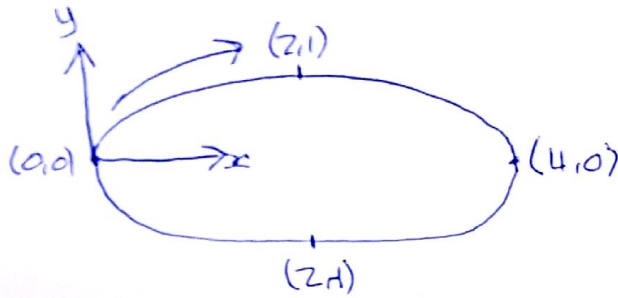


Question 1:



$$\begin{aligned}x &= 2 - 2\cos(2\pi s) \\ y &= \sin(2\pi s)\end{aligned}$$

$$s \in [0, 1]$$

~~Check the points~~ CHECK:

at $s = 0$:

$$(x, y) = (0, 0)$$

at $s = 0.25$:

$$(x, y) = (2, 1)$$

at $s = 0.5$:

$$(x, y) = (4, 0)$$

at $s = 0.75$:

$$(x, y) = (2, -1)$$

at $s = 1$:

$$(x, y) = (0, 0)$$

Question 2

$$X = (x, y, z)$$

$$x = \cos(2\pi s) \quad y = \sin(2\pi s) \quad z = 2s \quad s \in [0, 1]$$

$$s(t) = \frac{1}{4}t + \frac{1}{8}t^2 \quad t \in [0, 2]$$

$$\dot{X} = \frac{dX}{ds} \dot{s}$$

$$\frac{dx}{ds} \dot{s} = -2\pi \sin(2\pi s) \left(\frac{1}{4} + \frac{1}{4}t\right)$$

$$\frac{dy}{ds} \dot{s} = 2\pi \cos(2\pi s) \left(\frac{1}{4} + \frac{1}{4}t\right)$$

$$\frac{dz}{ds} \dot{s} = 2 \left(\frac{1}{4} + \frac{1}{4}t\right)$$

$$\ddot{X} = \frac{d^2X}{ds^2} \dot{s}^2 + \frac{d^2X}{ds^2} \ddot{s}$$

$$\frac{d^2x}{ds^2} \dot{s}^2 + \frac{d^2x}{ds^2} \ddot{s} = (-2\pi \sin(2\pi s) \left(\frac{1}{4}\right)) - 4\pi^2 \cos(2\pi s) \left(\frac{1}{4} + \frac{1}{4}t\right)^2$$

$$\frac{d^2y}{ds^2} \dot{s}^2 + \frac{d^2y}{ds^2} \ddot{s} = (2\pi \cos(2\pi s) \left(\frac{1}{4}\right)) - 4\pi^2 \sin(2\pi s) \left(\frac{1}{4} + \frac{1}{4}t\right)^2$$

$$\frac{d^2z}{ds^2} \dot{s}^2 + \frac{d^2z}{ds^2} \ddot{s} = 2 \left(\frac{1}{4}\right) + 0 = \frac{1}{2}$$

$$s(T)=1 \quad s(0)=\dot{s}(0)=\ddot{s}(0)=\ddot{s}(T)=\dot{s}(T)=0$$

Question 3

$$s(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

$$s(0) = a_0 = 0$$

$$\dot{s}(t) = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4$$

$$\dot{s}(0) = a_1 = 0$$

$$\ddot{s}(t) = 2a_2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3$$

$$\ddot{s}(0) = 2a_2 = 0$$

$$s(T) = a_3 T^3 + a_4 T^4 + a_5 T^5 = 1$$

$$\therefore a_3 = \frac{1}{T^3} - a_4 T - a_5 T^2 \quad \text{--- ①}$$

$$\dot{s}(T) = 3a_3 T^2 + 4a_4 T^3 + 5a_5 T^4 = 0 \quad \text{--- sub in eqn ①}$$

$$\therefore \frac{3}{T} - 3a_4 T^3 - 3a_5 T^4 + 4a_4 T^3 + 5a_5 T^4 = 0$$

$$\therefore \frac{3}{T} + a_4 T^3 + 2a_5 T^4 = 0$$

$$\therefore a_4 = \left[\frac{-\frac{3}{T} - 2a_5 T^4}{T^3} \right] \quad \text{--- ②}$$

