Homework #7

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Due: 03/01/11

1.

Of the 19 considered entries in the data sheet, N=9 of them are independent. The remaining 10 entries can be written as a function of the independent entries.

Independent Entries: N=9

1. Nominal Voltage, V_{nom}
2. Power Rating, P
3. No-Load Speed, ω_0
4. No-Load Current, I_0
5. Terminal Resistance, R
6. Terminal Inductance, L
7. Electrical Constant, k_e
8. Rotor Inertia, J
9. Short-Circuit Damping, B

Dependent Entries: 19-N=10

More explanations on the derivations of these equations are on part 2.

1. Max Mechanical Power: $P_{\max} = \left(\frac{1}{2}T_s\right)\left(\frac{1}{2}\omega_0\right) = \frac{k_e V_{nom}\omega_0}{4R}$
2. Max Continuous Current:

$$P = I_c^2 R;$$

$$I_c = \sqrt{\frac{P}{R}}$$

3. Starting Current: $I_s = \frac{V_{nom}}{R}$
4. Max Continuous Torque: $T_c = k_t I_c = k_e\sqrt{\frac{P}{R}}$
5. Stall Torque: $T_s = k_e I_s = \frac{k_e V_{nom}}{R}$
6. Max Efficiency: $\eta_{\max} = \left(1-\sqrt{\frac{I_0}{I_s}}\right)^2 = \left(1-\sqrt{\frac{I_0 R}{V_{nom}}}\right)^2$
7. Electrical Time Constant: $\tau_e = \frac{L}{R}$

8. Torque Constant: $k_t = k_e$

9. Speed Constant: $k_s = k_e^{-1}$

10. Mechanical Time Constant: $\tau_m = \frac{JR}{k_t^2} = \frac{JR}{k_e^2}$

2.

Motor Property	Symbol	Value	Units	Independent?
1. Nominal Voltage	V_{nom}	6	V	Y
2. Power Rating	P	5	W	Y
3. Max Mechanical Power	$P_{\max}$	1.507	W	
4. No-Load Speed	ω_0	189	$\frac{rad}{s}$	Y
5. No-Load Current	I_0	0.230	A	Y
6. Max Continuous Current	I_c	0.962	A	
7. Starting Current	I_s	1.11	A	
8. Max Continuous Torque	T_c	0.0276	Nm	
9. Stall Torque	T_s	0.0319	Nm	
10. Max Efficiency	$\eta_{\max}$	29.7	%	
11. Terminal Resistance	R	5.4	Ω	Y
12. Terminal Inductance	L	0.00107	H	Y
13. Electrical Time Constant	τ_e	1.98×10^{-4}	s	
14. Torque Constant	k_t	0.0287	$\frac{Nm}{A}$	
15. Electrical Constant	k_e	0.0287	$\frac{Vs}{rad}$	Y
16. Speed Constant	k_s	34.8	$\frac{rad}{Vs}$	
17. Mechanical Time Constant	τ_m	0.0551	s	
18. Rotor Inertia	J	8.4×10^{-6}	kgm^2	Y
19. Short-Circuit Damping	B	-	$\frac{Nms}{rad}$	Y
20. Friction	-	0	-	Y

Explanation of Different Entries

1. Nominal Voltage – The nominal voltage is given from the battery pack at 6V.

-**independent**

2. Power Rating – Because we can't actually measure the power rating without damaging the motor, we will assume a safe power rating of 5 W.

-**independent**

3. Max Mechanical Power – The maximum mechanical power can be calculated assuming that the speed-torque curve is linear.

$$P_{\max} = \left(\frac{1}{2}T_s\right)\left(\frac{1}{2}\omega_0\right) = \left(\frac{1}{2}\cdot 0.0319 N\cdot m\right)\left(\frac{1}{2}\cdot 189\frac{rad}{s}\right) = 1.507W$$

4. No-Load Speed –The no-load speed is experimentally calculated by measuring the angular speed of the shaft when the motor is powered by the battery pack and no load is added. The angular speed is calculated by looking at the encoder signals through NUScope and knowing that the rotary encoder has 100 lines.

