Robotics Manipulation_homework3
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1. Write the following functions for your robotics library. These functions build on the functions written for the last assignment.

FixedJacobian:

```
function [ Js ] = FixedJacobian( S,theta )
joints = size(S,2);
Js=[];
for i=1:joints
bracket_Si=VecTose3(S(:,i));
k=i-1;
if i==1
Ji=se3ToVec(bracket_Si);
else
Ji_=bracket_Si;
for j=1:k
Sj=S(:,k-j+1);
thetaj=theta(k-j+1);
Vj = Sj*thetaj;
Ji_=MatrixExp6(Vj)*Ji_*TransInv(MatrixExp6(Vj));
end
Ji=se3ToVec(Ji_);
end
Js=[Js,Ji];
end
end
%Takes a set of joint angles theta and screw axes Si for the robot joints
%expressed in the fixed space frame, and returns the space Jacobian Js(theta)
```

BodyJacobian:

```
function [ Jb ] = BodyJacobian( B,theta )
joints = size(B,2);
Jb=[];
for i=1:joints
    bracket_Bi=VecTose3(B(:,joints-i+1));
    k=i-1;
    if i==1
        Ji=se3ToVec(bracket_Bi);
    else
        Ji_=bracket_Bi;
        for j=joints-i+2:joints
            Bj=B(:,j);
            thetaj=theta(j);
            Vj = Bj*thetaj;
            Ji_=TransInv(MatrixExp6(Vj))*Ji_*MatrixExp6(Vj);
        end
         Ji=se3ToVec(Ji_);
    end
    Jb=[Ji,Jb];
end
end
%Takes a set of joint angles theta and screw axes Bi for the robot joints
%expressed in the end-effector body frame, and returns the body Jacobian
%Jb(theta)
```

IKinBody:

```
function [ matrix_jointangles ] = IKinBody( B, M, Tsd, theta0, ew, ev )
thetalist=[theta0];
i = 0;
Vb = MatrixLog6(TransInv(FKinBody(M, B, theta0))*Tsd);
wb = Vb(1:3);
vb = Vb(4:6);
while (norm(wb) > ew||norm(vb) > ev)&&(i < 100)
    if i == 0
        Jb = BodyJacobian(B,theta0);
    else
        Jb = BodyJacobian(B,thetaiplus1);
    end
    m = size(Jb,1);
    n = size(Jb,2);
    if n > m
        Jb_ = transpose(Jb)*pinv(Jb*transpose(Jb));
    else
        Jb_= pinv(transpose(Jb)*Jb)*transpose(Jb);
    end
    if i == 0
        thetaiplus1 = theta0+transpose(Jb_*Vb);
    else
        thetaiplus1 = thetaiplus1+transpose(Jb_*Vb);
    end
    thetalist = [thetalist;thetaiplus1];
    i = i + 1;
    Vb = MatrixLog6(TransInv(FKinBody(M, B, thetaiplus1))*Tsd);
    wb = Vb(1:3);
    vb = Vb(4:6);
end
thetalist
csvwrite('bodyq.csv',thetalist);
end

%Takes a set of screw axes Bi for the robot joints expressed in the
%end-effector body frame, the end-effector zero configuration M, the
%desired end-effector configuration Tsd, an initial guess theta0 that is
%"close" to satisfying T(theta0) = Tsd, and small scalar values ε_w > 0 and
%ε_v > 0 controlling how close the final solution thetak must be to the
%desired answer.There is a maximum number of iterations 'maxiterates'
%before stopping. Returns a matrix of joint angles.
```