$$\therefore a_4 = -\frac{3}{T^4} - 2a_5 T \quad \text{--- ②}$$

$$\ddot{s}(T) = 6a_3T + 12a_4T^2 + 20a_5T^3 = 0$$

Sub in ① & ②

$$\circ \circ 6T \left[\frac{1}{T^3} + \frac{3}{T^3} + 2a_5T^2 - a_5T^2 \right] + 12T^2 \left[-\frac{3}{T^4} - 2a_5T \right] + 20a_5T^3 = 0$$

$$\circ \circ \frac{6}{T^2} + \frac{18}{T^2} + 12a_5T^3 - 6a_5T^3 - \frac{36}{T^2} - 24a_5T^3 + 20a_5T^3 = 0$$

$$\circ \circ -\frac{12}{T^2} + 2a_5T^3 = 0$$

$$\circ \circ a_5 = \frac{6}{T^5}$$

Sub back into ②

$$a_4 = -\frac{3}{T^4} - 2T \left(\frac{6}{T^5} \right)$$

$$\circ \circ a_4 = -\frac{3}{T^4} - \frac{12}{T^4} = -\frac{15}{T^4}$$

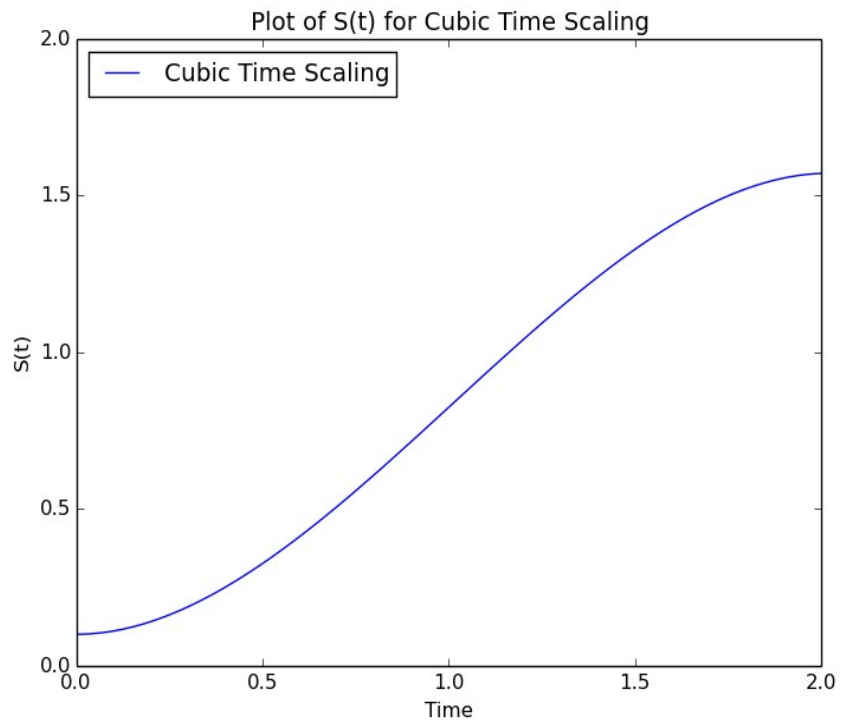
Sub back into ①

$$a_3 = \frac{1}{T^3} - T \left(-\frac{15}{T^4} \right) - T^2 \left(\frac{6}{T^5} \right)$$

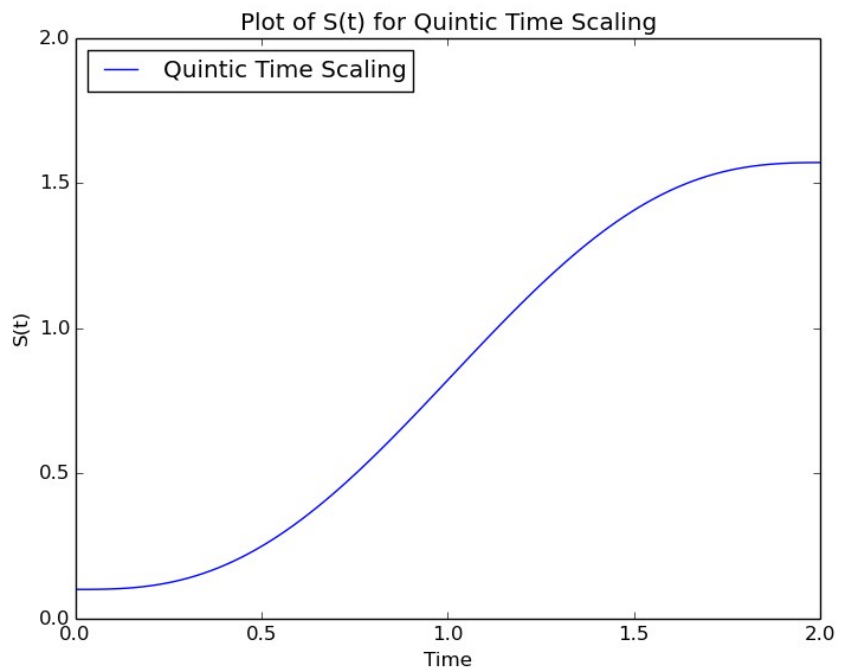
$$\circ \circ a_3 = \frac{1}{T^3} + \frac{15}{T^3} - \frac{6}{T^3} = \frac{10}{T^3}$$