-**independent**

5. No-Load Current – I used the current-sensing resistor to measure the no-load current. I assumed that the equation governing the current-sensing resistor was $V = aI + b$ where a and b are constants derived through experimentation. Changing the current to find the voltage across the resistor, I found that a=0.128 and $b \approx 0$. Using this, I found that I_0 = 230 mA.

-**independent**

6. Max Continuous Current – The maximum continuous current can be calculated using $P = I_c^2 R$.

$$I_c = \sqrt{\frac{P}{R}} = \sqrt{\frac{5W}{5.4\Omega}} = 0.962A$$

7. Starting Current – The starting current is can be found using the terminal resistance R.

$$I_s = \frac{V_{nom}}{R} = \frac{6V}{5.4\Omega} = 1.11A$$

8. Max Continuous Torque – The maximum continuous torque can be calculated from the maximum continuous current and the torque constant.

$$T_c = k_t I_c = \left(0.0287\frac{N\cdot m}{A}\right)(0.962A) = 0.0276 N\cdot m$$

9. <u>Stall Torque</u> – Stall torque can be calculated using the electrical constant and the starting current.

$$T_s = k_e I_s = \left(0.0287\frac{V\cdot s}{rad}\right)(1.11A) = 0.0319N\cdot m$$

10. <u>Max Efficiency</u> – Calculated from the no-load current and the starting current.

$$\eta_{\max} = \left(1-\sqrt{\frac{I_0}{I_s}}\right)^2 = \left(1-\sqrt{\frac{0.230A}{1.11A}}\right)^2 = 29.7\%$$

11. <u>Terminal Resistance</u> – The terminal resistance is found by running the motor from the battery pack with a current-sensing resistor attached in series. Using Ohm's law and the relation found in part 5 above, stalling the motor gave a current of 1.054 A. Measuring the voltage across the battery pack (since it is not exactly equal to 6 V) and subtracting the voltage across the current-sensing resistor yielded a voltage of 5.735 V. This gave a resistance R of 5.4 V.

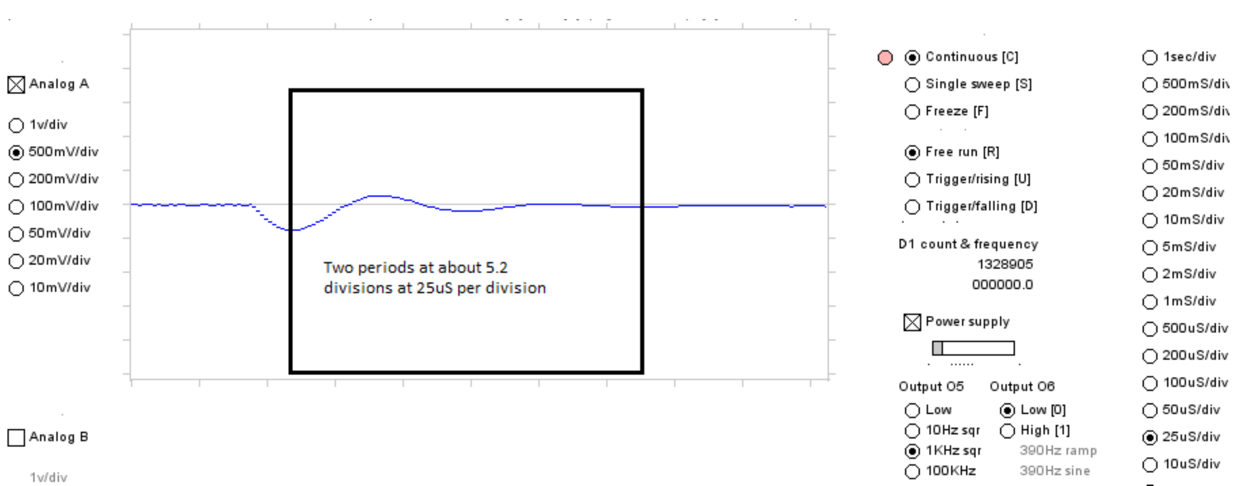
-**independent**

12. <u>Terminal Inductance</u> – The inductance of the motor is calculated using the oscillatory behavior of an RLC circuit. By attaching a capacitor in parallel to the motor, a decaying oscillating behavior can be seen. By measuring the period of the oscillations, the inductance of the motor can ultimately be calculated. A value of capacitance C = 0.1uF was used.

