MODERN ROBOTICS

MECHANICS, PLANNING, AND CONTROL

Practice Exercises

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Contents

Chapter 1

Introduction

These exercises are designed to give you practice with the concepts, the calculations, and the software associated with the book. To get the most out of these practice exercises, you are strongly encouraged not to look at the solutions until you have given your best effort to solve them. You are more likely to retain what you have learned when you work through the problem yourself instead of just reading the solution.

Chapter 2

Practice Exercises on Configuration Space

2.1 Practice Exercises

Practice exercise 2.1 Figure 2.1 shows a scissor jack. As you turn the screw, the jack goes up and down. The mechanical advantage provided by the mechanism allows a single person to jack up a car to change a tire.

Think about what rigid bodies and joints must be present in the scissor jack. You may not be able to see all of them in the image. Use Grübler's formula to calculate the number of degrees of freedom. Does your answer agree with what you know about how a scissor jack works? If not, can you explain why?

Practice exercise 2.2 Figure 2.2 shows a table lamp that moves only in the plane of the page. Use Grübler's formula to calculate the number of degrees of freedom.

Practice exercise 2.3 A unicycle is controlled moving on a rigid balance beam as shown in Figure 2.3. Suppose the wheel is always touching the beam with no sliding, answer the following questions in terms of $\mathbb{R}$, S, T, and I (a one-dimensional closed interval).

(a) Give a mathematical description of the C-space of the unicycle when it remains upright and is constrained to move in the 2-dimensional plane of the page.

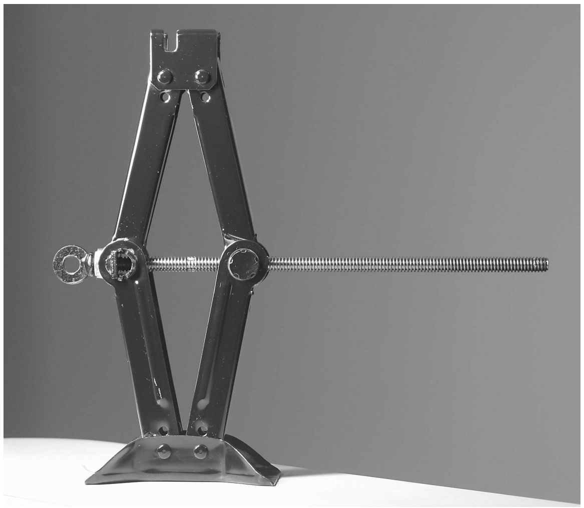

Figure 2.1: A scissor jack (also known as a scissor lift). Image courtesy of Wikipedia.

Figure 2.2: A table lamp that moves only in the plane of the page.

(b) Give a mathematical description of the C-space of the unicycle when it remains upright, it moves in a 3-dimensional space, and the beam has nonzero width.

Practice exercise 2.4 Explain why $S^1 \times S^1 = T^2$, not S^2. In other words,

Figure 2.3: A unicycle on a rigid balance beam.

explain why the C-space of a spherical pendulum (S^2) is not topologically equivalent to the C-space of a 2R robot (T^2), even though the configurations of both would typically be described by two angles. If you think the C-space of a 2R robot is topologically equivalent (*homeomorphic*) to S^2, propose a continuous mapping between points on a sphere and the independent joint angles of a 2R robot.

Practice exercise 2.5 Assume your arm has 7 dof and you constrain your hand to be at a fixed configuration (e.g., your palm is flat against a table).

(a) What is an explicit representation of the arm's configuration?
(b) What is an implicit representation?
(c) What does the set of feasible configurations look like in the 7-dimensional configuration space of the unconstrained arm?

Practice exercise 2.6 Imagine a C-space described as a circle in an (x, y) plane, of radius 2 centered at $(3, 0)$. What is an implicit representation of this one-dimensional C-space? If you were to decide to parameterize the one-dimensional C-space by the single parameter θ, give a mapping from θ to (x, y).

Practice exercise 2.7

Consider the 2D quadcopter and rod shown in Figure 2.4. The rod is attached to the quadcopter by a revolute joint, and you are given the task of bal-

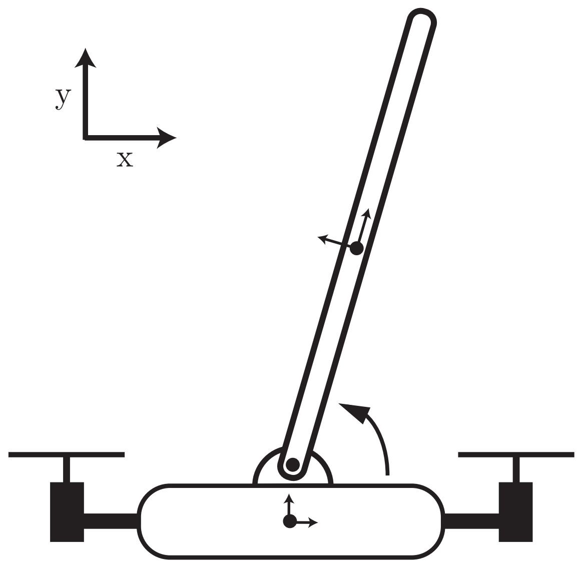

Figure 2.4: 2D quadcopter balancing a rod

ancing the rod upright (a flying version of the classic cart pendulum problem). Assume the configuration of the quadcopter center is described by (x_q, y_q, θ_q) and the configuration of the rod center is described as (x_r, y_r, θ_r) where θ_q and θ_r are measured with respect to the world x axis. The length of the rod is $2l$ and the height and width of the quadcopter body are $2h$ and $2w$ respectively.

(a) Solve for the configuration constraints that keep the rod and quadcopter connected.

(b) Express these as a Pfaffian constraint where $q = [x_q\ y_q\ \theta_q\ x_r\ y_r\ \theta_r]^{\mathrm{T}}$.

