

Engineering Analysis 3

Spring 2026

HW #6

Assigned May 7, 2026. Due May 14, 2026, at 5:00pm by submitting on Canvas.

#1 (15 pts)

1-1. (True/False; 8pts; 1 pt each) Which of the following functions, $y(t)$, are solutions to the differential equation $y'' - y' - 6y = 6t + 1$. For each function below, mark “T” if the function is a solution and “F” if the function is not a solution.

- a) $y(t) = -t$
- b) $y(t) = 6t + 1$
- c) $y(t) = e^{6t}$
- d) $y(t) = e^{-2t}$
- e) $y(t) = -t + e^{3t}$
- f) $y(t) = 6t + 1 + e^{-2t}$
- g) $y(t) = t + \frac{1}{6} + e^{6t}$
- h) $y(t) = -t + e^{3t} + e^{-2t}$

1-2. (4 pts) A system is described by a differential equation $\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -10 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ which can be written as $X' = AX$, where X is the state vector and A is the coefficient matrix. Two different solutions to the differential equation are $X = Y_1(t)$ and $X = Y_2(t)$ where

$$Y_1(t) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} e^{2t} \cos(4t) \\ e^{2t} (\cos(4t) - B \sin(4t)) \end{bmatrix}$$

$$Y_2(t) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} e^{2t} \sin(4t) \\ e^{2t} (\sin(4t) - C \cos(4t)) \end{bmatrix}$$

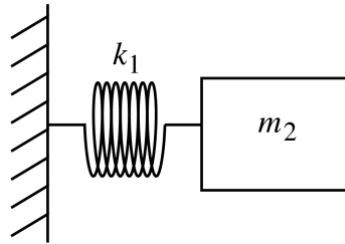
Find B and C such that both $Y_1(t)$ and $Y_2(t)$ are solutions to the differential equation.

1-3. (3 pts) If the initial conditions are $X(0) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ for the differential equation in Part 1-2, find a and b where the particular solution meeting the initial conditions is $X(t) = aY_1(t) + bY_2(t)$.

#2 (20 pts)

This problem uses MATLAB. Turn in your program and plots, as well as answers to the following parts. Please print your MATLAB code as a pdf to include in your submission.

The equation of motion for the spring-mass system below is derived in the webtext and solved both analytically and numerically. In this problem you will compare the analytic solution with that from first-order Euler integration and consider the impact of the initial conditions.



Let $k_1 = 1000$ N/m and $m_2 = 250$ kg. Let the initial conditions be $x_1(0) = 0.3$ m and $v_2(0) = 0$ $\frac{\text{m}}{\text{s}}$.

2-1. (4 pts) Write the state equations for this system and derive the analytic solution.

2-2. (10 pts) Simulate this system in MATLAB using the analytic solution and Euler's method with a timestep $dt = 0.1$ s for a total of 10 s. Plot the displacement of the spring x_1 and velocity of the mass v_2 over time.

2-3. Consider a different set of initial conditions. Let $x_1(0) = 0.3$ m and $v_2(0) = 0.8$ $\frac{\text{m}}{\text{s}}$.

- (4 pts)** Plot the original and new displacement of the spring x_1 and velocity of the mass v_2 over time in the same plot (use either Euler's method or analytic solution).
- (1 pt)** Is the period of oscillation the same for both sets of initial conditions? Why?
- (1 pt)** Is the amplitude of the oscillation the same for both sets of initial conditions? Why?

#3 (18 pts)

3-1. (6 pts; 1.5 pts each) Indicate whether the following differential equations are linear, homogeneous, and constant coefficients by checking the appropriate boxes.

	Linear	Homogeneous	Constant Coefficients
a) $x'' + \sin(t)x' + e^t x = 3$	[]	[]	[]
b) $x'' + 2x' + x^2 = 0$	[]	[]	[]
c) $x'' + e^t x = 0$	[]	[]	[]
d) $x'' + 3x = 3$	[]	[]	[]

3-2. (4 pts) Pick a nonlinear constant-coefficient homogeneous differential equation above and prove that you cannot superpose solutions. Hint: Use proof by contradiction.

3-3. (4 pts) Pick a linear constant-coefficient nonhomogeneous differential equation above and prove that you cannot superpose solutions. Hint: Use proof by contradiction.