$$\omega_n = \frac{2\pi}{T}; L = \frac{1}{\omega_n{}^2 C}$$

$$\omega_n = \frac{2\pi}{6.5(10)^{-5}} = 96664.4\frac{rad}{s}; L = \frac{1}{(96664\frac{rad}{s})^2(0.1\mu F)} = 1.07mH$$

A scope trace of the oscillatory behavior is seen below.



13. Electrical Time Constant – The electrical time constant is calculated from the terminal inductance and the terminal resistance.

$$\tau_e = \frac{L}{R} = \frac{0.00107H}{5.4\Omega} = 0.000198s$$

14. Torque Constant – The torque constant is the same as the electrical constant but derived in a different way. Approximating the torque constant using $\frac{V_{nom}}{\omega_0}$, we get 0.032 which is very close to the measured electrical constant of 0.0287.

15. Electrical Constant – The electrical constant was experimentally determined by measuring the back EMF when coupling two motors together and measuring their speeds.

16. Speed Constant – The speed constant is calculated directly from the electrical constant.

$$k_s = \frac{1}{k_e} = \frac{1}{0.0287} = 34.8\frac{rad}{V \cdot s}$$

17. Mechanical Time Constant – The mechanical time constant can be calculated from the rotor inertia, terminal resistance, and the torque constant.

$$\tau_m = \frac{JR}{k_t^2} = \frac{\left(8.4x10^{-6}kgm^2\right)\left(5.4\Omega\right)}{\left(0.0287\frac{N \cdot m}{A}\right)^2} = 0.0551s$$

18. Rotor Inertia – The rotor inertia is estimated by using the approximate mass and radius of the rotor. The mass of the motor is 250 g. The mass of the rotor is assumed to be 15% of the motor, making the mass of the rotor 37.5 g. The outer casing of the motor has a radius of 1.8 cm. It is reasonable to assume that the radius of the rotor is slightly less than that, about 1.5 cm. Finally, moment of inertia of a cylindrical shell is given by $J = mr^2$ while the moment of inertia of a solid cylinder is given by $J = \frac{1}{2}mr^2$. Thus, it is safe to assume that the motor has a moment of inertia somewhere in between those two equations, at say, $J = \frac{3}{4}mr^2$.

