

Where we are:

Chap 2	Configuration Space
Chap 3	Rigid-Body Motions
Chap 4	Forward Kinematics
Chap 5	Velocity Kinematics and Statics 5.1 Manipulator Jacobian
Chap 6	Inverse Kinematics
Chap 8	Dynamics of Open Chains
Chap 9	Trajectory Generation
Chap 11	Robot Control
Chap 13	Wheeled Mobile Robots

Important concepts, symbols, and equations

- “Geometric” forward kinematics

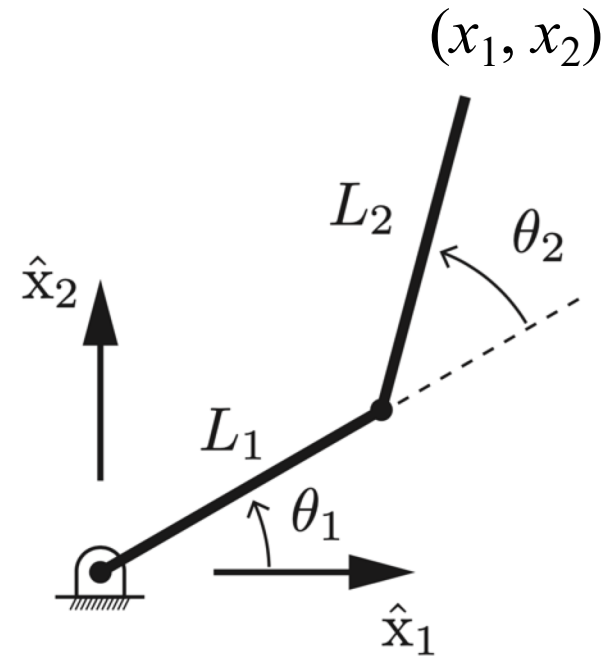
$$\theta(t) \in \mathbb{R}^n, \quad \mathcal{X}(t) = T_{sb}(\theta(t)) \in SE(3) \text{ via PoE}$$

- “Minimum coordinate” forward kinematics

$$\theta(t) \in \mathbb{R}^n, \quad x(t) = f(\theta(t)) \in \mathbb{R}^m$$

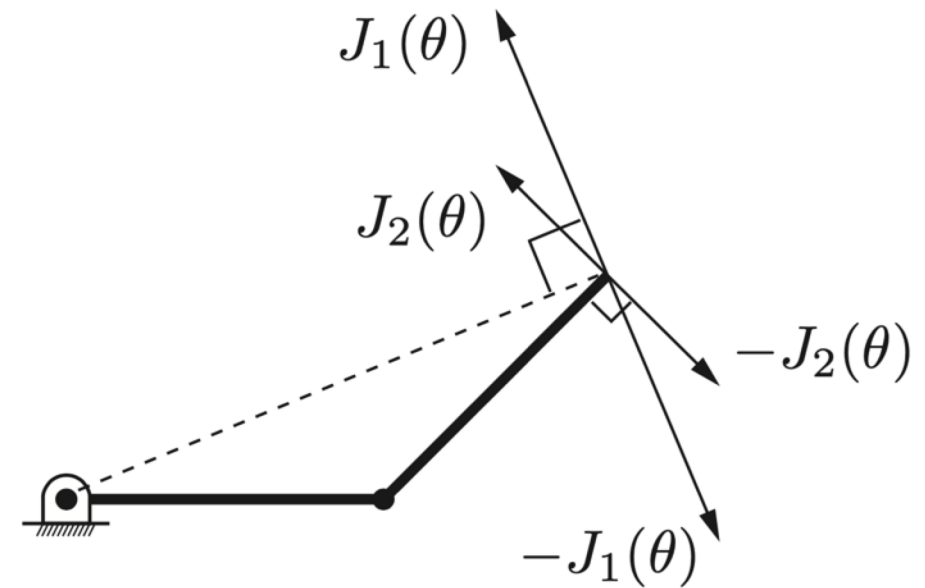
$$x_1 = L_1 \cos \theta_1 + L_2 \cos(\theta_1 + \theta_2)$$

$$x_2 = L_1 \sin \theta_1 + L_2 \sin(\theta_1 + \theta_2)$$



Important concepts, symbols, and equations (cont.)