QUESTION 5:

i)



ii)



iii)

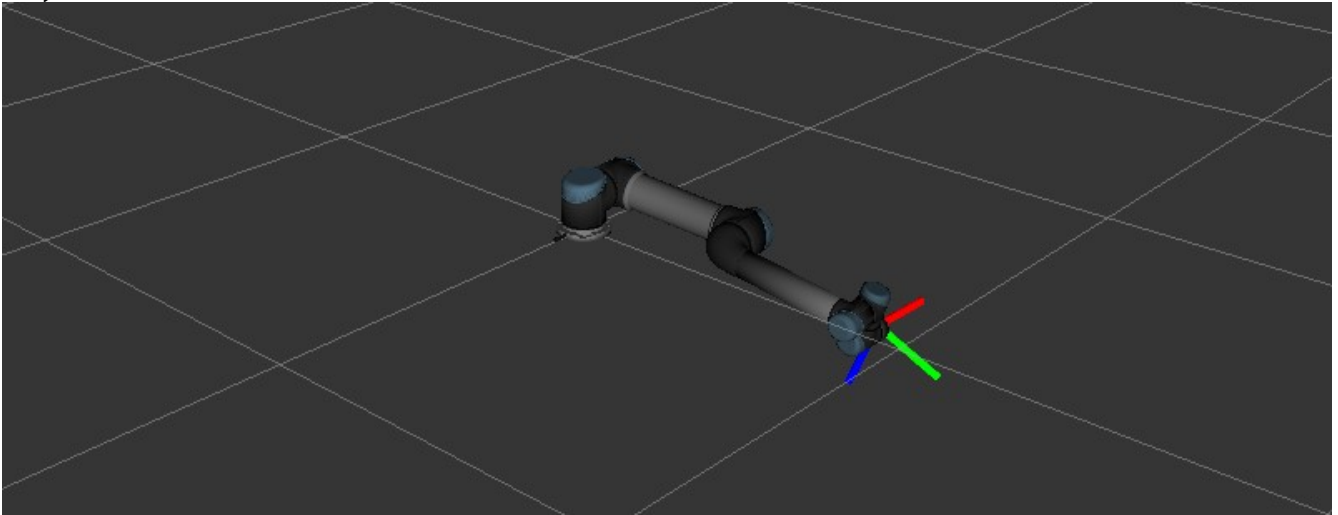


Illustration 1: First Angles

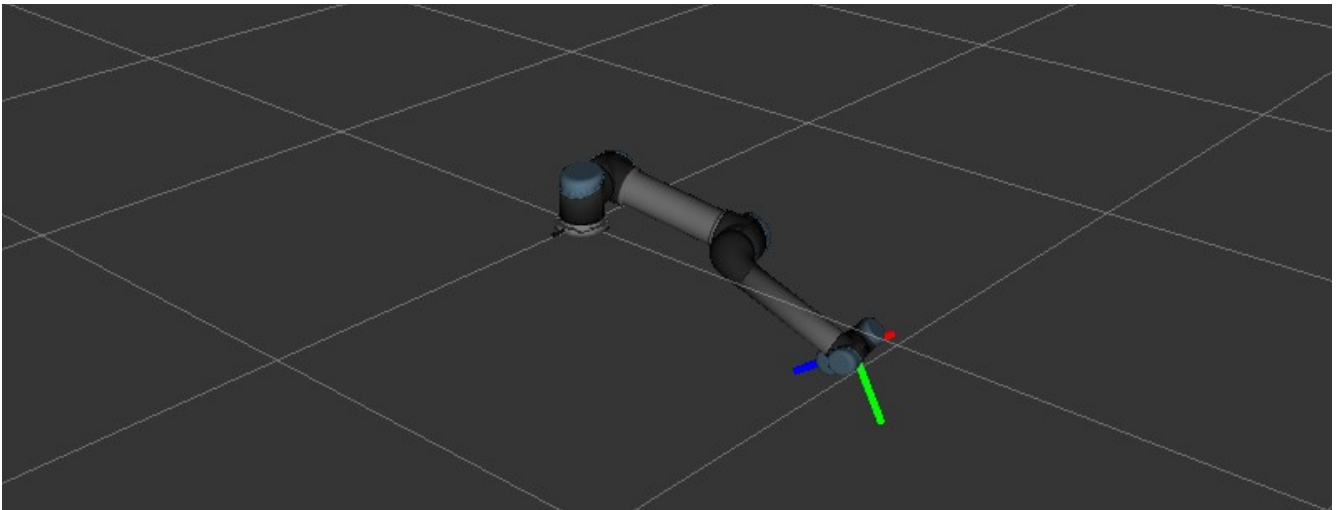