$$J = \frac{3}{4}mr^2 = \frac{3}{4}(0.0375kg)(0.015m)^2 = 8.4x10^{-6}kgm^2$$

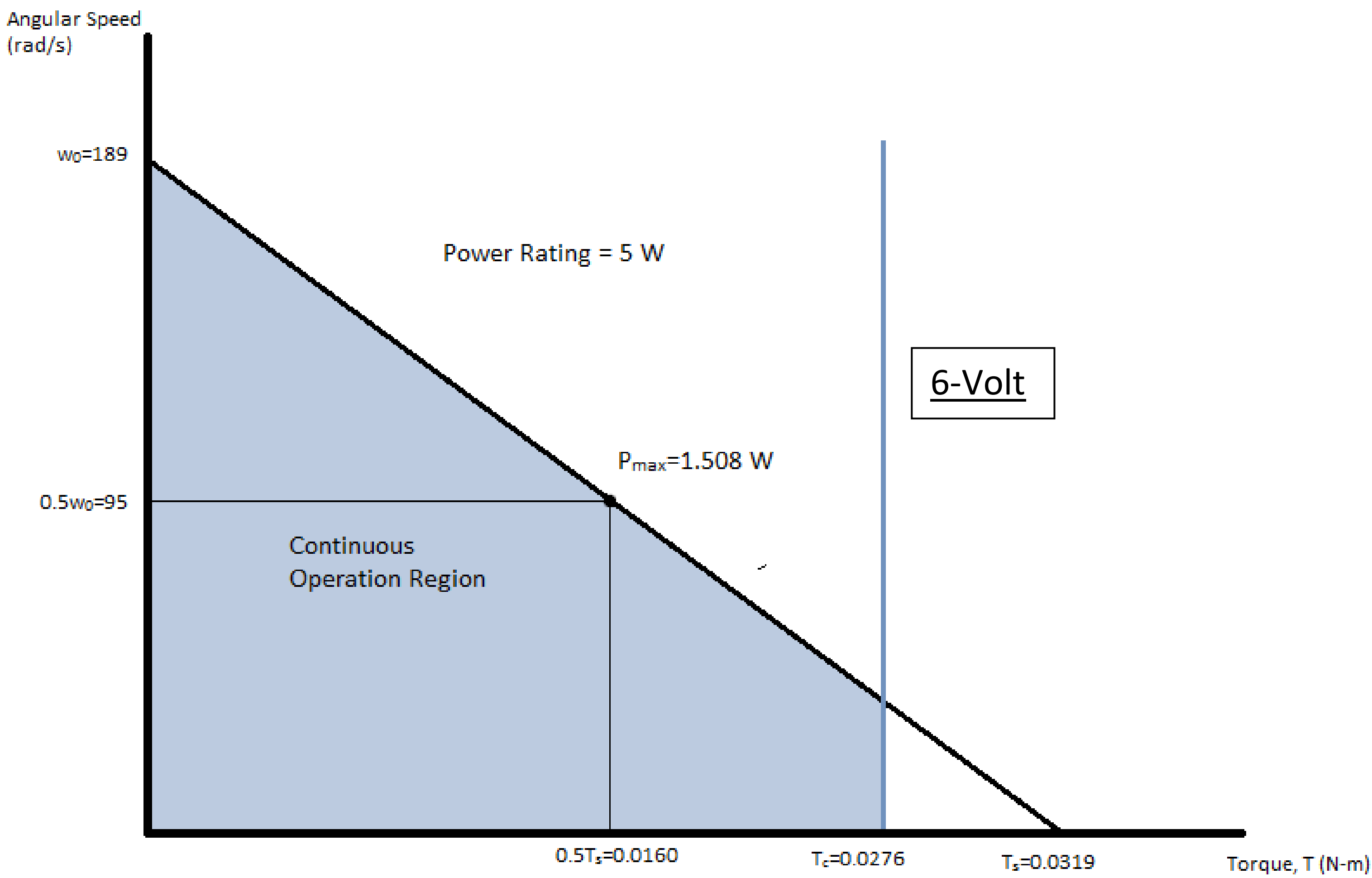
19. Short-Circuit Damping – Within the scope of the equipment used for this class, the short-circuit damping cannot be found. This is because the only way to spin the rotor when it is short-circuited is by spinning the shaft with our hands, which makes measuring the torque and the angular velocity very difficult.