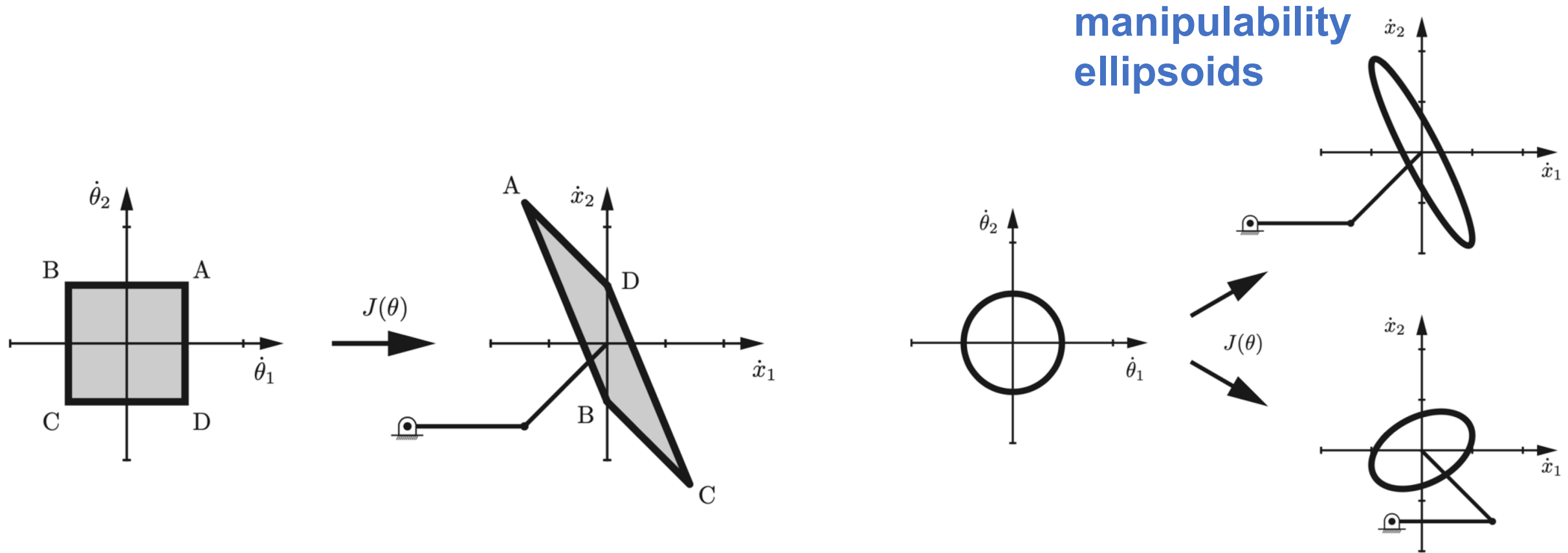
$$\begin{aligned}\dot{x} &= \frac{\partial f(\theta)}{\partial \theta} \frac{d\theta(t)}{dt} = \underbrace{\frac{\partial f(\theta)}{\partial \theta}}_{\text{(coordinate) Jacobian}} \dot{\theta} \\ &= J(\theta) \dot{\theta}\end{aligned}$$



$$\dot{x} = [J_1(\theta) \ J_2(\theta)] [\dot{\theta}_1 \ \dot{\theta}_2]^T = J_1(\theta) \dot{\theta}_1 + J_2(\theta) \dot{\theta}_2$$

