

1. Complete the motor power equation, where L is the motor inductance and R is the motor resistance.

①
$$V = \tau \omega + I^2 R + \frac{dI}{dt} LI$$

2. Write the motor voltage equation, derived from the power equation.

①
$$V = \frac{\tau}{I} \omega + IR + L \frac{dI}{dt}$$

3. Assume a motor with resistance 1Ω and torque constant 0.1 Nm/A . If the two motor leads are shorted to each other (current can flow through the motor, but there is no voltage across the motor leads), and if the output shaft of the motor is spun by an external source at a constant speed of 100 rad/s , how much torque must that external source provide?

② Short-circuit damping.

$$0 = k_t \omega + IR \quad \tau = \frac{-k_b}{R} \omega = \frac{-(0.1)^2}{1} \cdot 100 = -1 \text{ Nm (motor torque)}$$
1 Nm (external torque)

4. You are given a motor, but no specs. You apply 10 volts to it and find that, at stall, it provides 0.02 Nm of torque and draws 2 A . The motor coils can dissipate 1 W of power continuously before overheating. Fill in the rest of this partial data sheet.

1.5 each

nominal voltage (V): 10

resistance (Ω): At stall, $R = V/I_{\text{stall}} = 10/2 = \boxed{5 \Omega}$

torque constant (Nm/A): $k_t = \tau_{\text{stall}}/I_{\text{stall}} = \boxed{0.01 \text{ Nm/A}}$

electrical constant (Vs/rad): Same numerical val as k_t , $= \boxed{0.01 \text{ Vs/rad}}$

max continuous current (A): $I_{\text{cont}} = \sqrt{P/R} = \boxed{\sqrt{1/5} \text{ A}}$

max continuous torque (Nm): $\tau_{\text{cont}} = k_t I_{\text{cont}} = \boxed{0.01 \cdot \sqrt{1/5} \text{ Nm}}$

no-load speed (rad/s): $\omega_0 = V/k_t = \boxed{1000 \text{ rad/s}}$

max mechanical power out (W): $P_{\text{max}} = (\frac{1}{2} \tau_{\text{stall}})(\frac{1}{2} \omega_0) = \frac{1}{4} \cdot 0.02 \cdot 1000 = \boxed{5 \text{ W}}$

efficiency at max power out (%): $\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{P_{\text{max}}}{IV} = \frac{5 \text{ W}}{2 \text{ A} \cdot 10 \text{ V}} = \boxed{50\%}$
 ($I = \frac{1}{2} I_{\text{stall}}$)

5. Draw the **20 V** speed-torque curve for the motor in problem 4, indicating the continuous operating region. (NOTE: 20 V!) Also write the no-load speed and the stall torque at 20 V.