Practice exercise 2.8 Consider the parallel SCARA robot shown in Figure 2.5. The robot is controlled by two rotational motors located in the base, and one rotational and one prismatic motor at the end effector. Assume each of the links of the parallel mechanism are length 1 m, the prismatic joint has a maximum travel of 1 m, and the separation distance of the base motors is 0.5 m. Assume no collisions between the links, and that the end effector y-coordinate is constrained to be greater than zero.

(a) Sketch the workspace of the end effector.

(b) What are some benefits and drawbacks of making a parallel rather than a serial SCARA robot?

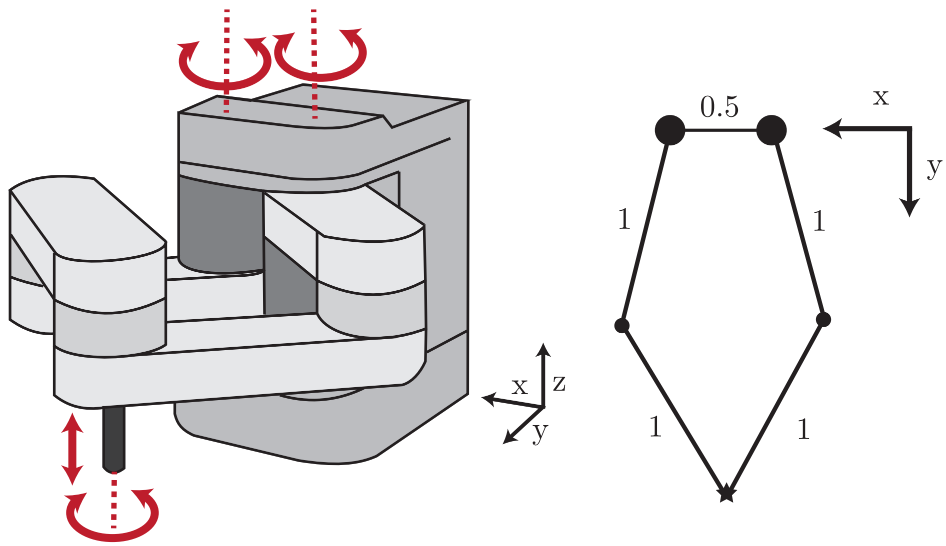

Figure 2.5: Parallel SCARA robot and a skeleton top view.

2.2 Solutions

Solution 2.1 See Figure 2.6 for work. Note that there are two extra cross pieces behind the two side joints that are not visible from the image. The result of Grübler's formula does NOT agree with the known solution of 1 DOF. This is due to the symmetry of this problem, causing certain constraints to not be independent. Instead, the formula provides a lower bound, and the known solution of 1 DOF is indeed above that lower bound.

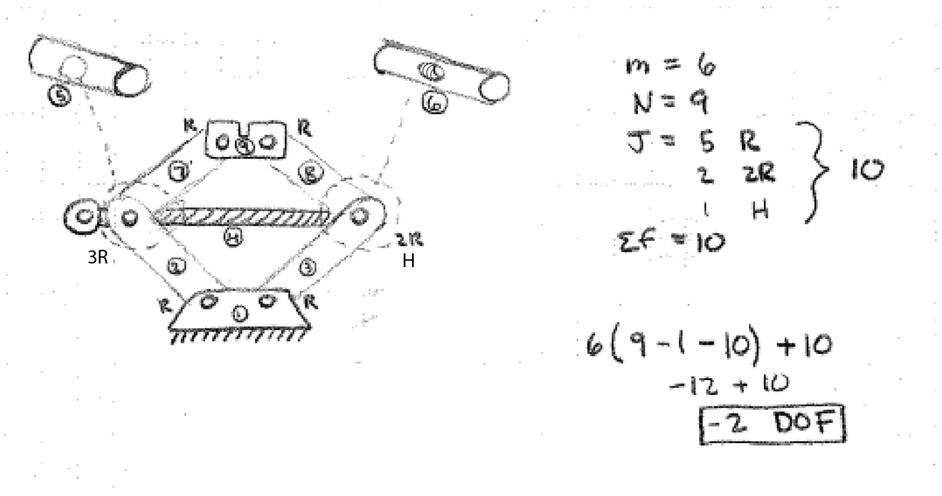

Figure 2.6: Written solution to scissor jack problem.

Solution 2.2 Despite all the links and revolute joints, this mechanical system behaves similarly to a 3R robot arm, since each set of two revolute joints acts as a single hinge.

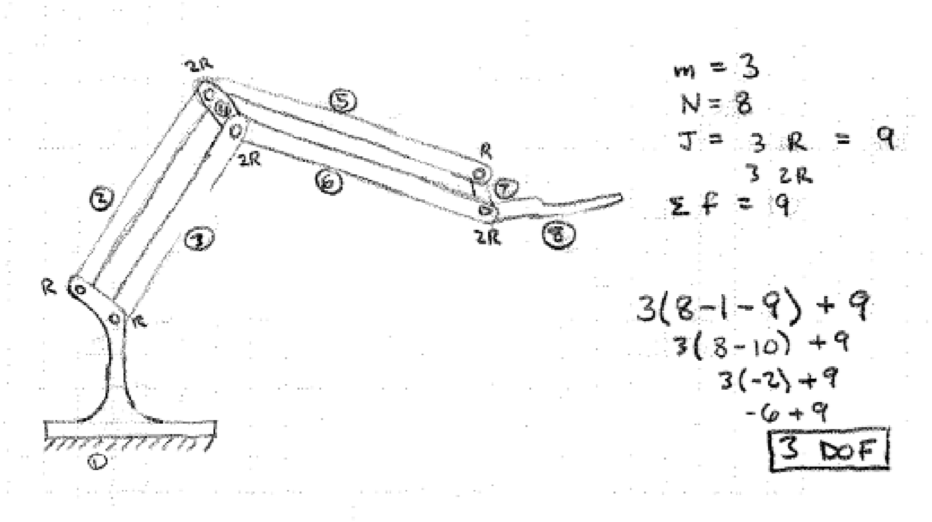

Figure 2.7: Written solution to lamp problem.