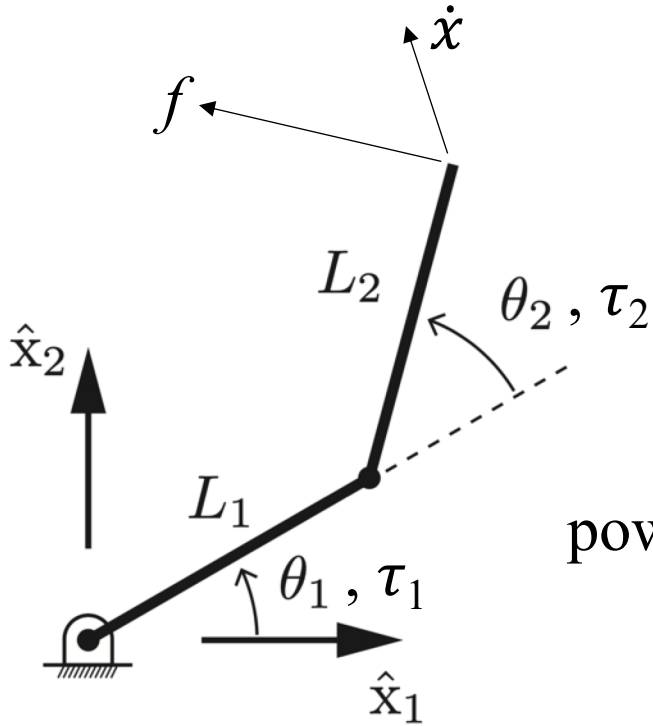
Column i is the end-effector velocity when $\dot{\theta}_i = 1$.

Important concepts, symbols, and equations (cont.)



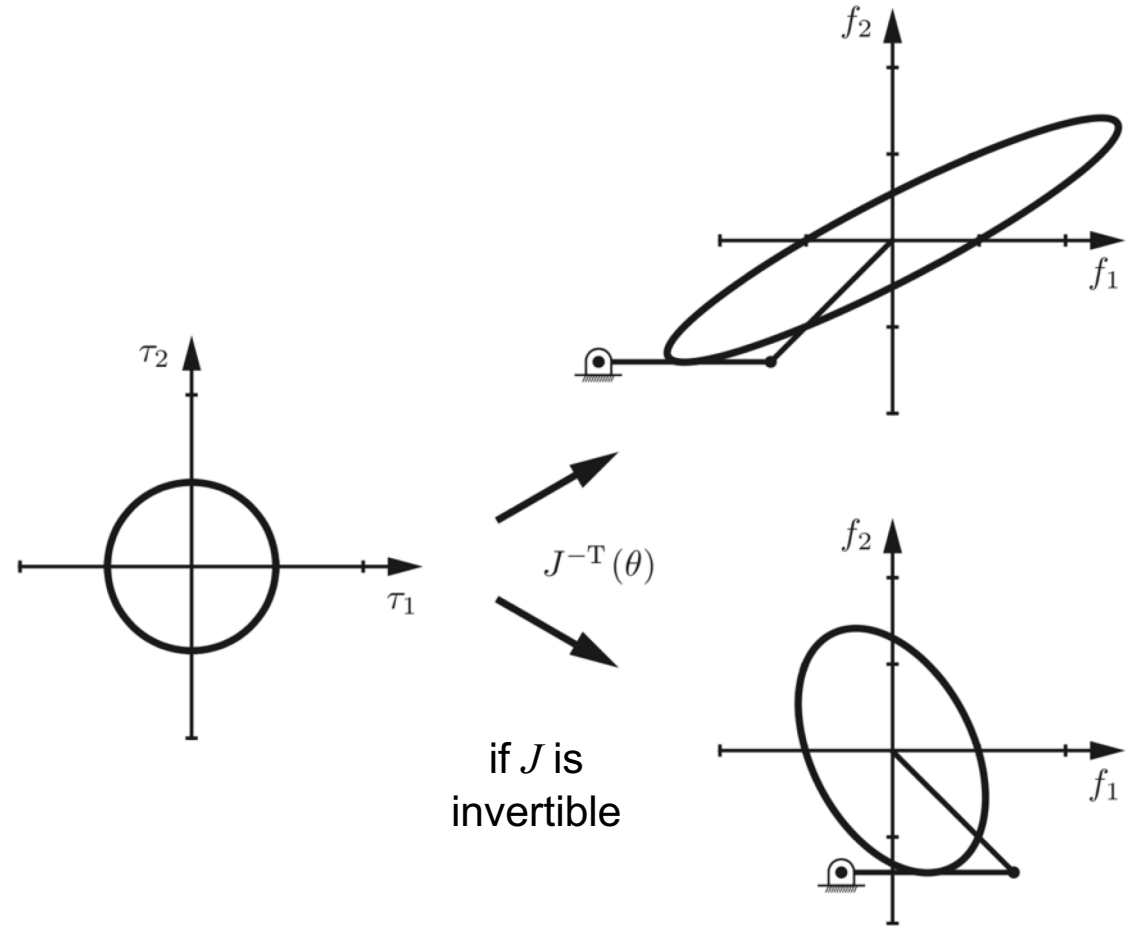
mapping joint velocity limits to end-effector velocity limits