Illustration 2: 26th Angles

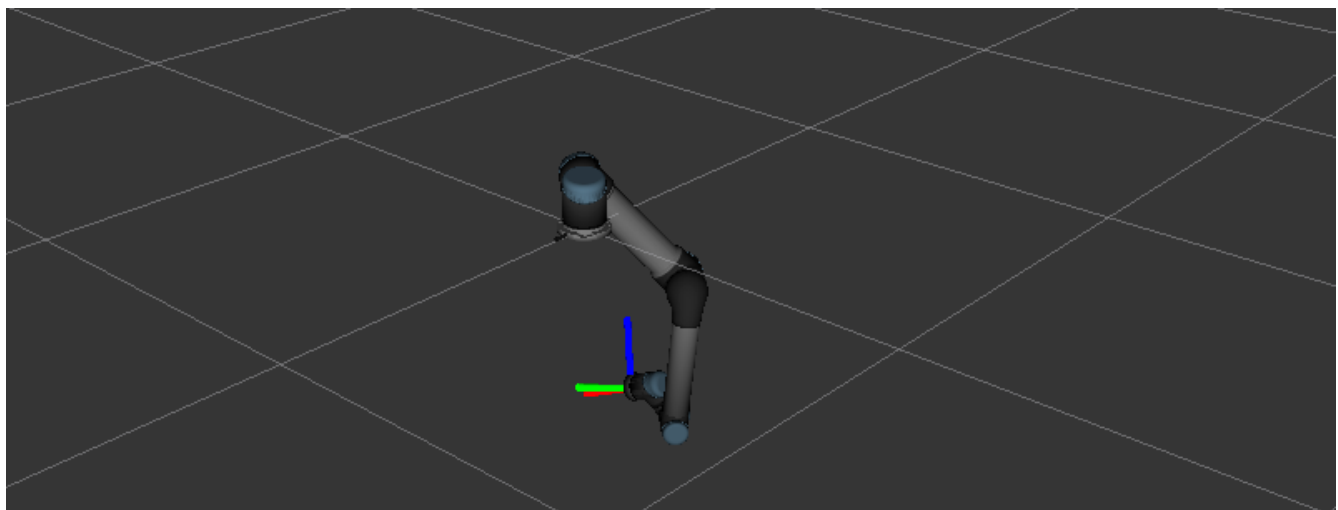


Illustration 3: 51st Angles

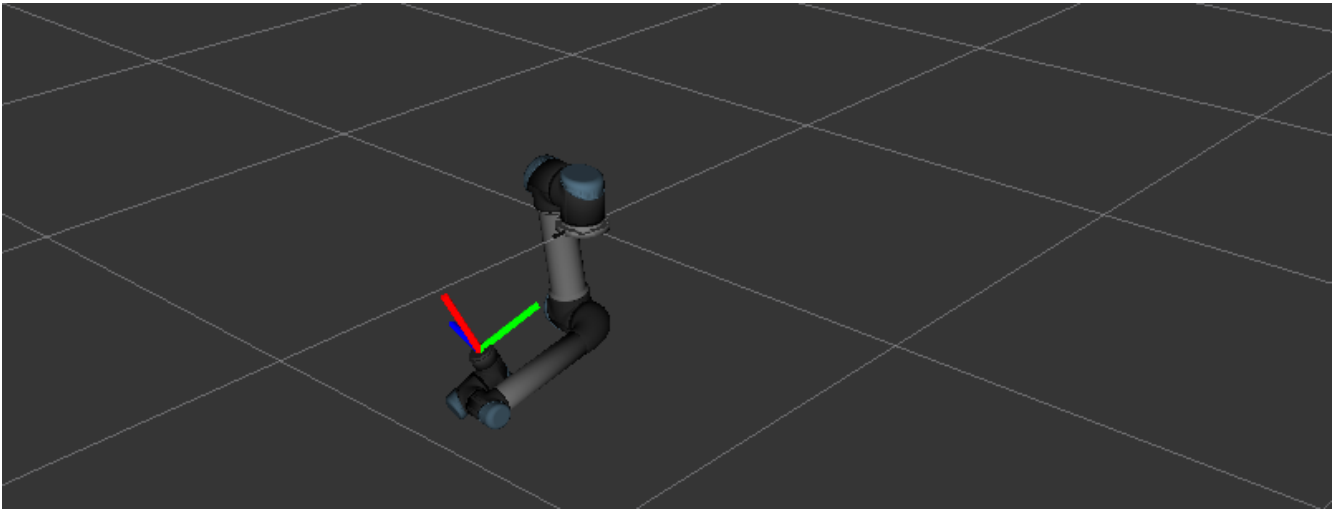