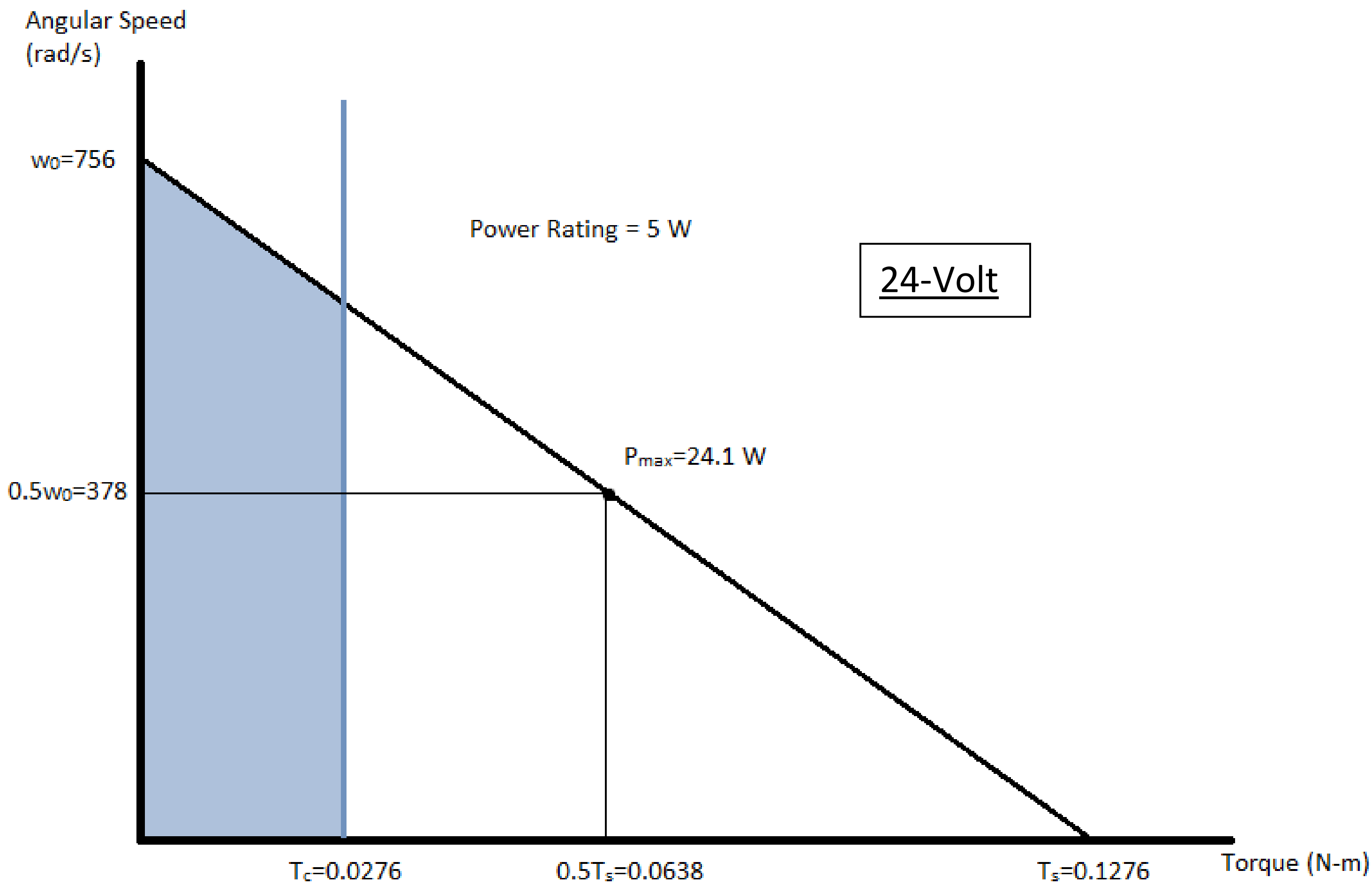
20. Friction – The friction term is calculated using the equation for motor modeling.