Important concepts, symbols, and equations (cont.)



$$\begin{aligned} \text{power} &= \dot{\theta}^T \tau = \dot{x}^T f \\ \dot{\theta}^T \tau &= (J \dot{\theta})^T f \\ \dot{\theta}^T \tau &= \dot{\theta}^T J^T f \end{aligned}$$

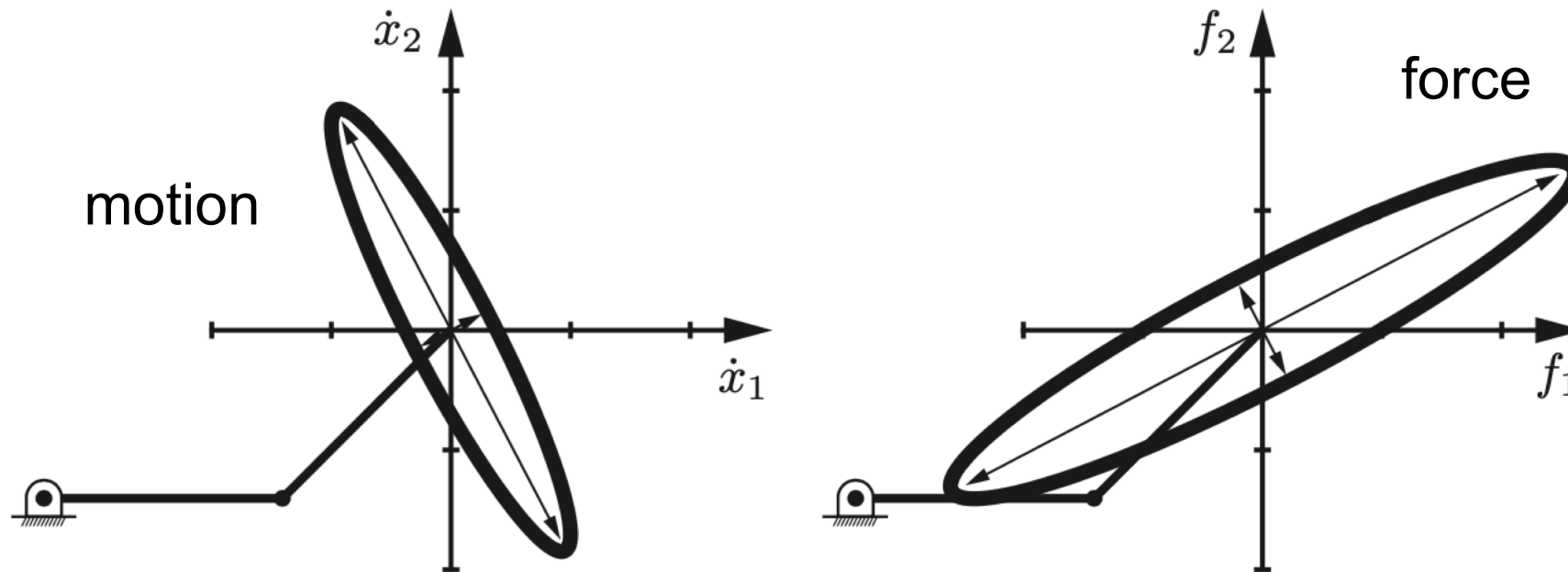
$$\tau = J^T f$$



Important concepts, symbols, and equations (cont.)

A configuration θ is **singular** if $J(\theta)$ loses rank.

- e-e motion capability becomes zero in one or more directions
- e-e can resist infinite force in one or more directions



approaching a singularity...

Important concepts, symbols, and equations (cont.)