Illustration 4: 76th Angles

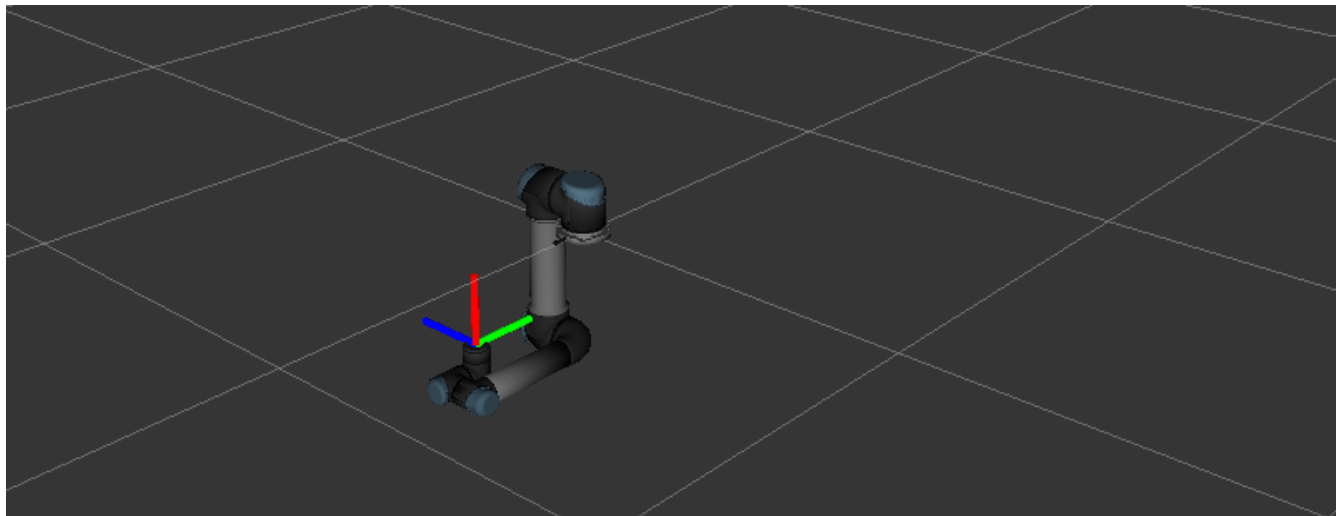


Illustration 5: Last Angles

Animation can be seen at this link:

<https://www.youtube.com/watch?v=fVElSuS1GgI>

QUESTION 6

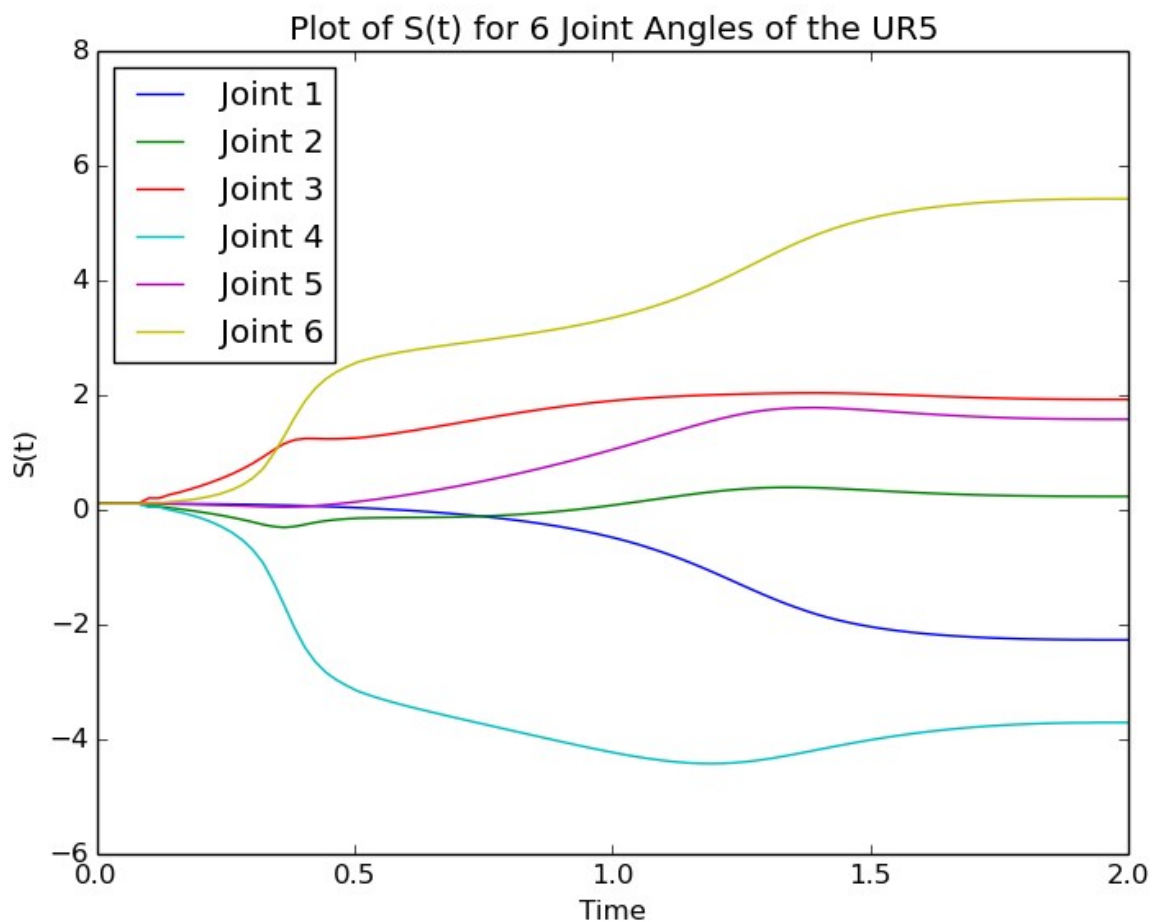
i)

```
File Edit View Search Terminal Help

Xstart rounded to 3 decimal places:
[[ 0.92 -0.392 0.005 -0.763]
 [-0.008 -0.029 -1. -0.269]
 [ 0.392 0.92 -0.03 -0.126]
 [ 0. 0. 0. 1.  ]]

Xend rounded to 3 decimal places:
[[ 0. -1. 0. 0.109]
 [ 1. 0. 0. 0.297]
 [-0. 0. 1. -0.254]
 [ 0. 0. 0. 1.  ]]
```

ii)



iii)