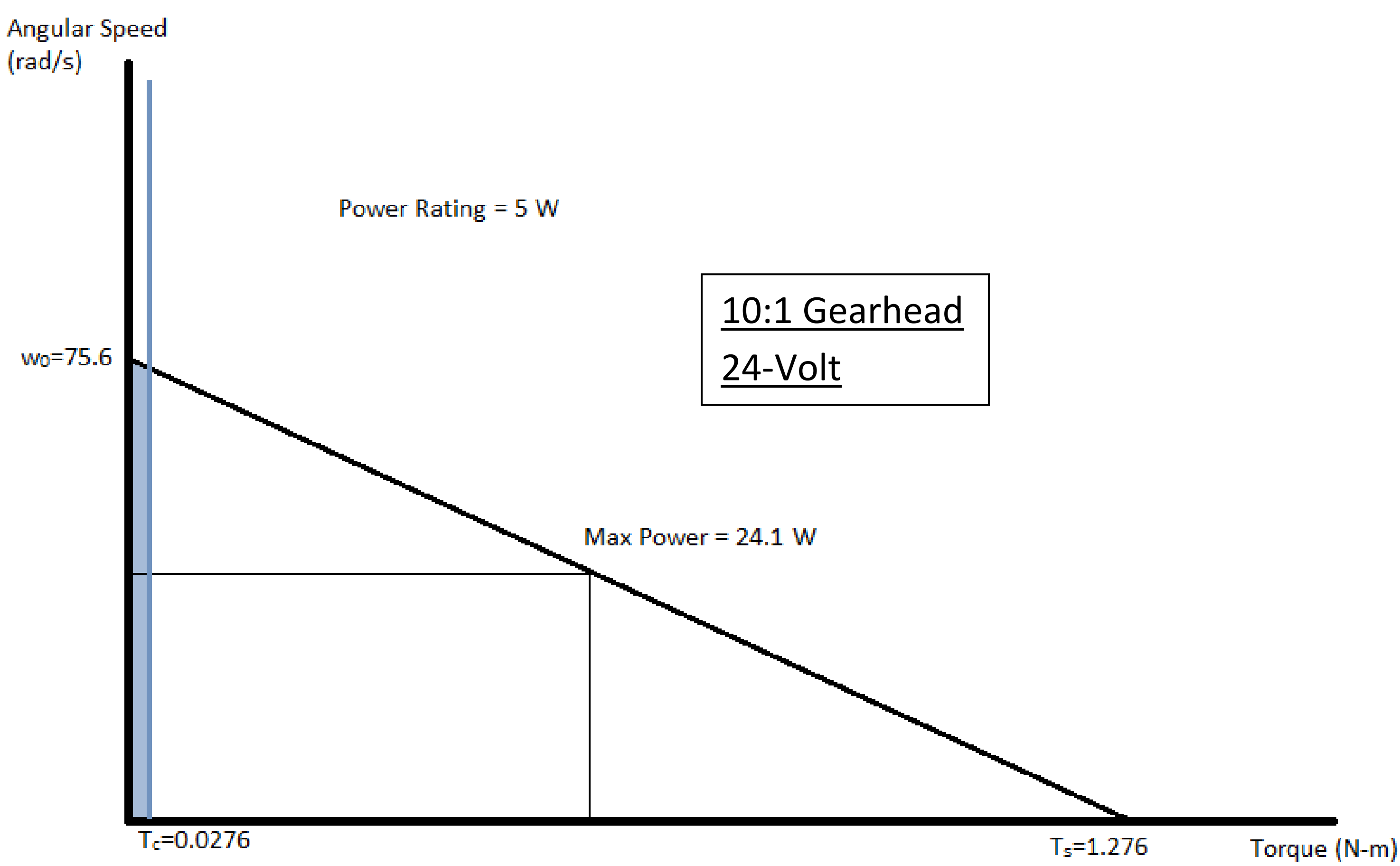
$$V_{nom} = IR + k_e\omega_0 + L\frac{dI}{dt} + friction$$