3-4. (4 pts) Pick a linear homogeneous equation above and prove that you can superpose solutions.

#4 (10 pts)

Consider a system described by the two coupled state equations:

$$x' = -6x + 25q$$

$$q' = -x$$

4-1. (5 pts) Express the two coupled first-order state equations using a single second-order equation using x , x' , and x'' .

4-2. (5 pts) Find the real-valued general solution for this system as a function of time.

#5 (28 pts)

5-1. (8 pts, 2 pts each) For the differential equation $ax' + bx = 0$, the *characteristic equation* is $ar + b = 0$, or $r + (b/a) = 0$. This second version is preferred. It uses a *monic polynomial* in r , where the leading coefficient is 1.

- a) What is the root r of the characteristic equation?
- b) What is the general solution $x(t)$?
- c) If $a = 2$, $b = 3$, and the initial condition is $x(0) = 2$, what is the particular solution $x(t)$?
- d) The solution of this differential equation is an exponential, and its *time constant* is $\tau = |1/r|$, with units of seconds. If r is positive, the solution $x(t) = Ae^{t/\tau}$ grows exponentially, and τ is a time constant of growth, indicating the solution increases by a factor of $e \approx 2.718$ every τ seconds. If r is negative, the solution $x(t) = Ae^{-t/\tau}$ decays exponentially, and τ is a time constant of decay, indicating the solution decreases by a factor of $(1/e) \approx 0.37$ every τ seconds. Is the answer to part c) a decaying exponential or a growing exponential, and what is the time constant τ ?

5-2. (20 pts, 2 pts each) You are given a system modeled by the homogeneous second-order differential equation $a_2x'' + a_1x' + a_0x = 0$.

- a) What is the monic polynomial version of the characteristic equation?
- b) What are the roots r_1, r_2 of the characteristic equation?
- c) The roots are complex in general. Assume $r_1 = c_1 + d_1j$ and $r_2 = c_2 + d_2j$. What conditions must (c_1, d_1, c_2, d_2) satisfy for any characteristic equation with real coefficients? (For example, a characteristic equation cannot have roots $r_1 = 1 + 2j$ and $r_2 = 3 + 4j$.)
- d) What conditions on (c_1, d_1, c_2, d_2) ensure that any particular solution $x(t)$, regardless of the initial conditions, decays over time, with or without oscillation? In other words, $x(t) \rightarrow 0$ as $t \rightarrow \infty$. We sometimes call such a system *stable*.
- e) What conditions on (c_1, d_1, c_2, d_2) ensure that the system will oscillate forever, without growing or shrinking, for any nonzero initial conditions? In terms of (c_1, d_1, c_2, d_2) , what is the general solution?
- f) What conditions on (c_1, d_1, c_2, d_2) ensure that the system is *overdamped*? In terms of (c_1, d_1, c_2, d_2) , what is the general solution?
- g) What conditions on (c_1, d_1, c_2, d_2) ensure that the system is *critically damped*? In terms of (c_1, d_1, c_2, d_2) , what is the general solution?
- h) What conditions on (c_1, d_1, c_2, d_2) ensure that the system is *underdamped*? In terms of (c_1, d_1, c_2, d_2) , what is the general solution?
- i) Assume that $c_1 = c_2 = -2$, $d_1 = 4$, $d_2 = -4$, and the initial conditions are $x(0) = -2$, $x'(0) = 1$. Is the system overdamped, underdamped, critically damped, or none of the above? What is the particular solution $x(t)$? If the solution oscillates, what is the frequency of oscillation, both in rad/s and in Hz? What is the time constant of decay (or growth)?
- j) The characteristic equation of an underdamped second-order system is sometimes written in the “standard” form $r^2 + 2\zeta\omega_n r + \omega_n^2 = 0$, where ζ is called the *damping ratio*, between 0 and 1, and ω_n is called the *natural frequency*. The actual frequency of oscillation is the *damped natural frequency*, $\omega_d = \omega_n \sqrt{1 - \zeta^2}$, a somewhat lower frequency than the natural frequency if $\zeta > 0$. If the roots of the characteristic equation are given by $c \pm dj$, express the damping ratio, natural frequency, damped natural frequency, and decay time constant in terms of c and d .