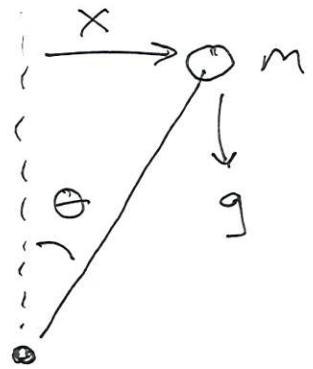
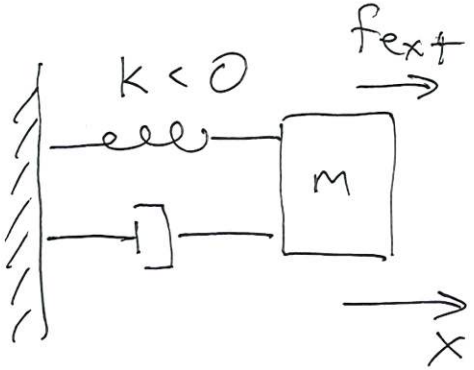
Control of an inverted pendulum



damped pivot

$\sin\theta \approx \theta$

→ by small angle approx, this is a linear equivalent →



$$m\ddot{x} = -kx - b\dot{x}$$

$$m\ddot{x} + b\dot{x} + kx = 0$$

$$4\ddot{x} + 8\dot{x} - 12x = 0$$

$$\ddot{x} + 2\dot{x} - 3x = 0$$

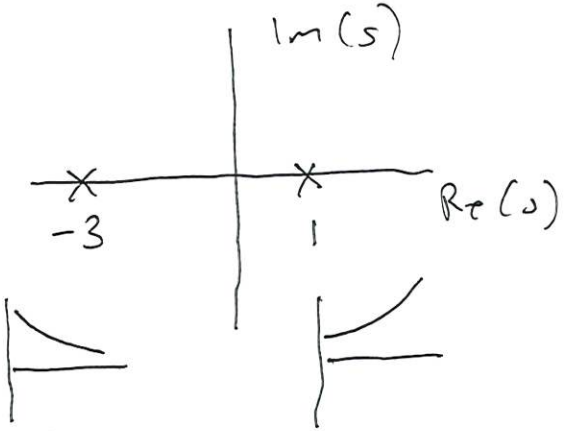
$$s^2 + 2s - 3 = 0 \quad \text{char eq}$$

$$(s+3)(s-1) = 0$$

$m = 4$

$b = 8$

$k = -12$



<u>unstable</u> solution grows with time

use a motor to apply a force, try a virtual spring

$f_{ext} = -K_p x$

(P) <u>proportional controller</u>

K_p = gain

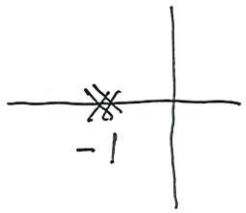
$$4\ddot{x} + 8\dot{x} + (K_p - 12)x = 0$$

$$s^2 + 2s + \left(\frac{K_p}{4} - 3\right) = 0 \quad \text{char eq}$$

goal: critical damping

$$s^2 + 2s + \left(\frac{K_p}{4} - 3\right) = 0 \quad = (s+a)(s+a) = (s+a)^2 = s^2 + 2as + a^2$$

$a = 1$, $\frac{K_p}{4} - 3 = 1 \rightarrow K_p = 16$

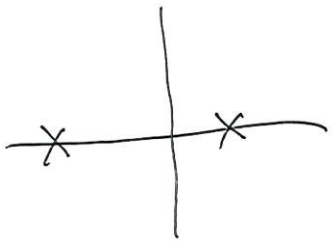


$$s^2 + 2s + \left(\frac{K_p}{4} - 3\right) = s^2 + 2\zeta\omega_n s + \omega_n^2$$

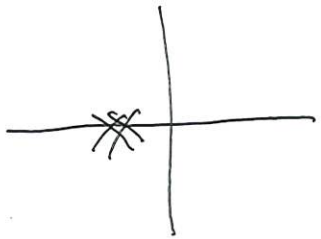
$\zeta = 1$ for critical damping

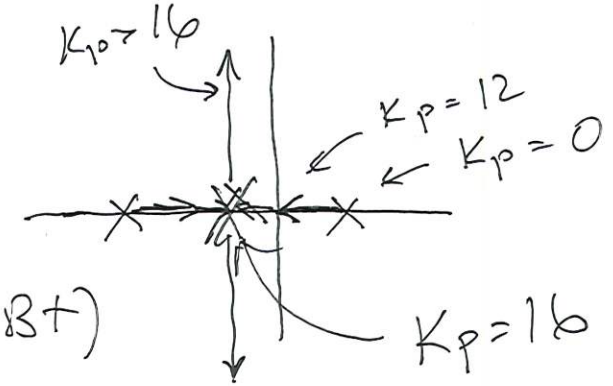
so $\omega_n = 1$, $K_p = 16$

Before



After





for $K_p = 16$, $x(t) = e^{-t}(A + Bt)$

time constant $\tau = \left|\frac{1}{c}\right| = 1\,\text{sec}$

To improve performance: $f_{ext} = -K_p x - \underbrace{K_D \dot{x}}_{\text{derivative control}}$ → virtual damper

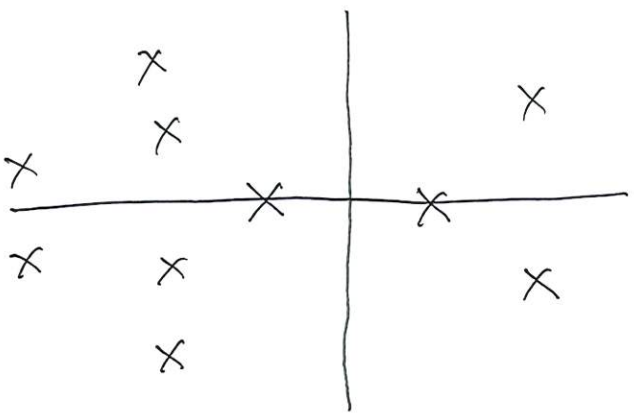
PD control

2 gains to play with, 2 roots, put roots anywhere

n^{th}-order char eq

$$s^n + a_{n-1}s^{n-1} + \dots + a_1 s + a_0 = 0$$

n roots



Each comp conj pair: $e^{ct}(A\cos dt + B\sin dt)$

real root: Ae^{ct} 2: $e^{ct}(A+Bt)$

repeated real root: Bte^{ct}

For stable underdamped 2^{nd}-order: $c \pm dj$

$$c = -\zeta\omega_n$$

$$d = \omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$\omega_n = \sqrt{c^2+d^2}$$

Your feedback needed! What works, what could be better?

Class format: Handwritten notes posted later as pdfs, lecture videos, occasional concept questions for discussion, shorter yell-out-your-answer questions

Web textbook: Are you reading/consulting it?

Homework: Helping you learn?

Discussion session: If you go, do you find it useful? Feedback for TAs?

Office hours: Helpful? Too crowded?

Other?