Solution 2.3

(a) I: the point of contact on the beam (which determines the angle of the wheel, since rolling is enforced). If we treat the allowed contact points on the beam as an open inerval, then the space is topologically equivalent to $\mathbb{R}$.

(b) $I^2 \times T^2$: intervals correspond to limited beam contact locations, S^1 for heading direction of wheel, and S^1 for the point of contact on the wheel.

Solution 2.4 For two spaces to be topologically equivalent, there must be a *homeomorphism* relating the two. A homeomorphism is a mapping from one space X (e.g., S^2) to another space Y that (1) is one-to-one, (2) "onto" (meaning the mapping from X to Y covers all of Y), (3) continuous, and (4) has a continuous inverse. A homeomorphism is the mathematical term for the functions that can only deform the space, not cut, glue, or change its dimension.

There is no homeomorphism between S^2 and T^2. When you poke a hole in S^2 to get T^2, for example, suddenly points that were neighbors to each other (at the point where you poked the hole) are no longer neighbors; this cannot occur with a continuous mapping.

Solution 2.5

(a) The explicit representation is θ, the angle to the elbow about a line connecting the shoulder to the palm.

(b) The implicit representation is $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7)$ in the 7-dimensional space, plus 6 equations constraining the position (3 dof) and orientation (3 dof) of the palm.

(c) A closed interval of a 1-dimensional curve in that 7-dimensional space.

Solution 2.6 Implicit: (x, y) such that $(x-3)^2 + y^2 = 4$. Explicit: $x = 3 + 2\cos\theta$, $y = 2\sin\theta$.

Solution 2.7 Note: opposite signs are also correct for the following solutions.

(a) Configuration constraints:

$$\begin{aligned} x_\mathrm{r} - \ell\cos(\theta_\mathrm{r}) &= x_\mathrm{q} - h\sin(\theta_\mathrm{q}) \\ y_\mathrm{r} - \ell\sin(\theta_\mathrm{r}) &= y_\mathrm{q} + h\cos(\theta_\mathrm{q}). \end{aligned} \tag{2.1}$$

(b) $A(q)\dot{q} = 0$, where $q = [x_q\ y_q\ \theta_q\ x_r\ y_r\ \theta_r]^\mathrm{T}$, $\dot{q} = [\dot{x}_q\ \dot{y}_q\ \dot{\theta}_q\ \dot{x}_r\ \dot{y}_r\ \dot{\theta}_r]^\mathrm{T}$

$$A(q) = \begin{bmatrix} 1 & 0 & -h\cos(\theta_\mathrm{q}) & -1 & 0 & -\ell\sin(\theta_\mathrm{r}) \\ 0 & 1 & -h\sin(\theta_\mathrm{q}) & 0 & -1 & \ell\cos(\theta_\mathrm{r}) \end{bmatrix} \tag{2.2}$$

Solution 2.8

(a) The top view of the workspace is shown by the shaded region in Figure 2.8, and is the intersection of two circles. To solve for the workspace area, sum the area of the two circle sectors and subtract the triangle area (formed when the arms are fully extended in the y position) that is counted twice. The workspace volume is then the 3D extrusion of this shape into the page by the reach of the prismatic joint.

(b) The parallel structure has the benefit of being more rigid and having more of the motor mass concentrated at the base. One drawback is that the parallel SCARA has a smaller workspace compared to a comparable serial SCARA arm.

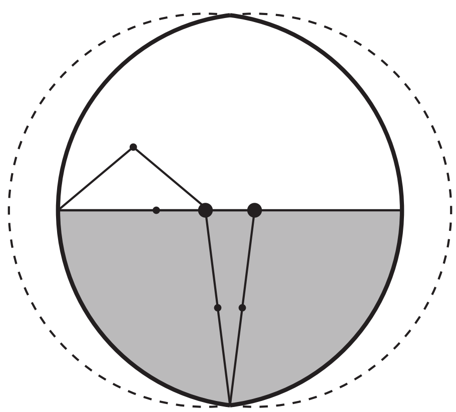

Figure 2.8: Parallel SCARA robot workspace solution

Chapter 3

Practice Exercises on Rigid-Body Motions

3.1 Practice Exercises

Practice exercise 3.1 The mobile manipulator in Figure 3.1 needs to orient its gripper to grasp the block. For subsequent placement of the block, we have decided that the orientation of the gripper relative to the block, when the gripper grasps the block, should be R_{eg}. Our job is to determine the rotation operator to apply to the gripper to achieve this orientation relative to the block.

Figure 3.1 shows the fixed world frame {a}, the mobile robot's chassis frame {b}, the gripper frame {c}, the RGBD camera (color vision plus depth, like the Kinect) frame {d}, and the object frame {e}. Because we put the camera at a known location in space, we know R_{ad}. The camera reports the configuration of {e} relative to {d}, so we know R_{de}. From the mobile robot's localization procedure (e.g., vision-based localization or odometry) we know R_{ab}. From the robot arm's forward kinematics we know R_{bc}.

(a) In terms of the four known rotation matrices R_{ad}, R_{de}, R_{ab}, and R_{bc}, and using only matrix multiplication and the transpose operation, express the current orientation of the gripper relative to the block, R_{ec}.

(b) To align the gripper properly, you could apply to it a rotation R_1 expressed in terms of axes in the gripper's {c} frame. What is R_1, in terms of the five known rotation matrices $(R_{ad}, R_{de}, R_{ab}, R_{bc}, R_{eg})$, matrix multiplication, and transpose?

