ME 449 Robotic Manipulation
Spring 2014
Problem Set 6
Due Wednesday June 11 at 11:00 AM **(no late hwks accepted!)**

1. (a) For the stationary contacts in Figure 1, draw the rotation centers that correspond to feasible motion of the planar object and label them with the appropriate contact modes. Put the contact label for the uppermost finger first, then the middle finger, then the bottom finger. (b) Add a fourth "finger" that puts the object in form closure.

2. (a) For the friction cones in Figure 2, draw the moment-labeling representation of all forces that can be applied to the object through the contacts. (b) Draw a force that cannot be resisted by the contacts and explain why in terms of the moment labeling regions.

3. Write a program that uses linear programming to determine if a set of contacts on a planar rigid body yields force closure. (This becomes first-order form closure when the friction coefficient is zero.) As input, the program takes the friction coefficient and the number of contacts, and for each contact, the program takes the (x, y) location of the contact and the angle of the contact normal pointing into the body. The program returns either "force closure" or "not force closure." Test the program for a right-triangle object with vertices at (0,0), (2,0), and (0,2), with two point-finger contacts at (1,0) and (1,1) and two different cases: in the first case, the friction coefficient is 0.5, and in the second case, the friction coefficient is 2.0. For each case, provide a printout of your program running (showing the input and output). Provide your commented code.

4. Explain how you would generalize the previous program to work for 3D rigid bodies.

5. Figure 3 shows an assembly of three identical planar blocks resting on a flat tray in gravity with $g = 9.8$ m/s^2. Each block has dimensions 8×2 cm, a mass of 1 kg with the center of mass in the center, and inertia about the center of mass of 9×10^{-4} kg/m^2. The friction coefficient at each contact is 0.25.

Write a program that takes as input the initial (x, y, θ) configuration of the tray, as well as its velocity and acceleration, and determines whether it is possible for the blocks to stay stationary relative to each other (i.e., stay assembled), and the assembly to stay stationary relative to the tray, under those conditions. Use your program in an iterative fashion to find the maximum sideways acceleration the assembly can sustain to the left ($-x$ direction) and to the right ($+x$) when the tray is horizontal. (Finding trajectories of the tray that satisfy assembly

constraints is sometimes called **the waiter's problem**.)

6. Explain how you would generalize the previous solution to arbitrary assemblies of planar rigid bodies.

7. Explain how you would generalize the previous solution to arbitrary assemblies of 3D rigid bodies.

8. Give an outline of planning methods to solve the problem of finding the fastest trajectory from one state to another that satisfies manipulator torque limits and assembly stability constraints in two different cases: (a) the path is pre-specified and (b) the path can be chosen.

9. In the paper "Using projected dynamics to plan dynamic contact manipulation," Srinivasa, Erdmann, and Mason, IROS 2005, a flat "tray" is moved to cause a block resting on the tray to stand up. The motion is parameterized by the angle $\theta \in [0, \pi/2]$ of the block relative to the tray. This paper uses ideas from this course of time-scaling of trajectories, dynamics of a single rigid body, and single-point-contact manipulation considering friction limits. Your job is to understand the main points of the paper and explain them to someone who has not taken the course. (Do not simply repeat figure captions; demonstrate your understanding!) Your answers to each part should be no more than one paragraph.

(i) Explain Figure 2. (Fully explain the two cones and the green shaded region in terms of the main problem of the paper.)

(ii) Explain how Figure 2 leads to Figure 1(ii).

(iii) Explain Figure 3.

(iv) Explain Figure 7.

(v) Explain how the trajectory planning problem differs from the time-optimal time-scaling problem we studied earlier, particularly noting that θ parameterizes the block's motion relative to the tray, but not the motion of the tray.

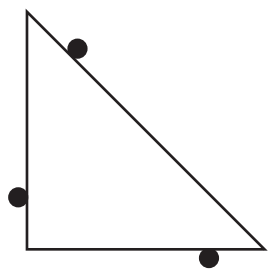

Figure 1: Three fingers contacting a triangular object.

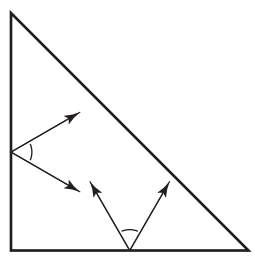

Figure 2: Two frictional contacts on an object.

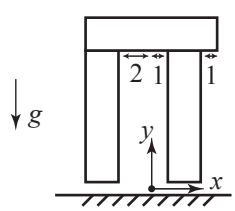

Figure 3: An assembly on a tray, with a frame attached to the tray. The tray frame is shown coincident with the fixed space frame.