ME 449 Assignment 5
Due 1:30 PM, Wednesday November 25, 2026

1. Exercise 6.7. Hand draw no more than three iterations which make it clear which root is being converged to.

2. Exercise 6.8.

3. Exercise 8.2.

4. You derive the Lagrangian $\mathcal{L}$ for a two-joint robot. The Lagrangian is the sum of several terms, $\mathcal{L} = \mathcal{L}^1 + \mathcal{L}^2 + \ldots$. One of these terms is $\mathcal{L}^1 = \mathfrak{m}_2 L_1 L_2 \dot{\theta}_2^2 \sin\theta_1$.

   The dynamics of the robot can be expressed in the form

   $$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = \begin{bmatrix}M_{11}(\theta) & M_{12}(\theta)\\ M_{21}(\theta) & M_{22}(\theta)\end{bmatrix}\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix} + \begin{bmatrix}c_1(\theta,\dot{\theta})\\ c_2(\theta,\dot{\theta})\end{bmatrix} + \begin{bmatrix}g_1(\theta)\\ g_2(\theta)\end{bmatrix}.$$

   Use the Euler-Lagrange equations to find the contributions of $\mathcal{L}^1$ to τ_1 and τ_2. For each term in your solution, indicate whether it would contribute to an $M_{11}, M_{12}, M_{21}, M_{22}, c_1, c_2, g_1$, or g_2 term. For each velocity-product term, indicate whether it is a Coriolis or centripetal term.

5. Describe the Newton-Euler inverse dynamics for a robot arm in intuitive plain English, assuming the reader understands the Newton-Euler dynamics of a rotating rigid body. There are three equations in the forward iterations and two equations in the backward iteration. For each term that appears on the right-hand side of each of the five equations, provide approximately one plain English sentence justifying why that term appears or makes sense.

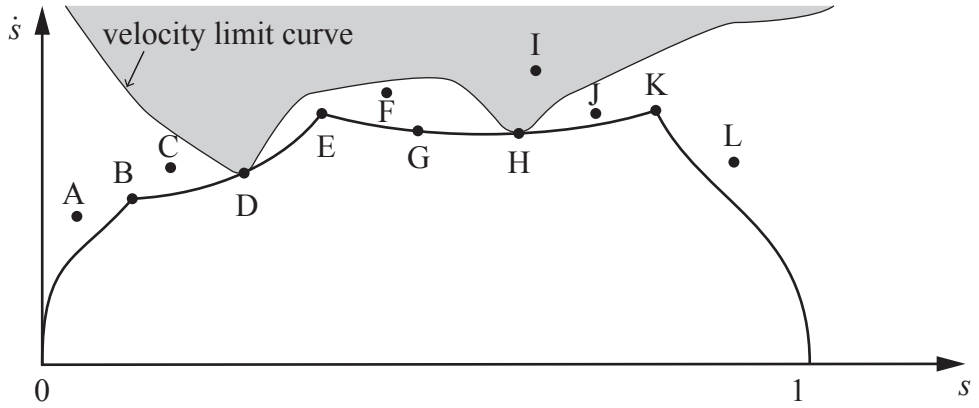
6. Exercise 9.2.

7. Exercise 9.3.

8. Exercise 9.22.

9. Exercise 11.6.

10. (This is a problem from quiz 2 in 2025.) The figure below shows a time-optimal time scaling, a velocity limit curve, and example $(s, \dot{s})$ states labeled as points A–L.



(a) Which labeled states on the time-optimal time scaling correspond to a switch from maximum deceleration to maximum acceleration? (List all of them.)

(b) Which labeled states certainly cannot be reached from the initial state $(s, \dot{s}) = (0, 0)$? (List all of them.) Explain.

(c) From which labeled states will the robot certainly be unable to stay on the path $\theta(s), s \in [0, 1]$, all the way to the end state $(s, \dot{s}) = (1, 0)$? (List all of them.) Explain.

(d) This robot has two actuators. If we pick a random state on the time-optimal trajectory, how many actuators are likely to be saturated (operating at full capacity) at that state, 0, 1, or 2? Explain.

(e) At a state $(s, \dot{s})$, $L_1(s, \dot{s}) = -1, L_2(s, \dot{s}) = 3, U_1(s, \dot{s}) = 1$, and $U_2(s, \dot{s}) = 4$. Which of the labeled points could be at this $(s, \dot{s})$? (List all of them.) Explain.

(f) Draw the motion cones (tangent vector cones) at each of the labeled states where it is possible to know the angles of the cone vectors. (Don't worry about the lengths of the cone vectors; focus on the angles.) Don't draw anything at other points.