- **Space Jacobian**

$$\mathcal{V}_s = J_s(\theta)\dot{\theta} \quad \text{where} \quad \begin{cases} J_{s1} = \mathcal{S}_1 \\ J_{si}(\theta) = \text{Ad}_{e^{[\mathcal{S}_1]\theta_1} \dots e^{[\mathcal{S}_{i-1}]\theta_{i-1}}}(\mathcal{S}_i) \quad i = 2, \dots, n \end{cases}$$

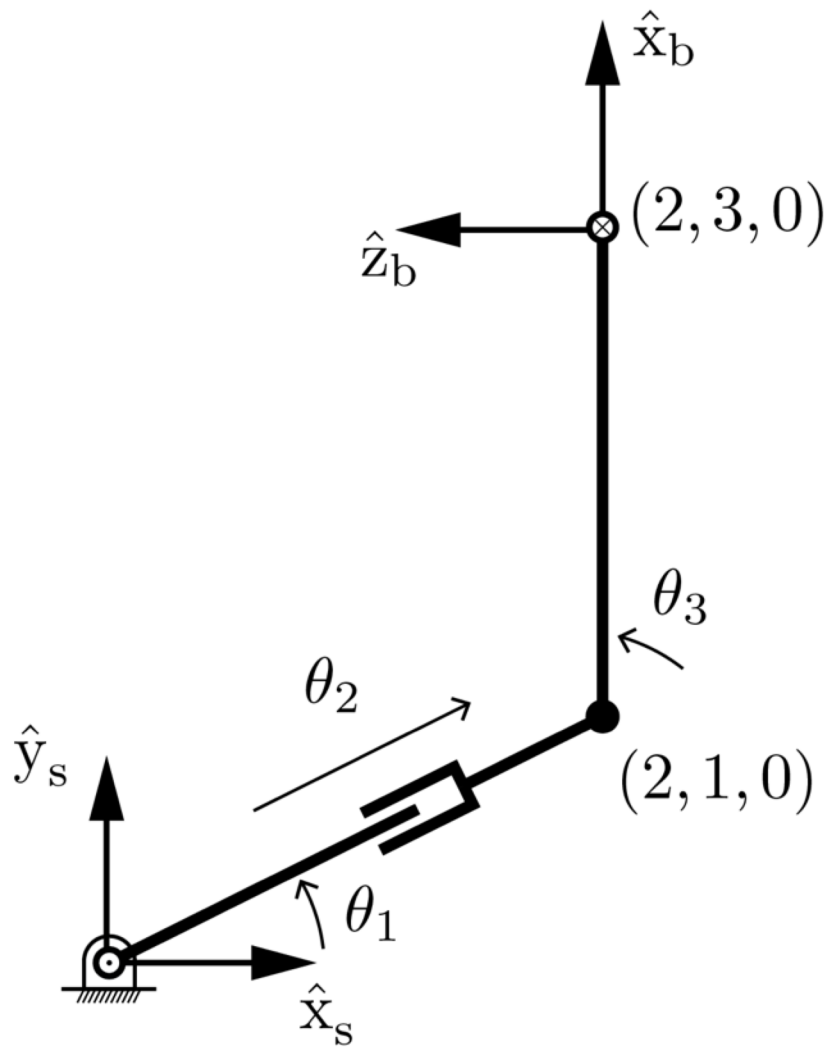
- **Body Jacobian**

$$\mathcal{V}_b = J_b(\theta)\dot{\theta} \quad \text{where} \quad \begin{cases} J_{bi}(\theta) = \text{Ad}_{e^{-[\mathcal{B}_n]\theta_n} \dots e^{-[\mathcal{B}_{i+1}]\theta_{i+1}}}(\mathcal{B}_i) \quad i = 1, \dots, n-1 \\ J_{bn} = \mathcal{B}_n \end{cases}$$

$$J_s(\theta) = [\text{Ad}_{T_{sb}}]J_b(\theta)$$

$$J_b(\theta) = [\text{Ad}_{T_{bs}}]J_s(\theta)$$

Column i is the end-effector twist when $\dot{\theta}_i = 1$.



$$J_s(\theta) =$$

$$J_b(\theta) =$$