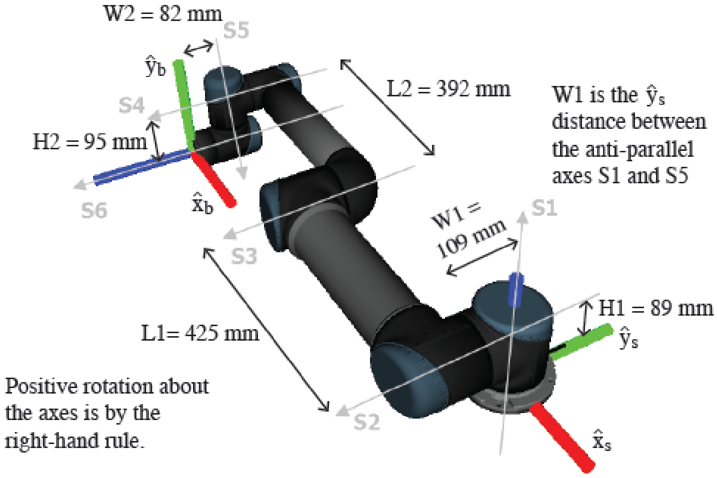
IKinFixed:

```
function [ matrix_jointangles ] = IKinFixed( S, M, Tsd, theta0, ew, ev )
thetalist=[theta0];
i = 0;
Tsb = FKinFixed(M, S, theta0);
Vb = MatrixLog6(TransInv(Tsb)*Tsd);
Vs = Adjoint(Tsb)*Vb;
ws = Vs(1:3);
vs = Vs(4:6);
while (norm(ws) > ew||norm(vs) > ev)&&(i < 100)
    if i == 0
        Js = FixedJacobian(S,theta0);
    else
        Js = FixedJacobian(S,thetaiplus1);

    end
    m = size(Js,1);
    n = size(Js,2);
    Js_ = pinv(Js);
    if i == 0
        thetaiplus1 = theta0+transpose(Js_*Vs);
    else
        thetaiplus1 = thetaiplus1+transpose(Js_*Vs);
    end
    thetalist = [thetalist;thetaiplus1];
    i = i + 1;
    Tsb = FKinFixed(M, S, thetaiplus1);
    Vb = MatrixLog6(TransInv(Tsb)*Tsd);
    Vs = Adjoint(Tsb)*Vb;
    ws = Vs(1:3);
    vs = Vs(4:6);
end
thetalist
csvwrite('fixedq.csv',thetalist);
end

%Takes a set of screw axes Si for the robot joints expressed in the
%space frame, the end-effector zero configuration M, the
%desired end-effector configuration Tsd, an initial guess theta0 that is
%"close" to satisfying T(theta0) = Tsd, and small scalar values ε_w > 0 and
%ε_v > 0 controlling how close the final solution thetak must be to the
%desired answer.There is a maximum number of iterations 'maxiterates'
%before stopping. Returns a matrix of joint angles.
```

2. For the Universal Robots UR5 six-joint robot arm shown in its zero configuration in Figure 1, write the end-effector configuration $M \in SE(3)$ and the six screw axes as (a) $\{\mathcal{B}_i\}$ in the end-effector frame and (b) $\{\mathcal{S}_i\}$ in the fixed space frame.

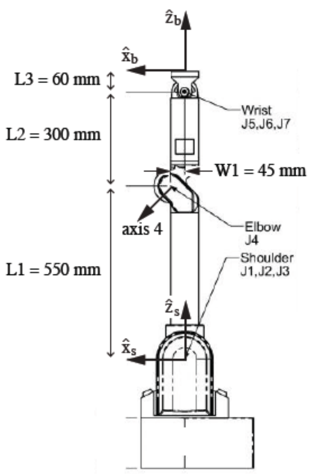


$$M = \begin{bmatrix} 1 & 0 & 0 & -(L1+L2) \\ 0 & 0 & -1 & -(W1+W2) \\ 0 & 1 & 0 & H1-H2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -817 \\ 0 & 0 & -1 & -191 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$S_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 89 \\ 0 \\ 0 \end{bmatrix}, \quad S_3 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 89 \\ 0 \\ 425 \end{bmatrix}, \quad S_4 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 89 \\ 0 \\ 817 \end{bmatrix}, \quad S_5 = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 109 \\ -817 \\ 0 \end{bmatrix}, \quad S_6 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ -6 \\ 0 \\ 817 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 191 \\ 0 \\ 817 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 95 \\ -817 \\ 0 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 95 \\ -392 \\ 0 \end{bmatrix}, \quad B_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 95 \\ 0 \\ 0 \end{bmatrix}, \quad B_5 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ -82 \\ 0 \\ 0 \end{bmatrix}, \quad B_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

3. For the redundant Barrett Technology seven-joint WAM robot arm shown in its home configuration in Figure 2, write the end-effector configuration $M \in SE(3)$ and seven screw axes as (a) $\{\mathcal{B}_i\}$ in the end-effector frame and (b) $\{\mathcal{S}_i\}$ in the fixed space frame.