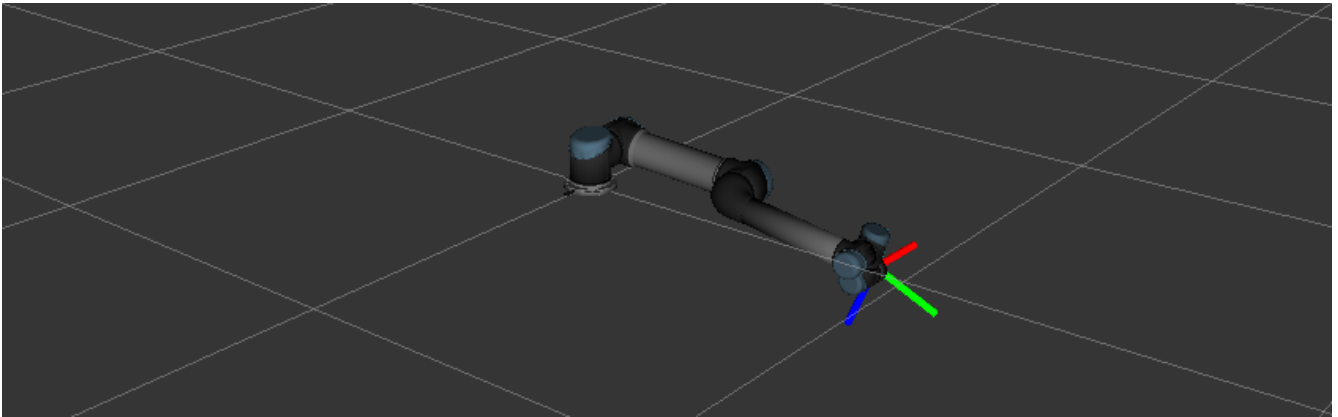


Illustration 6: First Angles

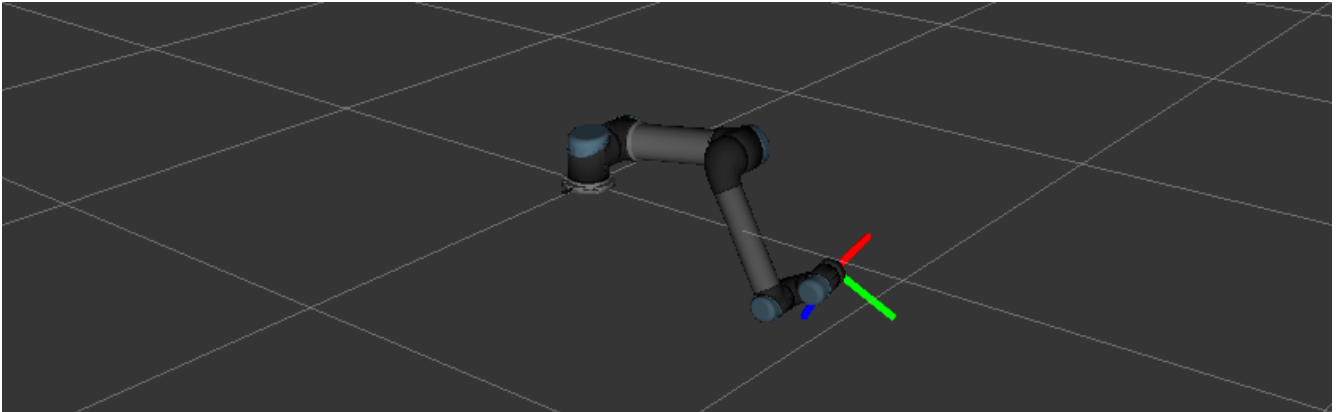


Illustration 7: 26th Angles

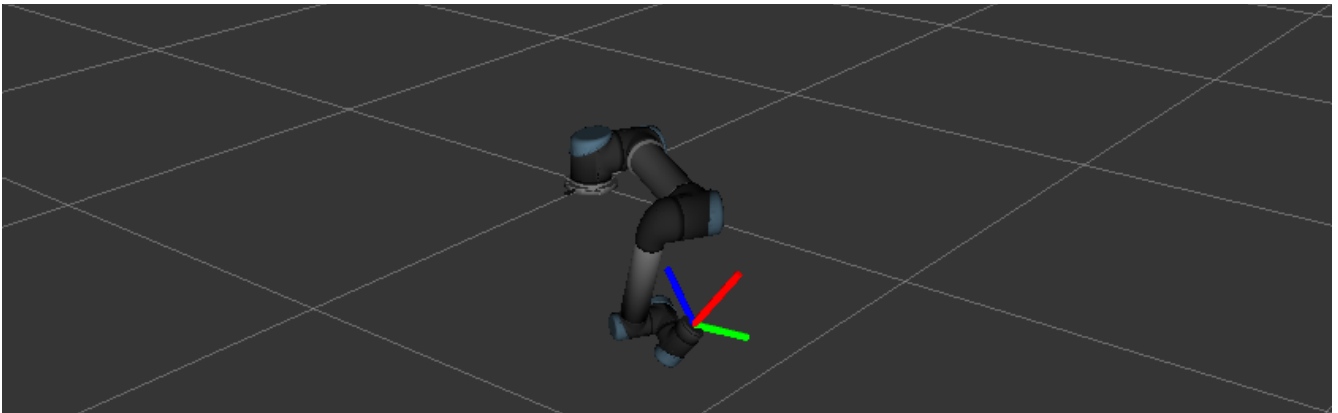


Illustration 8: 51st Angles