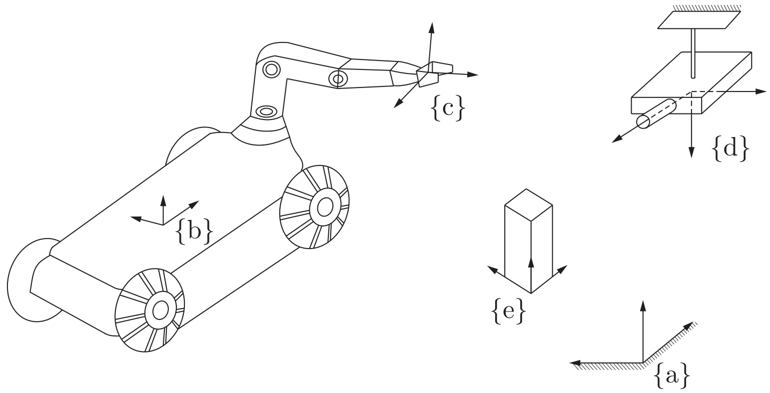

Figure 3.1: The fixed world frame {a}, the mobile robot's chassis frame {b}, the gripper frame {c}, the RGBD camera frame {d}, and the object frame {e}.

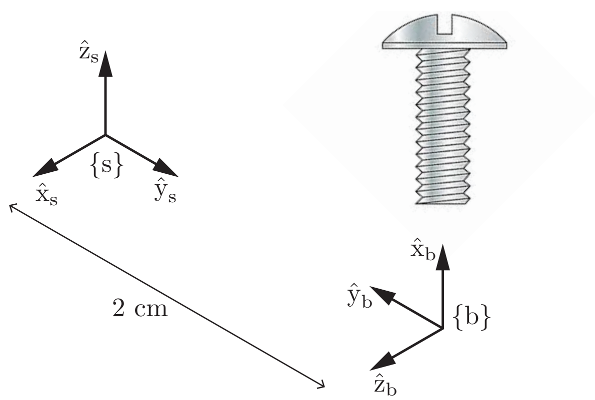

Figure 3.2: As the machine screw goes into a tapped hole, it advances linearly by 2 mm every rotation of the screw.

(c) The same rotation could be written R_2, in terms of the axes of the frame of the mobile base {b}. What is R_2?

Practice exercise 3.2 Figure 3.2 shows a screw, a frame {b}, and a frame {s}. The $\hat{x}_b$-axis of {b} is along the axis of the screw, and the origin of the frame {s} is displaced by 2 cm, along the $\hat{y}_b$-axis, from the {b} frame. The $\hat{z}_s$-axis is aligned with $\hat{x}_b$ and the $\hat{x}_s$-axis is aligned with $\hat{z}_b$.

Taking note of the direction of the screw's threads, as the machine screw goes

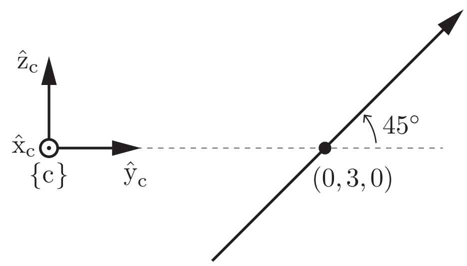

Figure 3.3: A screw axis in the $(\hat{y}_c, \hat{z}_c)$ plane.

into a tapped hole driven by a screwdriver rotating at 3 radians per second, what is the screw's twist expressed in {b}, $\mathcal{V}_b$? What is the screw axis expressed in {b}, $\mathcal{S}_b$? What is $\mathcal{V}_s$? What is $\mathcal{S}_s$? Give units as appropriate.

Practice exercise 3.3 A wrench F and a twist V are represented in {a} as $\mathcal{F}_a$ and $\mathcal{V}_a$, respectively, and they are represented in {b} as $\mathcal{F}_b$ and $\mathcal{V}_b$. Without consulting any other source, and using the facts that $(AB)^{\mathrm{T}} = B^{\mathrm{T}}A^{\mathrm{T}}$, that the adjoint of the transformation matrix T_{ab} can be used to change the frame of representation of a twist from the {a} frame to the {b} frame, and that the scalar power generated (or dissipated) by applying a wrench F along a twist V is independent of the frame of reference, show that $\mathcal{F}_a = [\mathrm{Ad}_{T_{ba}}]^{\mathrm{T}}\mathcal{F}_b$. (The ability to derive this result is useful for your understanding of it.)

Practice exercise 3.4 Figure 3.3 shows a screw axis in the $(\hat{y}_c, \hat{z}_c)$ plane, at a 45° angle with respect to the $\hat{y}_c$-axis. (The $\hat{x}_c$-axis points out of the page.) The screw axis passes through the point $(0, 3, 0)$.

(a) If the pitch of the screw is $h = 10$ linear units per radian, what is the screw axis $\mathcal{S}_c$? Make sure you can also write this in its $se(3)$ form $[\mathcal{S}_c]$, too.

(b) Using your answer to (a), if the speed of rotation about the screw axis is $\dot{\theta} = \sqrt{2}$ rad/s, what is the twist $\mathcal{V}_c$?

(c) Using your answer to (a), if a frame initially at {c} rotates by $\theta = \pi/2$ about the screw axis, yielding a new frame {c′}, what are the exponential coordinates describing the configuration of {c′} relative to {c}?

(d) What is $T_{cc'}$, corresponding to the motion in part (c)?

(e) Now imagine that the axis in Figure 3.3 represents a wrench: a linear force along the axis and a moment about the axis (according to the right-hand rule). The linear force in the direction of the axis is 20 and the moment about the axis is 10. What is the wrench $\mathcal{F}_c$?