$\hat{y}_s$ and $\hat{y}_b$ axes are aligned and out of the page. Axes 1, 2, and 3 intersect at the origin of {s}. Axes 5, 6, and 7 intersect at a common point 60 mm from the origin of {b}. Axes 1, 3, 5, and 7 are aligned with $\hat{z}_s$, and axes 2, 4, and 6 are out of the page at the zero configuration. Positive rotation about the axes is by the right-hand rule.

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 910 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$S_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad S_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad S_4 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -550 \\ 0 \\ 45 \end{bmatrix}, \quad S_5 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad S_6 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -850 \\ 0 \\ 0 \end{bmatrix}, \quad S_7 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 910 \\ 0 \\ 0 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad B_4 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 360 \\ 0 \\ 45 \end{bmatrix}, \quad B_5 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad B_6 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 60 \\ 0 \\ 0 \end{bmatrix}, \quad B_7 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

4. Use your function `IKinBody` to find the joint variables θ_d of the UR5 satisfying

$$T(\theta_d) = T_{sd} = \begin{bmatrix} 0 & 1 & 0 & -0.6 \\ 0 & 0 & -1 & 0.1 \\ -1 & 0 & 0 & 0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Distances are in meters. Use $\epsilon_\omega = 0.01$ (i.e., 0.57 degrees) and $\epsilon_v = 0.001$ (i.e., 1 mm). Use the zero configuration $\theta_0 = 0$ as your initial guess. If the configuration is outside the workspace, or if you find that

[matrix_jointangles] = IKinBody([0,0,0,0,0,0;1,0,0,0,-1,0;0,1,1,1,0,1;0.191,0.095,0.095,0.095,-0.082,0;0,-0.817,-0.392,0,0,0; 0.817,0,0,0,0,0], [1,0,0,-0.817;0,0,-1,-0.191;0,1,0,-0.006;0,0,0,1], [0,1,0,-0.6;0,0,-1,0.1;-1,0,0,0.1;0,0,0,1], [0,0,0,0,0,0], 0.01, 0.001)

theta_d :

```
        0         0         0         0         0         0
  -0.3562   -0.5351    0.4681    1.3934   -0.3562   -2.8971
  -0.3987   -1.0051    1.6764   -0.4338   -0.1000   -1.8008
  -0.5160   -1.0624    1.7307   -1.6301   -0.5019   -0.6074
  -0.4932   -0.9232    1.5078   -0.7303   -0.2893   -1.4203
  -0.4722   -0.8183    1.3652   -0.4548   -0.4672   -1.6624
  -0.4692   -0.8345    1.3953   -0.5611   -0.4673   -1.5706
```

5. Use your function `IKinFixed` to find the joint variables θ_d of the WAM satisfying

$$T(\theta_d) = T_{sd} = \begin{bmatrix} 1 & 0 & 0 & 0.4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.4 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Distances are in meters. Use $\epsilon_\omega = 0.01$ (i.e., 0.57 degrees) and $\epsilon_v = 0.001$ (i.e., 1 mm). Use the zero configuration $\theta_0 = 0$ as your initial guess. If the configuration is outside the workspace, or if you find that the zero configuration is too far from a final answer to converge, you may demonstrate `IKinFixed` using another T_{sd}.