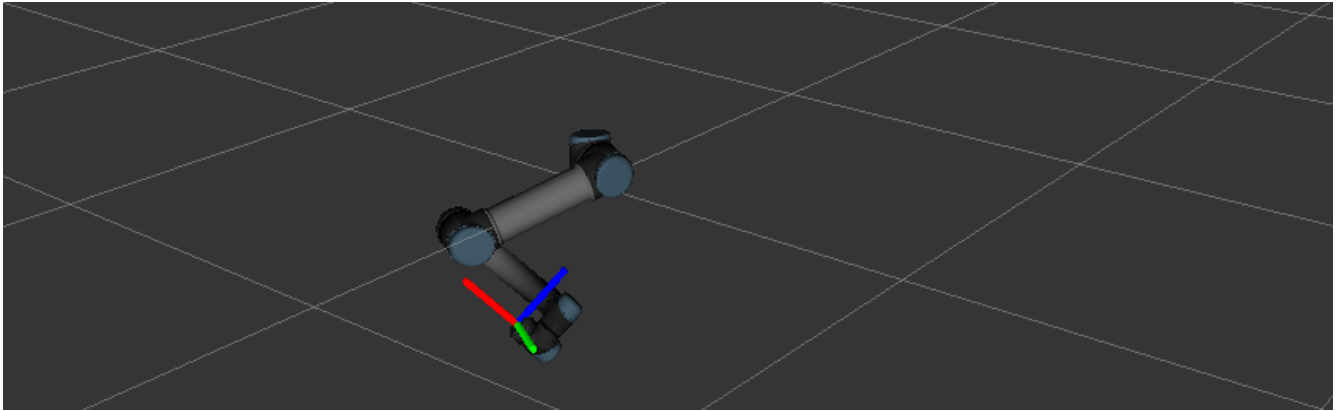


Illustration 9: 76th Angles

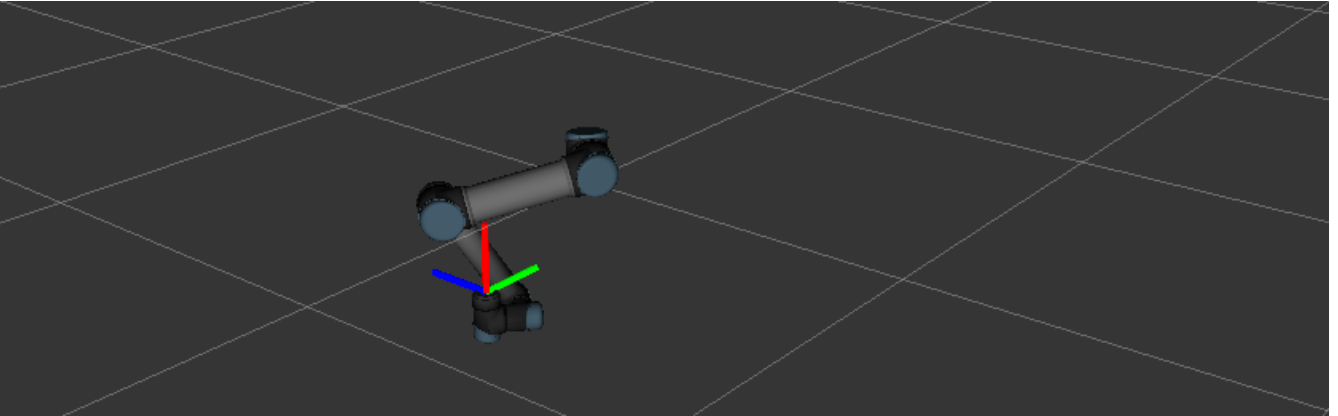


Illustration 10: Last Angles

Animation can be seen at this link:

https://www.youtube.com/watch?v=ycaGRk_0AE8