or the body-fixed $\hat{x}_b$-axis?

(vi) Use R_{sb} to change the representation of the point $p_b = (1, 2, 3)$ (in {b} coordinates) to {s} coordinates.

(vii) Choose a point p represented by $p_s = (1, 2, 3)$ in {s} coordinates. Calculate $p' = R_{sb}p_s$ and $p'' = R_{sb}^T p_s$. For each operation, did we change coordinates (from the {s} frame to {b}) or transform of the location of the point without changing the reference frame of the representation?

8. (This exercise is similar to the previous one, but now using transformation matrices instead of rotation matrices.) In terms of the $\hat{x}$-$\hat{y}$-$\hat{z}$ coordinates of a fixed space frame {s}, the frame {a} has an $\hat{x}_a$-axis pointing in the direction $(0, 0, 1)$ and a $\hat{y}_a$-axis pointing in the direction $(-1, 0, 0)$, and the frame {b} has an $\hat{x}$-axis pointing in the direction $(1, 0, 0)$ and a $\hat{y}_b$-axis pointing in the direction $(0, 0, -1)$. The origin of {a} is at (3,0,0) in {s} and the origin of {b} is at (2,0,0).

This exercise deals with some of the major uses of a rotation matrix: to represent the orientation of a frame relative to another, to change coordinate frames of the representation of a frame or point, and to rotate a frame or point.

(i) Give your best hand drawing of the three frames.

(ii) Write the rotation matrices R_{sa} and R_{sb}.

(iii) Given T_{sb}, how do you calculate T_{sb}^{-1} without using a matrix inverse? Write T_{sb}^{-1} and verify its correctness with your drawing.

(iv) Given T_{sa} and T_{sb}, how do you calculate T_{ab} (again no matrix inverses)? Compute the answer and verify its correctness with your drawing.

(v) Let $T = T_{sb}$. Calculate $T_1 = T_{sa}T$. Does T_1 correspond to a body-fixed or world-fixed transformation of T_{sa}, or neither? Now calculate $T_2 = TT_{sa}$. Does T_2 correspond to a body-fixed or world-fixed transformation of T_{sb}, or neither?

(vi) Use T_{sb} to change the representation of the point $p_b = (1, 2, 3)$ (in {b} coordinates) to {s} coordinates.

(vii) Choose a point p represented by $p_s = (1, 2, 3)$ in {s} coordinates. Calculate $p' = T_{sb}p_s$ and $p'' = T_{sb}^{-1}p_s$. For each operation, did we change coordinates (from the {s} frame to {b}) or transform of the location of the point without changing the reference frame of the representation?

9. (a) Find the general solution to the differential equation $\dot{x} = Ax$, where

$$A = \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix}.$$

What happens to the solution $x(t)$ as $t \to \infty$?
(b) Do the same for

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}.$$

What happens to the solution $x(t)$ as $t \to \infty$?

10. Find the exponential coordinates $\omega\theta \in \mathbb{R}^3$ corresponding to the $SO(3)$ matrix

$$\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}.$$

11. Assume that the body frame angular velocity is $\omega_b = (1, 2, 3)$ for a moving body at

$$R = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

in the world frame {s}. Calculate the angular velocity ω_s in {s}.

12. Two toy cars are moving on a round table as shown in Figure 5. Car 1 moves at a constant speed v_1 along the circumference of the table, while car 2 moves at a constant speed v_2 along a radius; the positions of the two vehicles at $t = 0$ are shown in the figure.
(a) Find T_{01}, T_{02} as a function of t.
(b) Find T_{12} as a function of t.

13. A cube undergoes two different screw motions from frame {1} to frame {2} as shown in Figure 6. In both cases (a) and (b), the initial configuration of the cube is

$$T_{01} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

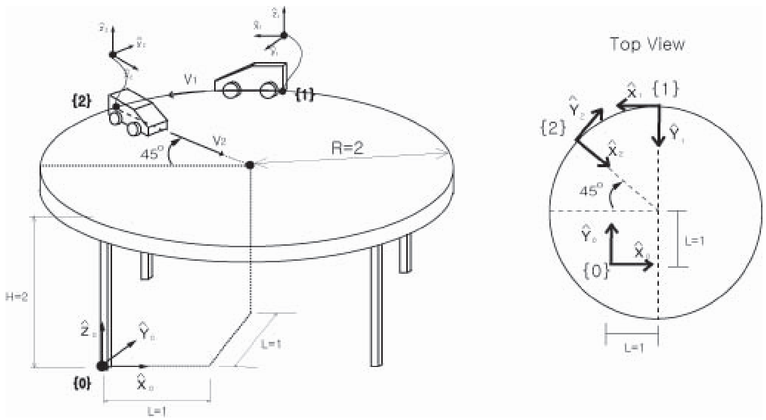
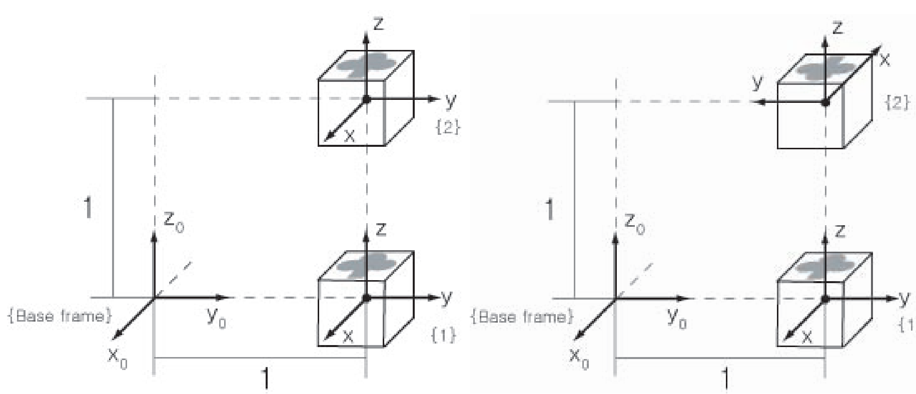


Figure 5: Two toy cars on a round table.



(a) a

(b) b

Figure 6: A cube undergoing two different screw motions.

(a) For each case (a) and (b), find the screw parameter $S = (\omega, v)$ such that $T_{02} = e^{[S]}T_{01}$, where no constraints are placed on ω or v.
(b) Repeat (a), this time with the constraint that $\|\omega\| \in [-\pi, \pi]$.