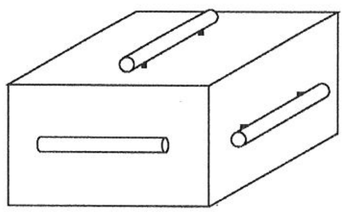
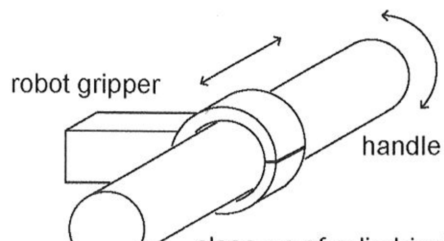
1. (2 pts) The box shown on the left has handles rigidly attached to each of its faces. (The handles cannot move relative to the box.) When a robot gripper closes on a handle, it forms a cylindrical joint with the handle: the gripper can rotate around and translate along the axis of the handle, as shown in the figure on the right.

Assume the box is jointly manipulated by n 7R robot arms, each of which is mounted to the floor. If each of the n robots grasps a handle, forming a cylindrical joint with the box, how many degrees of freedom does the entire system (robots plus box) have?



box with handles on each face



close-up of cylindrical joint
formed by robot grasp of a handle

$$N = 7n + \underbrace{1}_{\text{ground}} + \underbrace{1}_{\text{box}} = 7n+2$$

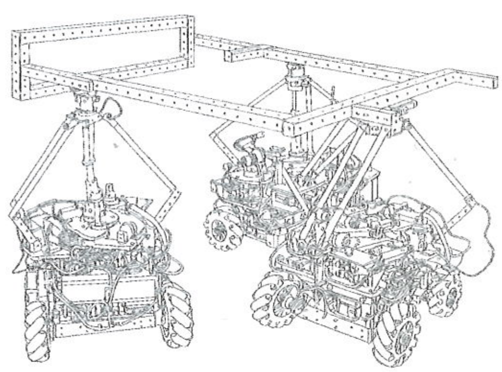
$$J = (7+1)n = 8n$$

$$\sum_i f_i = (7+2)n = 9n$$

cylindrical grasp joint

$$\text{dof} = 6(7n+2-1-8n) + 9n$$
$$= 3n+6$$

2. (2 pts) The image below shows three Omnid mobile manipulators jointly carrying a rigid object. Each mobile manipulator has 9 dof: a 6-dof manipulator mounted on a wheeled mobile base, which is considered to have 3 dof (rotation and translation of the chassis in the plane of the floor). Each robot manipulator's end-effector "grasps" the object rigidly: no relative motion is possible between the end-effector and the object. What is the total number of degrees of freedom of the system (robots plus rigidly-grasped object)?

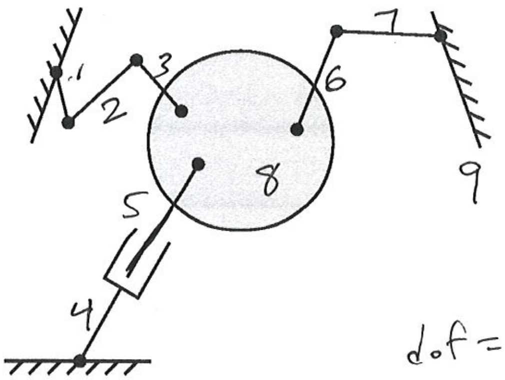


9 dof for Omnid 1 and its rigidly grasped object

object's location places 6 constraints on configuration of end-effector for robots 2 & 3, so each provides $9-6 = 3$ dof

$9+3+3 = 15$ dof

3. (2 pts) The planar mechanism below has one prismatic joint, several revolute joints (represented as filled circles), and several links, including the gray disk. How many degrees of freedom does this mechanism have?



$N = 8 + \underset{\text{ground}}{1} = 9$

$M = 3$ (planar)

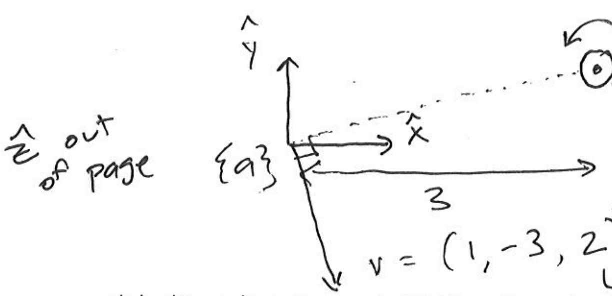
$J = 10 \qquad \sum_i f_i = 10$

$\text{dof} = 3(9 - 1 - 10) + 10$

$= 4$

4. We will represent a screw axis S in two different frames in parts (a) and (c).

(a) (2 pts) The screw axis can be represented in a frame {a} by the point $q = (3, 1, 0)$, a direction $\hat{s} = (0, 0, 1)$, and a pitch $h = 2$. Write the screw axis as $\mathcal{S}_a$. This should be a six-vector of numbers only.