Practice exercise 3.5 Let $T_{sb} \in SE(3)$ represent the configuration of the frame {b} relative to {s}. (We sometimes write this simply as T.) If {b} moves over time, you could represent its velocity as $\dot{T}_{sb}$ (or simply $\dot{T}$), the time derivative of T_{sb}. You should think of this velocity as a twist of the entire space, to which the moving frame is attached. But we know that the velocity should be representable by only six values, and $\dot{T}_{sb}$ could have 12 unique nonzero values in (the top three rows of the 4×4 matrix; the bottom row will be all zeros, since the bottom row of a transformation matrix is always the constant [0 0 0 1]).

Instead, we could post-multiply $\dot{T}_{sb}$ by T_{bs}, i.e., $\dot{T}_{s\cancel{b}}T_{\cancel{b}s} = \dot{T}T^{-1} = \dot{T}_{ss}$. This post-multiplication has the effect of representing the velocity in the {s} frame, getting rid of the dependence on the current {b} frame. What do we call the quantity $\dot{T}T^{-1}$? How many values are needed to uniquely specify it?

We could also pre-multiply $\dot{T}_{sb}$ by T_{bs} to get $T_{b\cancel{s}}\dot{T}_{\cancel{s}b} = T^{-1}\dot{T} = \dot{T}_{bb}$. This pre-multiplication has the effect of representing the velocity in the {b} frame, getting rid of the dependence on the {s} frame. What do we call the quantity $T^{-1}\dot{T}$?

Practice exercise 3.6

Consider the satellite and Earth shown in Figure 3.4. Let $\omega_b = (0, 1, 1)$ be the angular velocity of the satellite expressed in the satellite body frame {b}. Assume a fixed Earth frame {e} (a geocentric view of the universe like the ancient Greeks had).

(a) Solve for the coordinate axis velocities of {b} ($\dot{\hat{\mathrm{x}}}_\mathrm{b}$, $\dot{\hat{\mathrm{y}}}_\mathrm{b}$, and $\dot{\hat{\mathrm{z}}}_\mathrm{b}$) represented in the {b} frame. Sketch the velocity vectors on the figure above to confirm that your solutions make sense.

(b) The orientation of the {b} frame is equivalent to the {e} frame after it has been rotated -90 degrees about its $\hat{\mathrm{z}}_\mathrm{e}$-axis. Solve for ω_e, the satellite angular velocity represented in {e}. Sketch the velocity vectors on the figure above to confirm that your solution makes sense.

(c) Solve for $\dot{R}_{eb}$, the time derivative of the body orientation expressed in {e}.

(d) Give the $so(3)$ representation of the angular velocity in both the Earth and the body frame.

Practice exercise 3.7 Consider again the satellite and Earth shown in Figure 3.4.

(a) If the given rotational velocity $\omega_b = (0, 1, 1)$ was instead the exponential coordinates for a rotation, solve for the axis-angle representation and the corresponding rotation matrix.

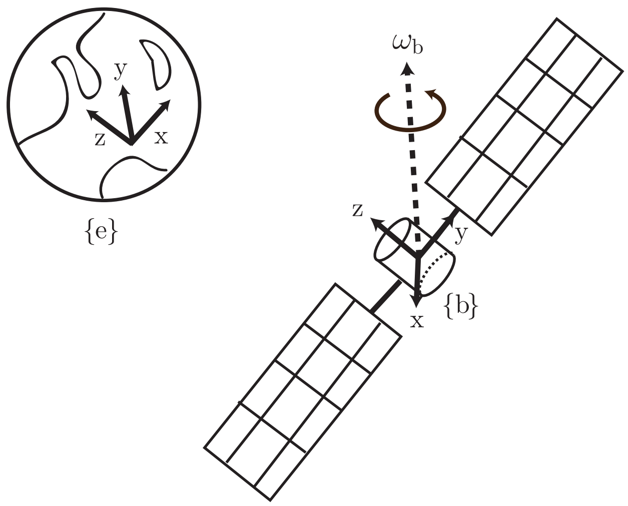

Figure 3.4: Satellite rotating in space.

(b) After rotating and orbiting the Earth for some time, the relative orientation of the Earth and satellite is given as

$$R_{eb'} = \begin{bmatrix} 0 & -\sqrt{2}/2 & \sqrt{2}/2 \\ 1 & 0 & 0 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}.$$

Find the axis-angle representation that describes the rotation from the initial body frame {b} to the new body frame {b'}.

Practice exercise 3.8 Consider the scene in Figure 3.5 of a once peaceful park overrun by robots. Frames are shown attached to the tree {t}, robot chassis {c}, manipulator {m}, and quadcopter {q}. The distances shown in the figure are $d_1 = 4$ m, $d_2 = 3$ m, $d_3 = 6$ m, $d_4 = 5$ m, $d_5 = 3$ m. The manipulator is at a position $p_{\text{cm}} = (0, 2, 1)$ m relative to the chassis frame {c}, and {m} is rotated from {c} by 45 degrees about the $\hat{x}_c$-axis.

(a) Give the transformation matrices representing the quadcopter frame {q}, chassis frame {c}, and manipulator frame {m} in the tree frame {t}.
(b) Assume that the position controller for the manipulator on the mobile robot is referenced to the chassis frame {c}. What position should you