I chose the initial angles as [0.01, 0.01, 0.01, 0.01, 0.01, 0.01, 0.01],
[matrix_jointangles] = IKinFixed([0,0,0,0,0,0,0;0,1,0,1,0,1,0;1,0,1,0,1,0,1;0,0,0,-0.55,0,-0.85,0;0,0,0,0,0,0,0;0,0,0,0.045,0,0,0], [1,0,0,0;0,1,0,0;0,0,1,0.91;0,0,0,1], [1,0,0,0.4;0,1,0,0;0,0,1,0.4;0,0,0,1], [0.01, 0.01, 0.01, 0.01, 0.01, 0.01, 0.01], 0.01, 0.001)
theta_d:

0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100
5.2469	4.6557	-6.9319	-11.8566	-4.4208	7.2003	6.1063
3.0783	2.4629	-5.4078	-2.0940	-1.3219	7.6042	-1.7010
0.6488	1.7268	-6.6251	-4.5297	0.6080	6.2355	-5.3106
-0.0260	1.9126	-8.8616	-5.4043	0.5097	5.2571	-5.4609
0.9853	1.8039	-8.4446	-4.0572	1.3264	4.8978	-4.7912
0.7494	1.1442	-8.5459	-3.8744	0.8434	4.9328	-4.3514
0.6675	1.2112	-8.5464	-4.1121	0.8488	5.0115	-4.5356
0.6588	1.2015	-8.5460	-4.1121	0.8475	5.0036	-4.5256

To verify it, I took the function :
FKinFixed [1,0,0,0;0,1,0,0;0,0,1,0.91;0,0,0,1], [0,0,0,0,0,0,0;0,1,0,1,0,1,0;1,0,1,0,1,0,1;0,0,0,-0.55,0,-0.85,0;0,0,0,0,0,0,0;0,0,0,0.045,0,0,0],[0.6588,1.2015,-8.5460,-4.1121,0.8475,5.0036,-4.5256]
and I got this :

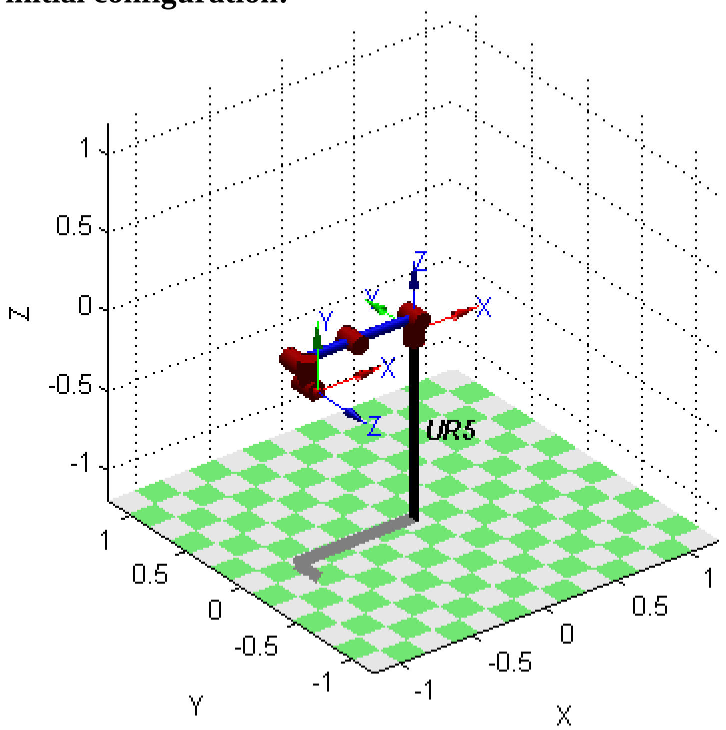
1.0000	0.0000	-0.0000	0.4000
-0.0000	1.0000	0.0000	0.0000
0.0000	-0.0000	1.0000	0.4000
0	0	0	1.0000

which is equals to Tsd.