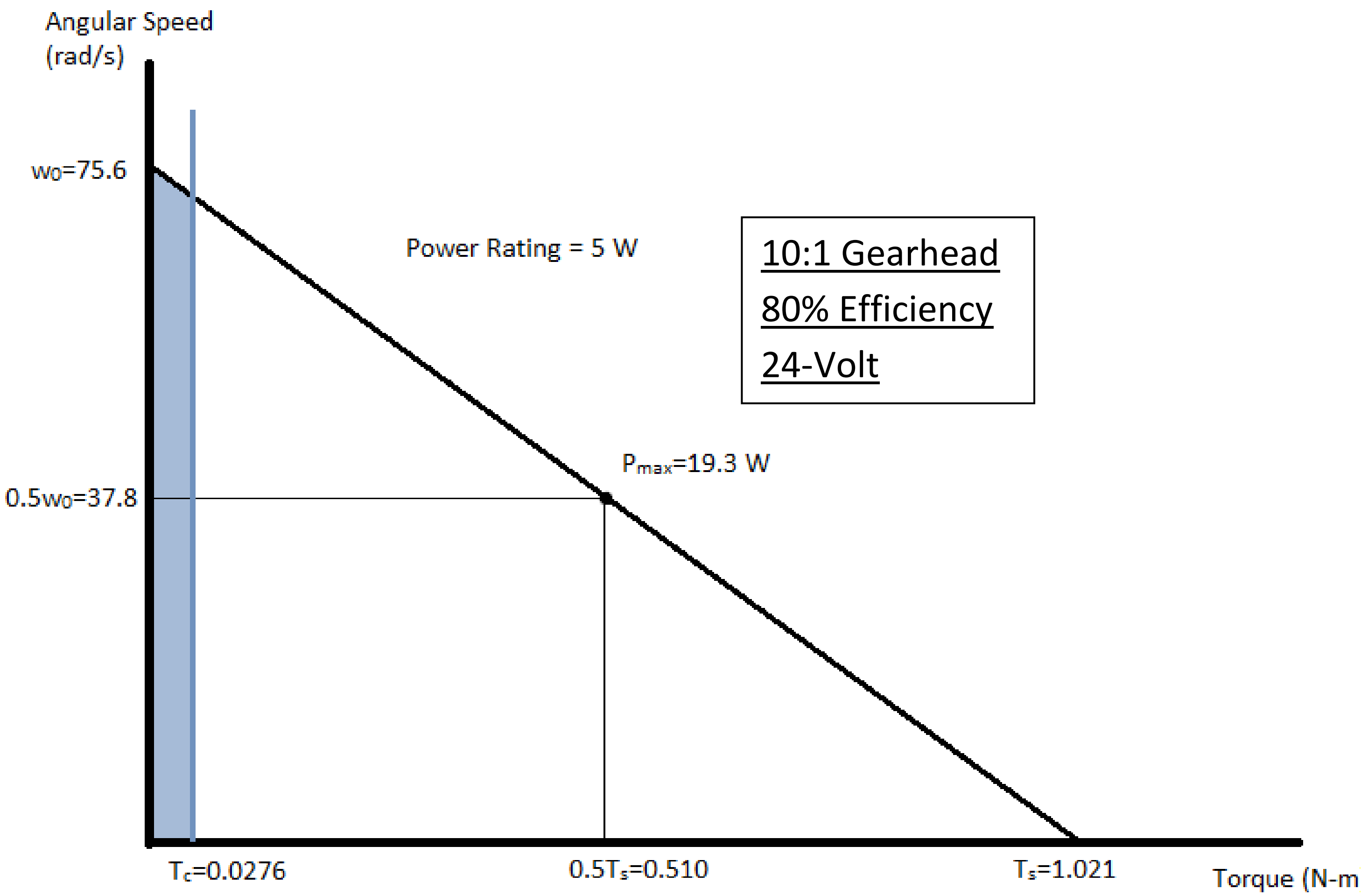
Assuming that V is constant, thus dI/dt=0, and plugging in the values for the other values, we get value of friction loss of -0.5 V, which is impossible. This can be attributed to measurement error. As a result, it is safe to assume that friction losses are zero.

3. **Speed – Torque Curves**




Angular Speed
(rad/s)
w0=756
Power Rating = 5 W
24-Volt
Pmax=24.1 W
0.5w0=378
Tc=0.0276
0.5Ts=0.0638
Ts=0.1276
Torque (N-m)


Angular Speed
(rad/s)
Power Rating = 5 W
10:1 Gearhead
24-Volt
w0=75.6
Max Power = 24.1 W
Tc=0.0276
Ts=1.276
Torque (N-m)


Angular Speed (rad/s)
w0=75.6
Power Rating = 5 W
10:1 Gearhead
80% Efficiency
24-Volt
Pmax=19.3 W
0.5w0=37.8
Tc=0.0276
0.5Ts=0.510
Ts=1.021
Torque (N-m)