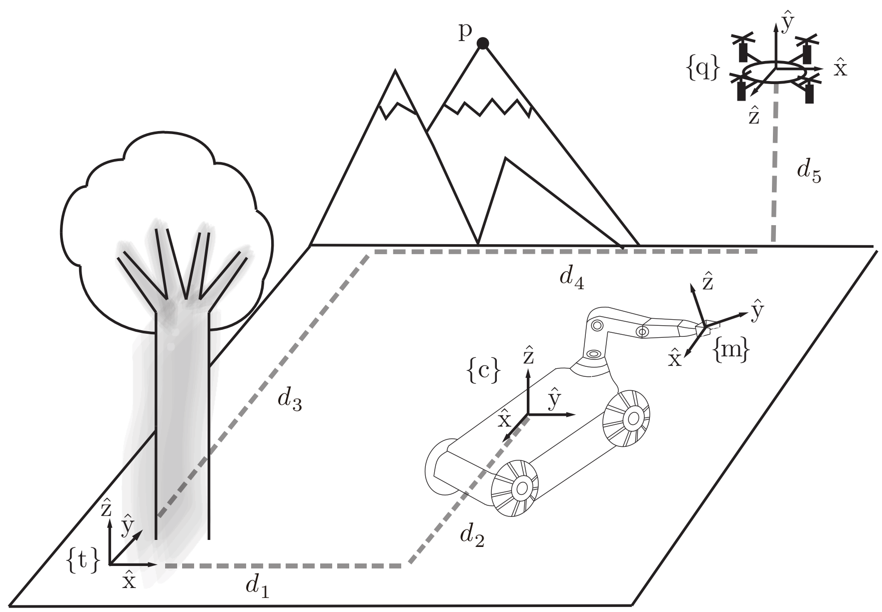

Figure 3.5: A tree, mobile manipulator, and flying robot in a park and the corresponding reference frames.

command the gripper to go to if you would like to snatch the quadcopter out of the sky?

(c) You are tasked to move the mobile robot so that the chassis origin is directly underneath the quadcopter and its frame is aligned with the tree frame. Assume the mobile robot chassis controller takes transformation matrices in the chassis frame as inputs. What tranformation should you command the robot to follow?

Practice exercise 3.9 Consider the scene in Figure 3.6 of a quadcopter {q} flying near a tree {t} and house {h}. The quadcopter is at a position $p_{\mathrm{tq}} = (10, 5, 5)$ m expressed in the tree frame {t}, and the house is at a position $p_{\mathrm{th}} = (0, 10, 10)$ m expressed in the tree frame {t}. The quadcopter is flying upwards with a velocity of 1 m/s, and rotating with a velocity of 1 rad/s.

(a) Calculate the quadcopter's twist in {q} and {t}.

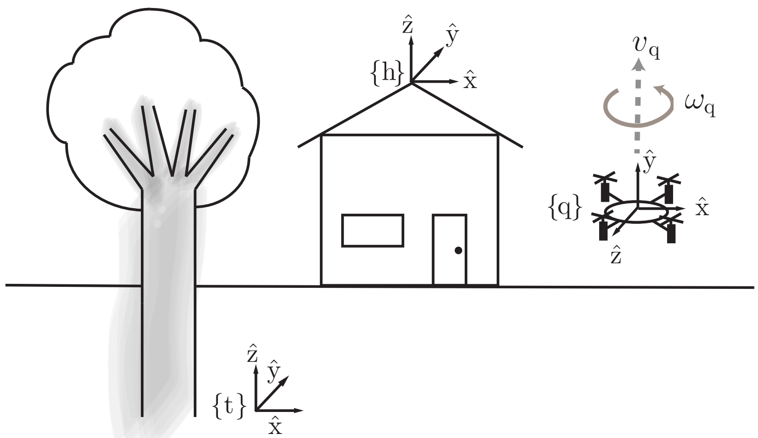

Figure 3.6: A tree, and flying robot in a park and the corresponding reference frames.

(b) Use the adjoint map to express the twist in the house frame {h}.

Practice exercise 3.10 Consider the cube with side lengths $l = 2$ m and the ant shown in Figure 3.7. Frames {b} and {c} show the ant at the midpoint of the cube edges.

(a) Solve for the screw axis $\mathcal{S}_w$ and angle θ corresponding to the transformation from {b} to {c}.
(b) Sketch the location of the screw axis on the figure.
(c) Use the appropriate adjoint map to find $\mathcal{S}_b$, the screw axis representation in the {b} frame.

Practice exercise 3.11 Consider the scene in Figure 3.8 of a tree {t} and a frame {b} attached to its branch. The figure shows a strong wind that applies a force of 100 N at the center of frame {b}. Assume the branch has a mass of 50 kg centered at frame {b} as well. The position of the branch frame {b} in the tree frame {t} is $p_{\mathrm{tb}} = (2, 1, 3)$ m.

(a) What is the wrench $\mathcal{F}_b$ due to the wind and the branch's weight?
(b) What is this wrench in the tree frame {t}?

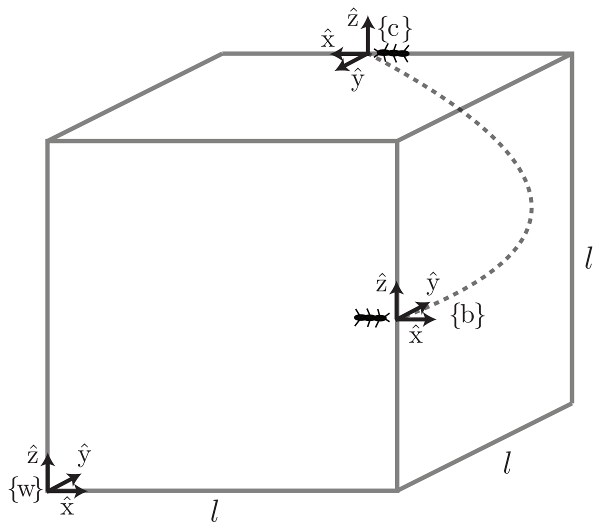

Figure 3.7: An ant shown at different positions on a cube.

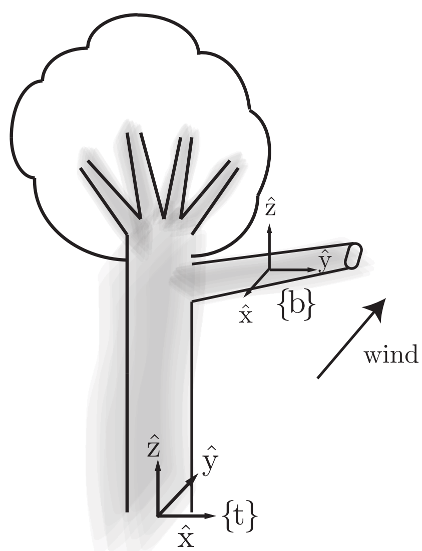

Figure 3.8: A tree and a frame attached to its branch.

