Figure 1: A table lamp that moves only in the plane of the page.

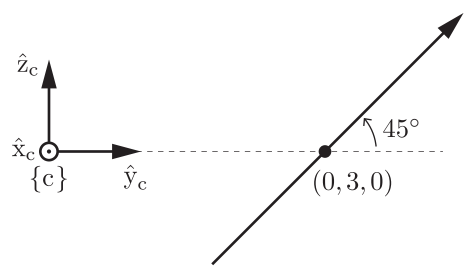

Figure 2: A screw axis in the $(\hat{y}_c, \hat{z}_c)$ plane.

Submitted by: Your name here

Practice exercise 1 Figure 1 shows a table lamp that moves only in the plane of the page. Use Grübler's formula to calculate the number of degrees of freedom.

Practice exercise 2 Figure 2 shows a screw axis in the $(\hat{y}_c, \hat{z}_c)$ plane, at a 45° angle with respect to the $\hat{y}_c$-axis. (The $\hat{x}_c$-axis points out of the page.) The screw axis passes through the point $(0, 3, 0)$.

(a) If the pitch of the screw is $h = 10$ linear units per radian, what is the screw axis $\mathcal{S}_c$? Make sure you can also write this in its $se(3)$ form $[\mathcal{S}_c]$,

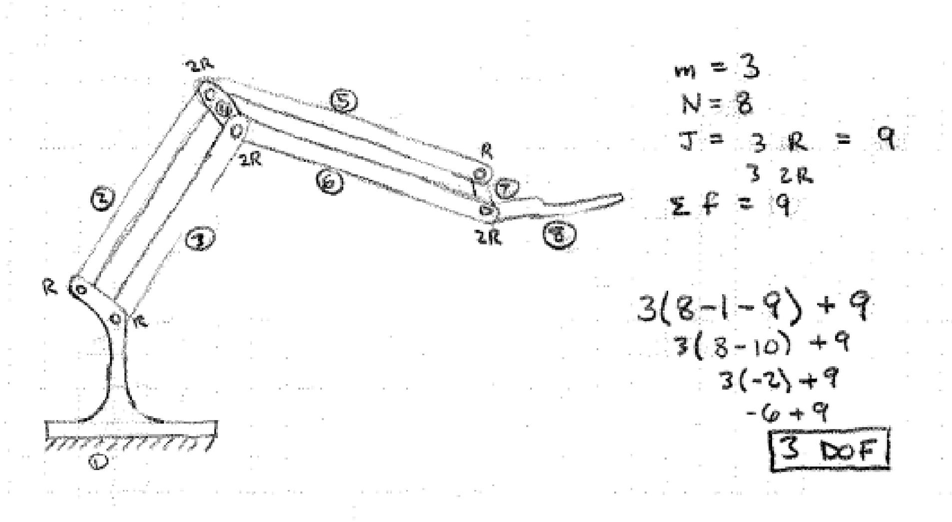

Figure 3: Written solution to lamp problem.

too.

(b) Using your answer to (a), if the speed of rotation about the screw axis is $\dot{\theta} = \sqrt{2}$ rad/s, what is the twist $\mathcal{V}_c$?

(c) Using your answer to (a), if a frame initially at {c} rotates by $\theta = \pi/2$ about the screw axis, yielding a new frame {c′}, what are the exponential coordinates describing the configuration of {c′} relative to {c}?

(d) What is $T_{cc'}$, corresponding to the motion in part (c)?

(e) Now imagine that the axis in Figure 2 represents a wrench: a linear force along the axis and a moment about the axis (according to the right-hand rule). The linear force in the direction of the axis is 20 and the moment about the axis is 10. What is the wrench $\mathcal{F}_c$?

Solution 1 Despite all the links and revolute joints, this mechanical system behaves similarly to a 3R robot arm, since each set of two revolute joints acts as a single hinge.

Solution 2

(a) Since the screw axis $\mathcal{S}_c = (\mathcal{S}_{c_\omega}, \mathcal{S}_{c_v})$ has a rotational component, $\mathcal{S}_{c_\omega}$ is a unit vector aligned with the axis, i.e., $\mathcal{S}_{c_\omega} = \hat{s} = (0, \cos 45°, \sin 45°) =$

$(0, 1/\sqrt{2}, 1/\sqrt{2})$. The linear component is $\mathcal{S}_{c_v} = h\hat{s} - \hat{s} \times q$ (a linear component due to linear motion along the screw plus a linear component due to rotation about the screw), where $q = (0, 3, 0)$ and $h = 10$, i.e., $\mathcal{S}_{c_v} = (0, 10/\sqrt{2}, 10/\sqrt{2}) + (3/\sqrt{2}, 0, 0) = (3, 10, 10)/\sqrt{2}$.

(b) $\mathcal{V}_c = \mathcal{S}_c\dot{\theta} = (0, 1, 1, 3, 10, 10)$.

(c) $\mathcal{S}_c\theta = (0, 1, 1, 3, 10, 10)\pi/(2\sqrt{2})$.

(d) You can use the MR code library to do the calculation. Use `VecTose3` to convert the exponential coordinates $\mathcal{S}_c\theta$ to their $se(3)$ representation $[\mathcal{S}_c\theta]$ and then use `MatrixExp6` to calculate

$$T_{cc'} = e^{[\mathcal{S}_c\theta]} = \begin{bmatrix} 0 & -0.71 & 0.71 & 2.12 \\ 0.71 & 0.5 & 0.5 & 12.61 \\ -0.71 & 0.5 & 0.5 & 9.61 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

(e) The wrench is written $\mathcal{F}_c = (m_c, f_c)$. The linear component f_c has a magnitude of 20 and is aligned with the axis shown, so $f_c = (0, 10\sqrt{2}, 10\sqrt{2})$. If the axis passed through the origin of {c}, the moment (which has magnitude 10) would be $(0, 5\sqrt{2}, 5\sqrt{2})$, but since it is displaced from the origin of {c}, there is an extra moment component due to the linear component, $q \times f_c = (0, 3, 0) \times (0, 10\sqrt{2}, 10\sqrt{2}) = (30\sqrt{2}, 0, 0)$, so the total moment is $m_c = (0, 5\sqrt{2}, 5\sqrt{2}) + (30\sqrt{2}, 0, 0) = \sqrt{2}(30, 5, 5)$.
You can verify that you get the same answer using $\mathcal{F}_c = [\mathrm{Ad}_{T_{ac}}]^{\mathrm{T}}\mathcal{F}_a$, where {a} is a frame aligned with {c} and with an origin at $(0, 3, 0)$.