$v = (1, -3, 2)^T$ → from the pitch

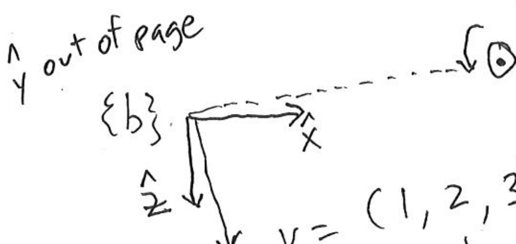
$$\mathcal{S}_a = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ -3 \\ 2 \end{bmatrix}$$

(b) (2 pts) A frame initially aligned with {a} follows a twist $\mathcal{V}_a = (\pi, 0, 0, 0, 0, 0)^{\mathsf{T}}$ for 1/2 second, ending at the frame {b}. What is the exponential coordinate representation of T_{ab}?

$$\frac{1}{2}\mathcal{V}_a = \begin{bmatrix} \pi/2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(c) (2 pts) Represent the screw axis S in {b}, i.e., $\mathcal{S}_b$. Your answer should be a six-vector of numbers only.

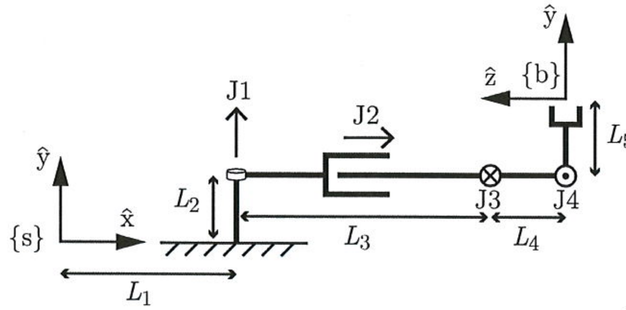
{b} obtained by rotating {a} by $\pi/2$ about $\hat{x}$-axis.



$v = (1, 2, 3)^T$ → from the pitch

$$\mathcal{S}_b = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

5. The figure below shows a schematic for a RPRR robot arm at its home configuration, $\theta = 0$. The axis of J1 points upward on the page, the axis of the prismatic joint J2 is to the right, the axis of J3 is into the page, and the axis of J4 is out of the page.



$\hat{x}$ out of page

$\hat{z}$ out of page

(a) (2 pts) Give $M = T_{sb}(0)$.

$$M = \begin{bmatrix} 0 & 0 & -1 & L_1+L_3+L_4 \\ 0 & 1 & 0 & L_2+L_5 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) (4 pts) Write the space Jacobian $J_s(0)$.

$$J_s(0) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 1 & -L_2 & L_2 \\ 0 & 0 & L_1+L_3 & -L_1-L_3-L_4 \\ L_1 & 0 & 0 & 0 \end{bmatrix}$$

(c) (2 pts) Is this home configuration a singularity? Explain briefly.

No. Each column of $J_s(0)$ is linearly independent of the others.

6. (2 pts) Let T_{ab} represent the {b} frame relative to the {a} frame and T_{ac} represent the {c} frame relative to the {a} frame. Consider the twist that takes a frame initially aligned with {a} to alignment with {b} in 4 seconds. If the same twist is applied to {c} for 2 seconds, moving it to {c'}, give the expression for $T_{ac'}$ as a function of T_{ab} and T_{ac}.

$$4[\mathcal{V}_a] = \log T_{ab} \Rightarrow [\mathcal{V}_a] = \tfrac{1}{4}\log T_{ab}$$

$$T_{ac'} = \exp(2[\mathcal{V}_a])T_{ac} = \exp\left(\tfrac{1}{2}\log T_{ab}\right)T_{ac}$$

7. (2 pts) Let T_{ab} represent the configuration of {b} relative to {a}. A twist takes a frame attached to a rigid body, initially aligned with {a}, to {b} in t seconds. At the beginning of the execution of this twist (when the body is at {a}), a wrench $\mathcal{F}_b$ is applied to the body. What is the power due to the wrench on the moving body as a function of T_{ab} and $\mathcal{F}_b$? You may use the vec($\cdot$) operation, which is the inverse of the [$\cdot$] operation, i.e., $\text{vec}([\mathcal{V}]) = \mathcal{V}$.

$$[\mathcal{V}_a] = \tfrac{1}{t}\log T_{ab} \Rightarrow \mathcal{V}_a = \text{vec}\left(\tfrac{1}{t}\log T_{ab}\right)$$

power $= \mathcal{F}_b^T\mathcal{V}_b$ or $\mathcal{F}_a^T\mathcal{V}_a$ or $\mathcal{V}_b^T\mathcal{F}_b$ or $\mathcal{V}_a^T\mathcal{F}_a$, for example...

$$\mathcal{F}_b^T\mathcal{V}_b = \mathcal{F}_b^T[\text{Ad}_{T_{ab}^{-1}}]\,\text{vec}\left(\tfrac{1}{t}\log T_{ab}\right)$$