3.2 Solutions

Solution 3.1

(a)

$$\begin{aligned} R_{ec} &= R_{ed}R_{da}R_{ab}R_{bc} \\ &= R_{de}^{\mathrm{T}}R_{ad}^{\mathrm{T}}R_{ab}R_{bc}. \end{aligned}$$

(b)

$$\begin{aligned} R_{ec}R_1 = R_{eg} \rightarrow R_1 &= R_{ec}^{\mathrm{T}}R_{eg} \\ &= (R_{de}^{\mathrm{T}}R_{ad}^{\mathrm{T}}R_{ab}R_{bc})^{\mathrm{T}}R_{eg} \\ &= R_{bc}^{\mathrm{T}}R_{ab}^{\mathrm{T}}R_{ad}R_{de}R_{eg} \ \ (= R_{cg}). \end{aligned}$$

(c)

$$R_2 = R_{bc}R_1 = R_{bc}R_{cg} = R_{bc}R_{bc}^{\mathrm{T}}R_{ab}^{\mathrm{T}}R_{ad}R_{de}R_{eg} = R_{ab}^{\mathrm{T}}R_{ad}R_{de}R_{eg} \ \ (= R_{bg}).$$

Solution 3.2 The threads of this screw are the typical right-handed threads, which means that the screw, when viewed from the top, rotates clockwise when it advances into a tapped hole. In other words, the fingers of your right hand curl in the direction of rotation of the screw when your right thumb points downward on the page, in the negative direction of the upward-pointing $\hat{\mathrm{x}}_{\mathrm{b}}$-axis. Since the screwdriver rotates at 3 rad/s, the screw also rotates at 3 rad/s, so the angular component of the twist, expressed in {b}, is $\omega_b = (-3 \text{ rad/s}, 0, 0)$. Since radians and seconds are the SI units for angle and time, respectively, you could write $(-3, 0, 0)$ and assume the default SI units. You could also write $(-3(180/\pi) \text{ deg/s}, 0, 0)$, but that would be unusual.

The pitch of the screw is 2 mm/rev, or $(1/\pi)$ mm/rad. So as the screw is rotated at 3 rad/s, it moves linearly in the $-\hat{\mathrm{x}}_{\mathrm{b}}$ direction at $(2 \text{ mm/rad})(3 \text{ rad/s}) = 6$ mm/s. So the linear component of the twist expressed in {b} is $(-6 \text{ mm/s}, 0, 0)$, or, in SI units, $v_b = (-0.006 \text{ m/s}, 0, 0)$. So, in SI units, the entire twist is $\mathcal{V}_b = (\omega_b, v_b) = (-3, 0, 0, -0.006, 0, 0)$.

The corresponding screw axis expressed in {b} is the normalized version of $\mathcal{V}_b$ where the magnitude of the angular velocity is unit. The magnitude of ω_b is 3, so divide the twist by 3 to get $\mathcal{S}_b = (-1, 0, 0, -0.002, 0, 0)$. We can write $\mathcal{V}_b = \mathcal{S}_b\dot{\theta}$ where $\dot{\theta} = \|\omega_b\| = 3$.

The screw axis could also be represented in the {b} frame by the collection $\{q_b, \hat{s}_b, h\}$, where a point q_b on the axis is $(0,0,0)$ (expressed in {b}), the axis direction is $\hat{s}_b = (-1,0,0)$, and the pitch is $h = 0.002$.

In the {s} frame, the axis of rotation is aligned with the $-\hat{\mathrm{z}}_\mathrm{s}$-axis, so $\omega_s = (0,0,-3)$. A point at the origin of {s}, rigidly attached to the advancing screw, has a downward linear component of -0.006 m/s in the $-\hat{\mathrm{z}}_\mathrm{s}$ direction (i.e., $(0,0,-0.006)$) from the downward motion of the screw. But it also has a linear component in the $-\hat{\mathrm{x}}_\mathrm{s}$ direction from the rotation of the screw. The point at the origin of {s} can be expressed as $q_b = (0, 0.02, 0)$ in terms of {b} coordinates, so the linear motion at {s} due to the rotation of the screw is $\omega_b \times q_b = (0,0,-0.06)$. In the {s} frame, this is $(-0.06, 0, 0)$. (Imagine a turntable rotating about the screw axis and the resulting motion of a point at {s}.) So the total linear motion at {s}, expressed in {s}, is $v_s = (0,0,-0.006) + (-0.06,0,0) = (-0.06, 0, -0.006)$. Therefore, $\mathcal{V}_s = (0,0,-3,-0.06,0,-0.006)$. The screw axis is $\mathcal{S}_s = (0,0,-1,-0.02,0,-0.002)$ and $\mathcal{V}_s = \mathcal{S}_s\dot{\theta}$.

The screw axis could also be represented in the {s} frame by the collection $\{q_s, \hat{s}_s, h\}$, where a point q_s on the axis is $(0, 0.02, 0)$, the axis direction is $\hat{s}_s = (0,0,-1)$, and the pitch is $h = 0.002$. Note that $\mathcal{S}_s = (\hat{s}_s, -\hat{s}_s \times q_s + h\hat{s})$, where $h\hat{s}$ is the linear velocity due to the linear motion of the screw and $-\hat{s}_s \times q_s$ is the linear velocity due to the rotation of the screw.

Solution 3.3 See Chapter 3.4 of the textbook.