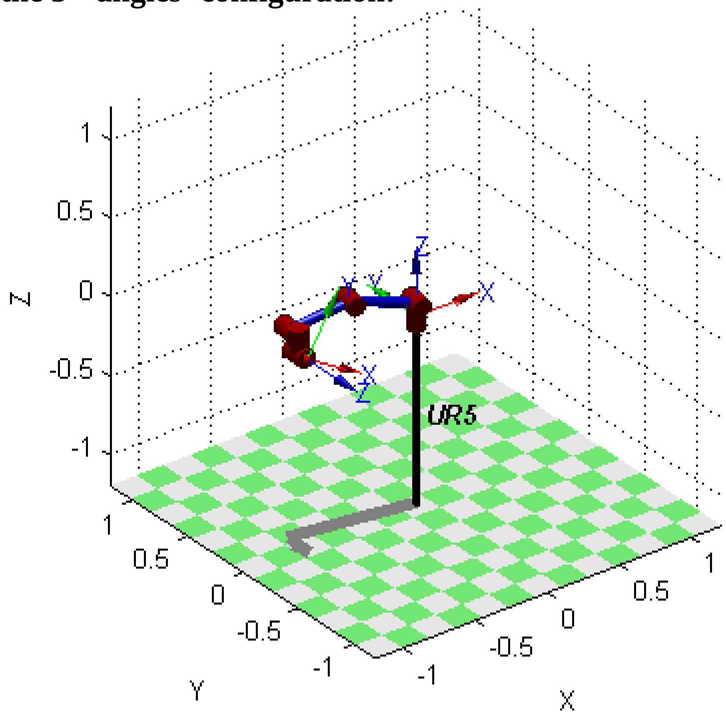
6. Use either the Matlab robotics toolbox or ROS/rviz to animate your solutions to Exercises 4 and 5. The matrices of joint angles returned by `IKinBody` and `IKinFixed` should be displayed as consecutive arm configurations, so you can see a (choppy) "movie" of the converging iterations. For each of the UR5 and WAM examples, turn in snapshots of the robot at its initial configuration, one or two intermediate configurations, and the final configuration.

For ur5:

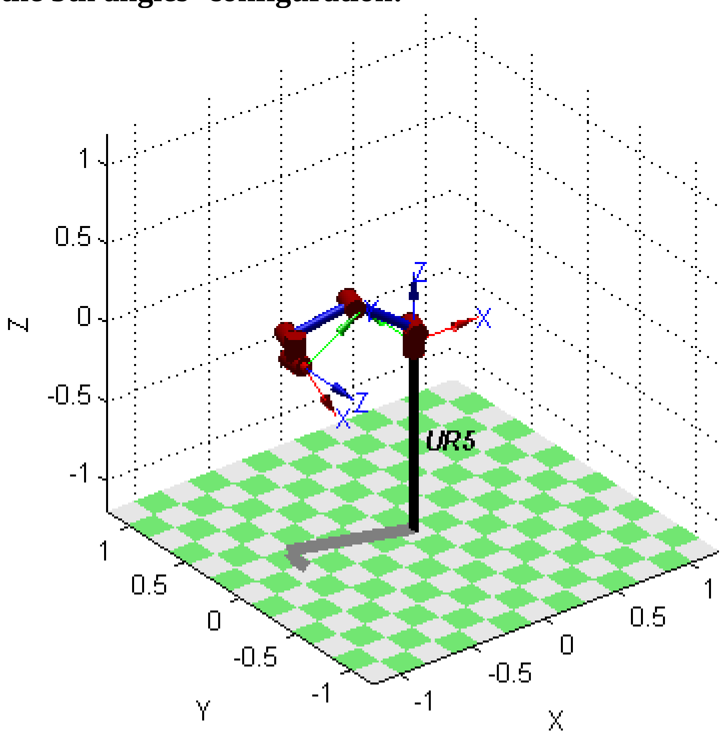
initial configuration:



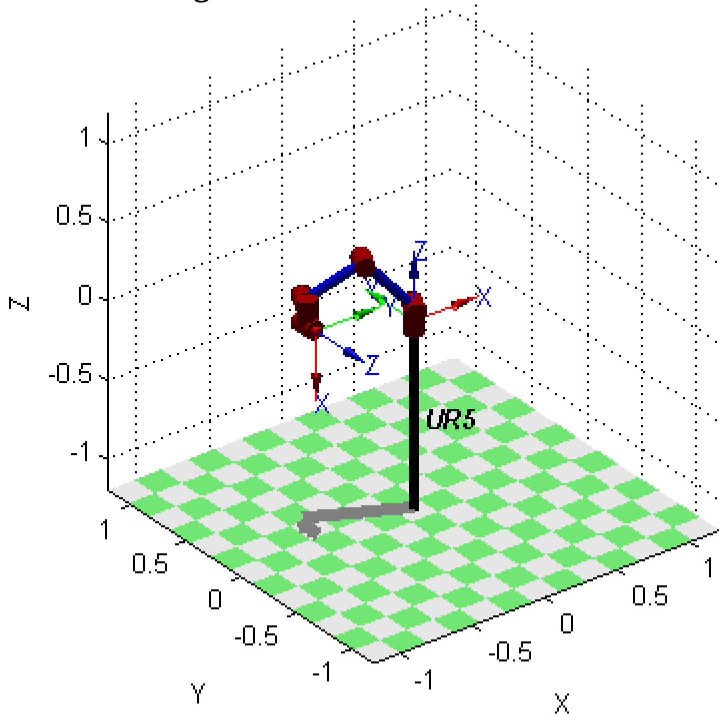
the 3^rd^ angles' configuration:



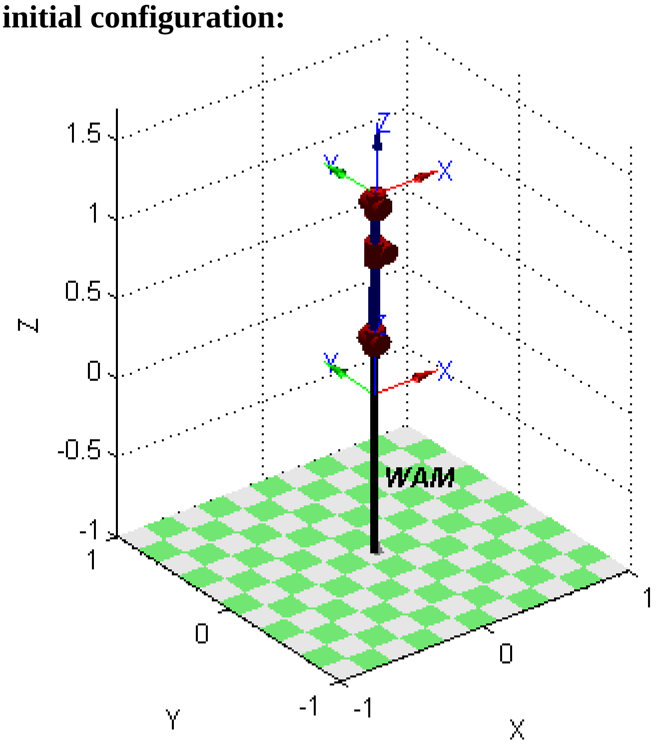
the 5th angles' configuration:



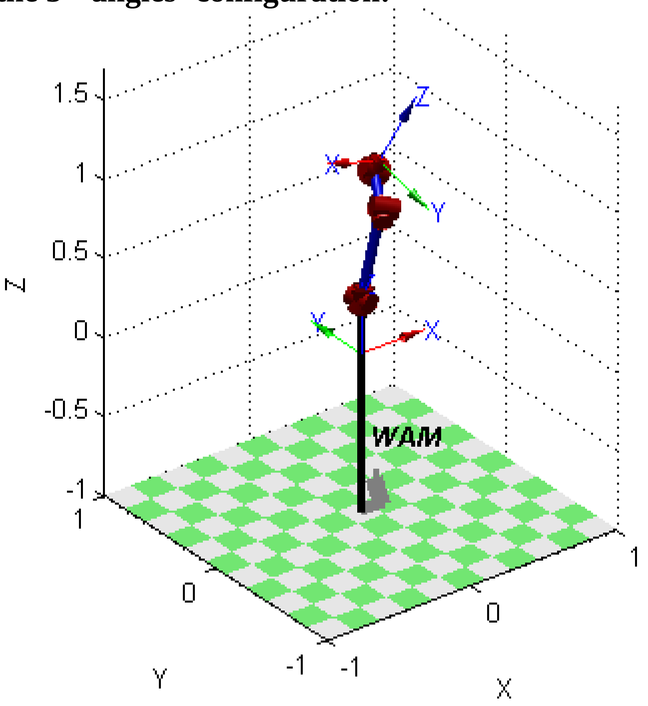
the final configuration:



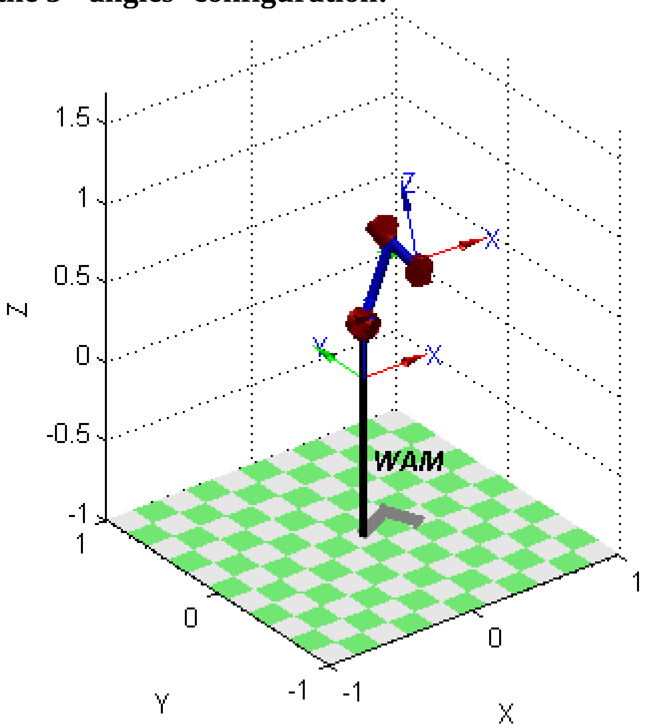
For WAM:
initial configuration:



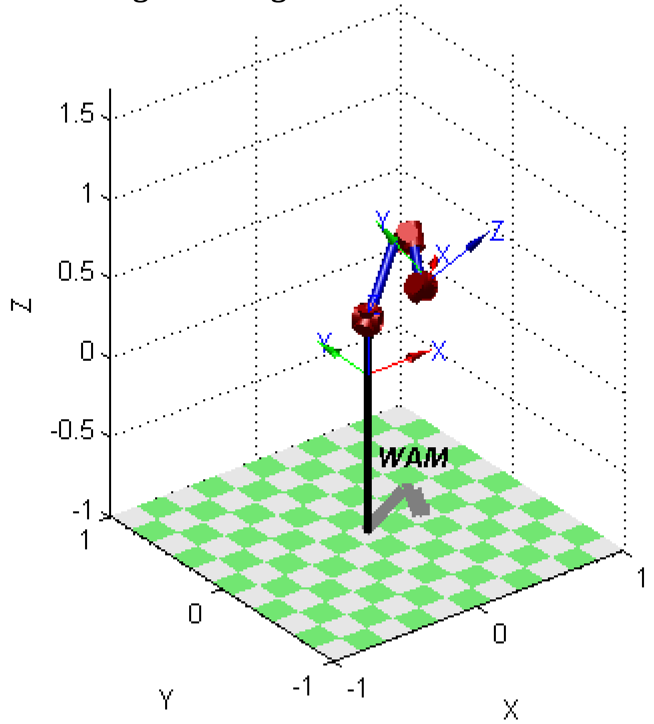
the 3rd angles' configuration:



the 5th angles' configuration:



the 8th angles' configuration:



the final configuration: