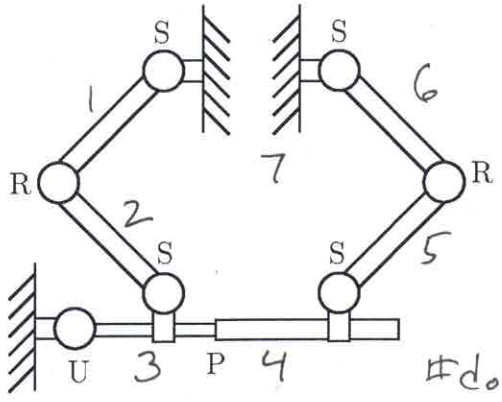
1. (4 pts) Two anthropomorphic SRS robot arms firmly grasp and manipulate a UP mechanism, as shown in the image below. The firm grasps mean that the hands of the robot arms are fixed relative to the links they grasp. How many degrees of freedom does the combined arms-plus-mechanism system have?



$N = 7 \quad J = 8 \quad \sum f_i = 17$

4 spherical joints: $4 \times 3 = 12$

2 revolute joints: $2 \times 1 = 2$

1 universal joint: $1 \times 2 = 2$

1 prismatic joint: $1 \times 1 = 1$

17

$\#dof = 6(N-1-J) + \sum f_i = 6(-2) + 17 = 5$

2. (2 pts) Three spheres are free to move in space, subject only to the constraints that each sphere must always touch the other two. How many degrees of freedom does the three-sphere system have?

sphere 1 moves with 6 dof

sphere 2 has 1 distance constraint from sphere 1

$6 - 1 = 5$ dof

sphere 3 has 2 distance constraints

$6 - 2 = 4$ dof

$\#dof = 6 + 5 + 4 = 15$

3. (4 pts) The configuration of a frame {d} relative to a frame {c} is written T_{cd}. A frame initially coincident with {d} follows a screw axis $\mathcal{S}_d$ (expressed in the {d} frame) a distance θ, then follows a twist $\mathcal{V}_c$ (expressed in the {c} frame) for time t, ending at the frame {e}. What is T_{ce}?

after motion 1: $T_{cd} \exp([S_d]\theta)$

after motion 2: $\exp([\mathcal{V}_c]t)\left[T_{cd} \exp([S_d]\theta)\right]$

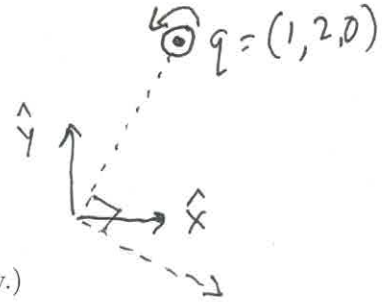
$T_{ce} = \exp([\mathcal{V}_c]t)\, T_{cd} \exp([S_d]\theta)$

4. A frame {d} starts out aligned with a frame {c}, $T_{cd} = I$. It then follows a screw axis — represented in {c} by $q = (1, 2, 0)^\intercal$, $\hat{s} = (0, 0, 1)^\intercal$, and $h = -2$ — at a constant speed for time $t = 10$, rotating a total distance π and ending up at {d′}. **You should not do any significant calculations for this problem. A drawing may be helpful but is not required.**

(a) (2 pts) Write the screw axis in {c}, i.e., $\mathcal{S}_c$. (Numbers only.)

$S_c = (0, 0, 1, 2, -1, -2)^T$ (transpose optional)

↑ due to pitch $h = -2$



(b) (2 pts) Write the twist $\mathcal{V}_c$ while the frame is moving. (Numbers only.)

$\mathcal{V}_c = \frac{\pi}{10} S_c = \left(0, 0, \frac{\pi}{10}, \frac{2\pi}{10}, \frac{-\pi}{10}, \frac{-2\pi}{10}\right)^T$

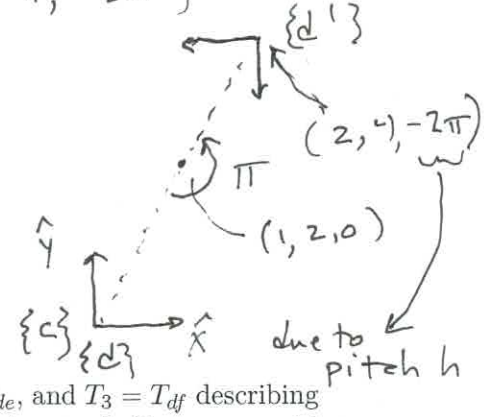
$N_c t = 10 \mathcal{V}_c = S_c \theta = \pi S_c$ $\quad \frac{\pi}{5} \quad \frac{-\pi}{5}$

(c) (2 pts) Give the exponential coordinate representation of $T_{cd'}$. (Numbers only.)

$S_c \pi$ or $\mathcal{V}_c(10) = (0, 0, \pi, 2\pi, -\pi, -2\pi)^T$

(d) (4 pts) Give $T_{cd'} \in SE(3)$. (Numbers only.)

$$T_{cd'} = \begin{bmatrix} -1 & 0 & 0 & 2 \\ 0 & -1 & 0 & 4 \\ 0 & 0 & 1 & -2\pi \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



5. (3 pts) You are given the transformation matrices $T_1 = T_{ec}, T_2 = T_{de}$, and $T_3 = T_{df}$ describing the relative configurations of the frames {c}, {d}, {e}, and {f}. A wrench $\mathcal{F}_c$ is expressed in {c}. What is this wrench expressed in {f}? Write it in terms of $\mathcal{F}_c, T_1, T_2, T_3$, and any operations needed on them.

$\mathcal{F}_f = [Ad_{T_{cf}}]^T \mathcal{F}_c$

$T_{cf} = T_{ce} T_{ed} T_{df} = T_1^{-1} T_2^{-1} T_3$

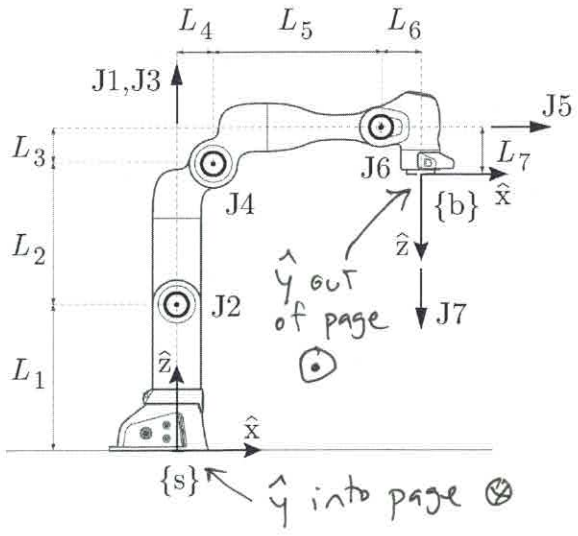
$\mathcal{F}_f = [Ad_{(T_1^{-1} T_2^{-1} T_3)}]^T \mathcal{F}_c$

6. (3 pts) There are three frames, {c}, {d}, and {e}, with configurations T_{cd} and T_{ce}. A frame initially at {e} follows the screw axis that takes {c} to {d}, but follows it three times the distance needed to take {c} to {d}. The frame ends up at {e′}. Give $T_{ce'}$ in terms of T_{cd}, T_{ce}, and any operations needed on them.

$T_{ce'} = \exp(3 \log T_{cd}) T_{ce}$

$\log T_{cd}$ is the motion that takes {c} to {d} in time 1, expressed in {c} frame

7. Consider the 7R Franka Research 3 (FR3) robot arm shown at the configuration below.



(a) (2 pts) Give T_{sb} for this configuration.

$$T_{sb} = \begin{bmatrix} 1 & 0 & 0 & L_4+L_5+L_6 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & L_1+L_2+L_3-L_7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) (7 pts) Give the space Jacobian at this configuration. The axes of rotation of joints 1 and 3 are coincident and point up on the page, joint axis 5 points right on the page, joint axis 7 points down on the page, and joint axes 2, 4, and 6 point out of the page at the locations indicated by the out-of-page arrowheads $\odot$. Dimensions are given by $L_1, \ldots, L_7$.

$$J_s = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & L_1 & 0 & L_1+L_2 & 0 & L_1+L_2+L_3 & 0 \\ 0 & 0 & 0 & 0 & L_1+L_2+L_3 & 0 & L_4+L_5+L_6 \\ 0 & 0 & 0 & -L_4 & 0 & -L_4-L_5 & 0 \end{bmatrix}$$