Solution 3.4

(a) Since the screw axis $\mathcal{S}_c = (\mathcal{S}_{c_\omega}, \mathcal{S}_{c_v})$ has a rotational component, $\mathcal{S}_{c_\omega}$ is a unit vector aligned with the axis, i.e., $\mathcal{S}_{c_\omega} = \hat{s} = (0, \cos 45°, \sin 45°) = (0, 1/\sqrt{2}, 1/\sqrt{2})$. The linear component is $\mathcal{S}_{c_v} = h\hat{s} - \hat{s} \times q$ (a linear component due to linear motion along the screw plus a linear component due to rotation about the screw), where $q = (0,3,0)$ and $h = 10$, i.e., $\mathcal{S}_{c_v} = (0, 10/\sqrt{2}, 10/\sqrt{2}) + (3/\sqrt{2}, 0, 0) = (3, 10, 10)/\sqrt{2}$.

(b) $\mathcal{V}_c = \mathcal{S}_c\dot{\theta} = (0,1,1,3,10,10)$.

(c) $\mathcal{S}_c\theta = (0,1,1,3,10,10)\pi/(2\sqrt{2})$.

(d) You can use the MR code library to do the calculation. Use `VecTose3` to convert the exponential coordinates $\mathcal{S}_c\theta$ to their $se(3)$ representation $[\mathcal{S}_c\theta]$ and then use `MatrixExp6` to calculate

$$T_{cc'} = e^{[\mathcal{S}_c\theta]} = \begin{bmatrix} 0 & -0.71 & 0.71 & 2.12 \\ 0.71 & 0.5 & 0.5 & 12.61 \\ -0.71 & 0.5 & 0.5 & 9.61 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

(e) The wrench is written $\mathcal{F}_c = (m_c, f_c)$. The linear component f_c has a magnitude of 20 and is aligned with the axis shown, so $f_c = (0, 10\sqrt{2}, 10\sqrt{2})$. If the axis passed through the origin of {c}, the moment (which has magnitude 10) would be $(0, 5\sqrt{2}, 5\sqrt{2})$, but since it is displaced from the origin of {c}, there is an extra moment component due to the linear component, $q \times f_c = (0, 3, 0) \times (0, 10\sqrt{2}, 10\sqrt{2}) = (30\sqrt{2}, 0, 0)$, so the total moment is $m_c = (0, 5\sqrt{2}, 5\sqrt{2}) + (30\sqrt{2}, 0, 0) = \sqrt{2}(30, 5, 5)$.
You can verify that you get the same answer using $\mathcal{F}_c = [\mathrm{Ad}_{T_{ac}}]^{\mathrm{T}} \mathcal{F}_a$, where {a} is a frame aligned with {c} and with an origin at $(0, 3, 0)$.

Solution 3.5 $\dot{T}T^{-1}$ is the $se(3)$ representation of the twist represented in {s}, i.e., $[\mathcal{V}_s] \in se(3)$. Only six values (the six elements of $\mathcal{V}_s$) are needed to specify it.

$T^{-1}\dot{T}$ is the $se(3)$ representation of the twist represented in {b}, i.e., $[\mathcal{V}_b] \in se(3)$. Only six values (the six elements of $\mathcal{V}_b$) are needed to specify it.

Solution 3.6

(a) $\dot{\hat{\mathrm{x}}}_{\mathrm{b}} = (0, 1, -1)$, $\dot{\hat{\mathrm{y}}}_{\mathrm{b}} = (-1, 0, 0)$, $\dot{\hat{\mathrm{z}}}_{\mathrm{b}} = (1, 0, 0)$

(b) $\omega_e = (1, 0, 1)$.

(c) $$\dot{R}_{eb} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$

(d) $$[\omega_b] = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}, \quad [\omega_e] = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

Solution 3.7

(a) $\hat{\omega} = (0, \sqrt{2}/2, \sqrt{2}/2)$, and $\theta = \sqrt{2}$.

$$R = \begin{bmatrix} 0.1559 & -0.6985 & 0.6985 \\ 0.6985 & 0.5780 & 0.4220 \\ -0.6985 & 0.4220 & 0.5780 \end{bmatrix}$$

(b) $$R_{bb'} = R_{eb}^{\mathrm{T}} R_{eb'} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\sqrt{2}/2 & \sqrt{2}/2 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix},$$

$\hat{\omega} = (0, 0.3827, 0.9239)$, and $\theta = \pi$.

Solution 3.8

(a)

$$T_{tq} = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 0 & -1 & 6 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{tc} = \begin{bmatrix} 0 & 1 & 0 & 4 \\ -1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{tm} = \begin{bmatrix} 0 & \sqrt{2}/2 & -\sqrt{2}/2 & 6 \\ -1 & 0 & 0 & 3 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b)

$p_{cq} = (-3, 1, 3)$

(c)

$p_{cc'} = (-3, 1, 0)$

$R_{cc'} = R_{ct} = R'_{tc}$

$$T_{cc'} = \begin{bmatrix} 0 & -1.0 & 0 & -3.0 \\ 1.0 & 0 & 0 & 1.0 \\ 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 1.0 \end{bmatrix}$$

Solution 3.9

(a) $\mathcal{V}_b = (0, 1, 0, 0, 1, 0)$. $\mathcal{V}_t = (0, 0, 1, 5, -10, 1)$.

(b) $\mathcal{V}_h = (0, 0, 1, -5, -10, 1)$.

Solution 3.10

(a) $\mathcal{S}_w = (0, 0, 1, 1, -1.5, 1/\pi)$, $\theta = \pi$.

(b) Axis points in in the world z direction intersecting the (x,y) coordinates (1.5,1) in the {w} frame.

(c) $\mathcal{S}_b = (0, 0, 1, 1, 0.5, 1/\pi)$, $\theta = \pi$.

Solution 3.11

(a) $\mathcal{F}_b = (0, 0, 0, -100, 0, -500)$

(b) $\mathcal{F}_t = (-800, 1000, 200, 0, 100, -500)$