Where we are:

# Important concepts, symbols, and equations

- "Geometric" forward kinematics

$$\theta(t) \in \mathbb{R}^n, \quad X(t) = T_{sb}(\theta(t)) \in SE(3) \text{ via PoE}$$

- "Minimum coordinate" forward kinematics

$$\theta(t) \in \mathbb{R}^n, \quad x(t) = f(\theta(t)) \in \mathbb{R}^m$$

$$
\begin{aligned}
x_1 &= L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2) \\
x_2 &= L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)
\end{aligned}
$$
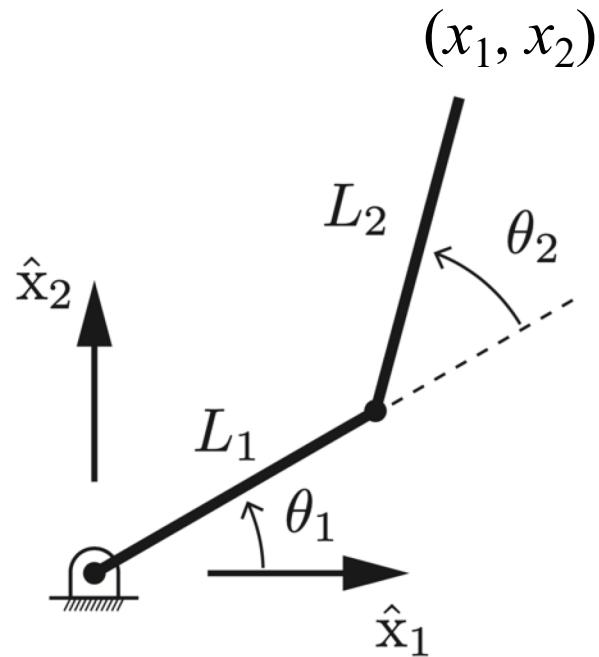
Important concepts, symbols, and equations (cont.)

$$\dot{x} = \frac{\partial f(\theta)}{\partial \theta} \frac{d\theta(t)}{dt} = \frac{\partial f(\theta)}{\partial \theta} \dot{\theta}$$

$$= J(\theta)\dot{\theta}$$

(coordinate)

Jacobian

$$\dot{x} = [\, J_1(\theta) \;\; J_2(\theta) \,] \; [\, \dot{\theta}_1 \;\; \dot{\theta}_2 \,]^{\mathrm{T}} = J_1(\theta)\,\dot{\theta}_1 + J_2(\theta)\,\dot{\theta}_2$$
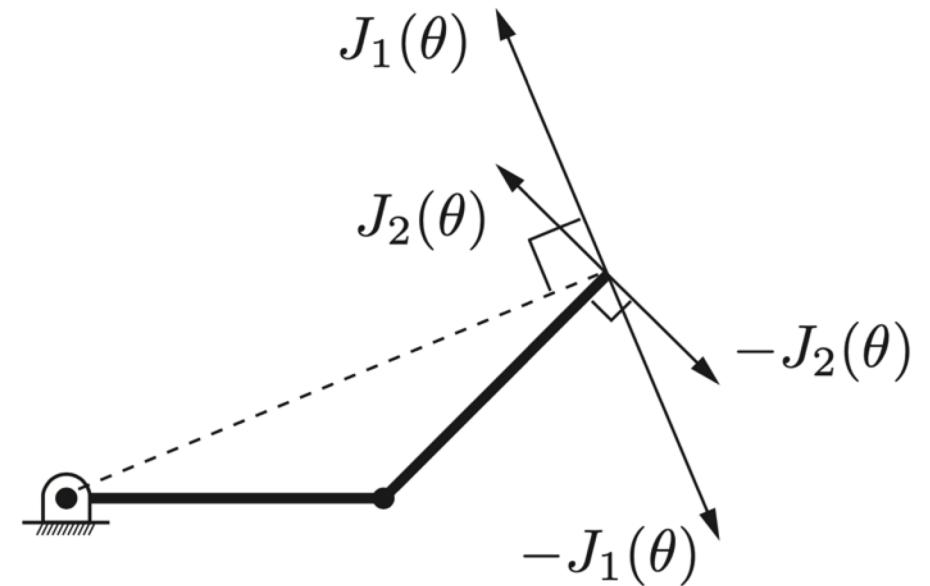
Column i is the end-effector velocity when $\dot{\theta}_i = 1$.

Important concepts, symbols, and equations (cont.)
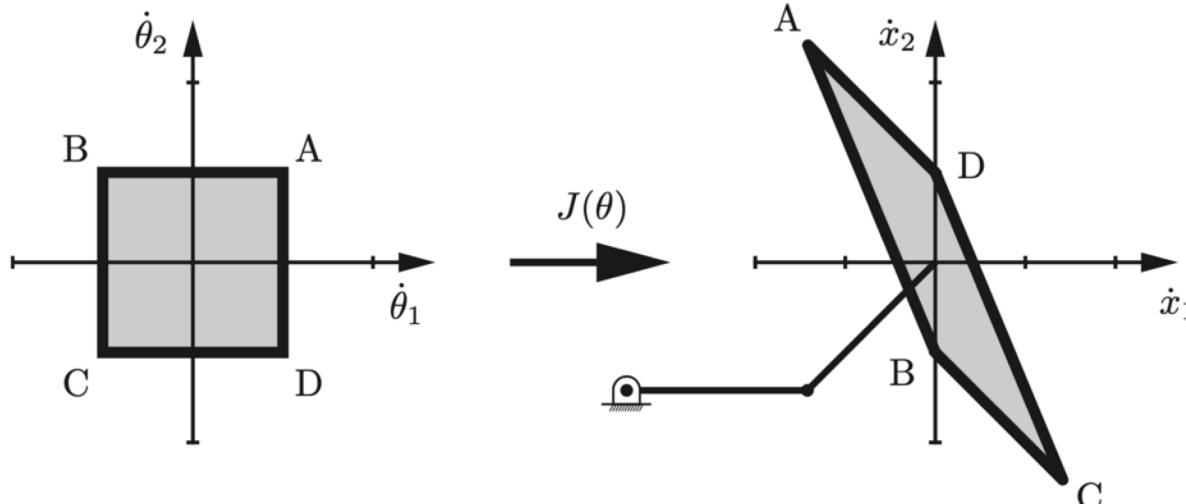
manipulability ellipsoids
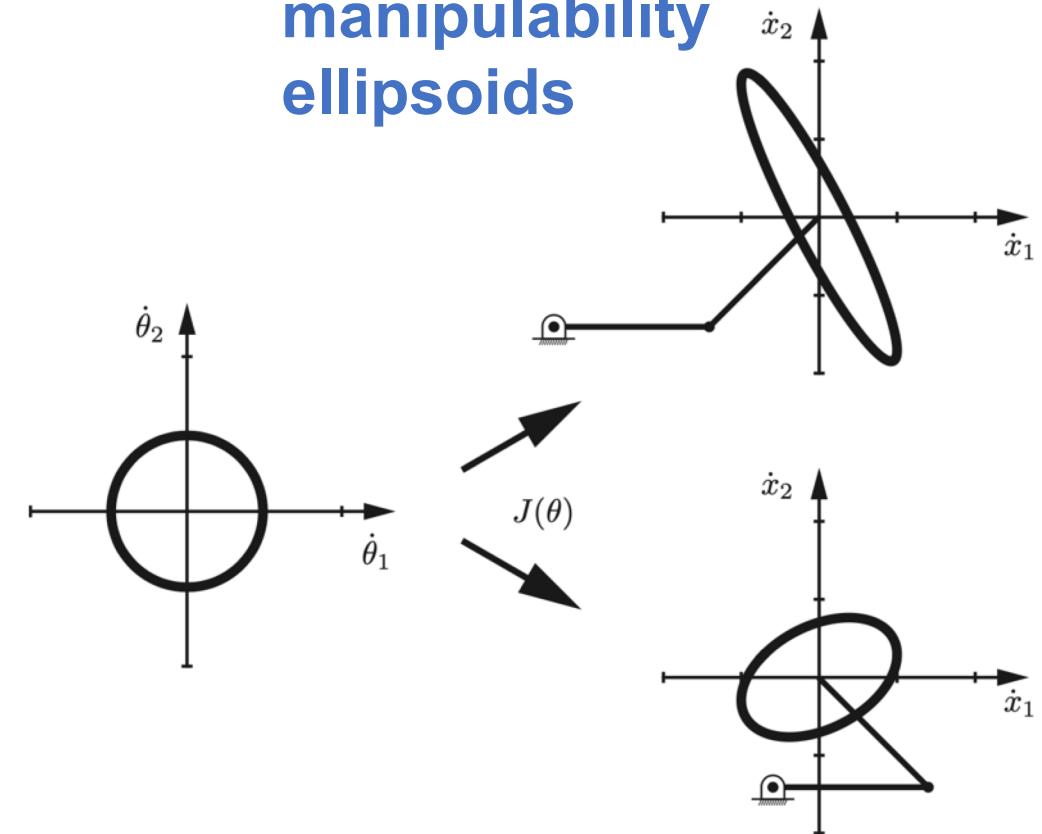
mapping joint velocity limits to end-effector velocity limits

Important concepts, symbols, and equations (cont.)
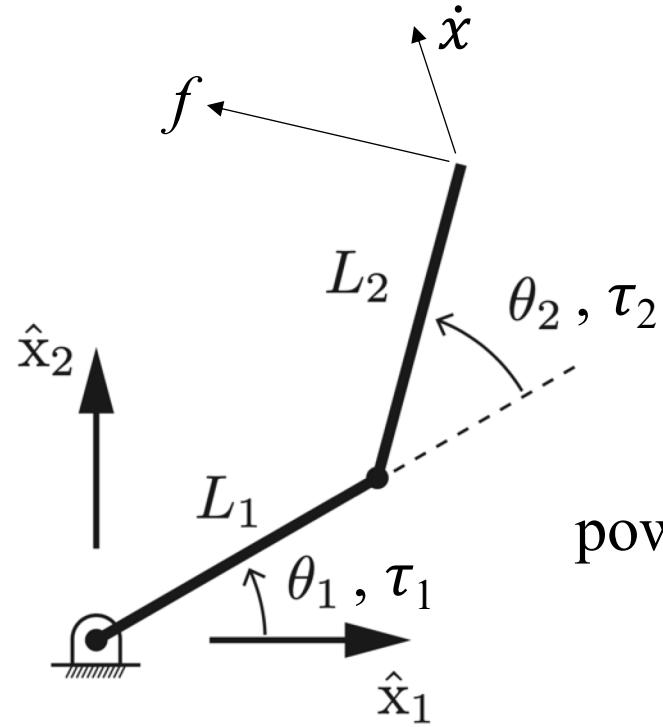
$$\text{power} = \dot{\theta}^{\text{T}} \tau = \dot{x}^{\text{T}} f$$
$$\dot{\theta}^{\text{T}} \tau = (J \dot{\theta})^{\text{T}} f$$
$$\dot{\theta}^{\text{T}} \tau = \dot{\theta}^{\text{T}} J^{\text{T}} f$$

$$\boxed{\tau = J^{\text{T}} f}$$
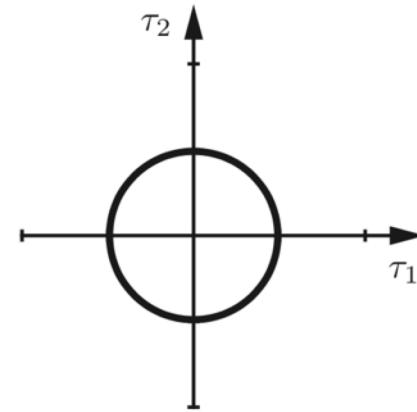
if J is invertible
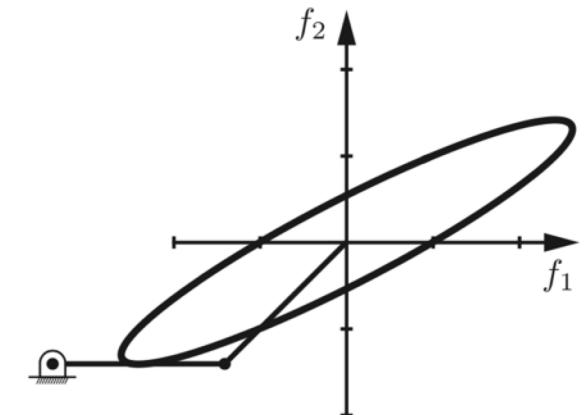
$J^{-\text{T}}(\theta)$
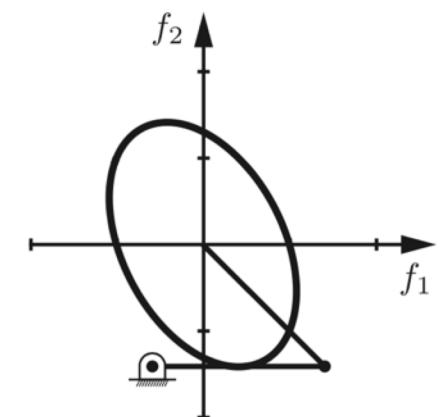
Important concepts, symbols, and equations (cont.)

A configuration θ is **singular** if $J(\theta)$ loses rank.

- e-e motion capability becomes zero in one or more directions
- e-e can resist infinite force in one or more directions
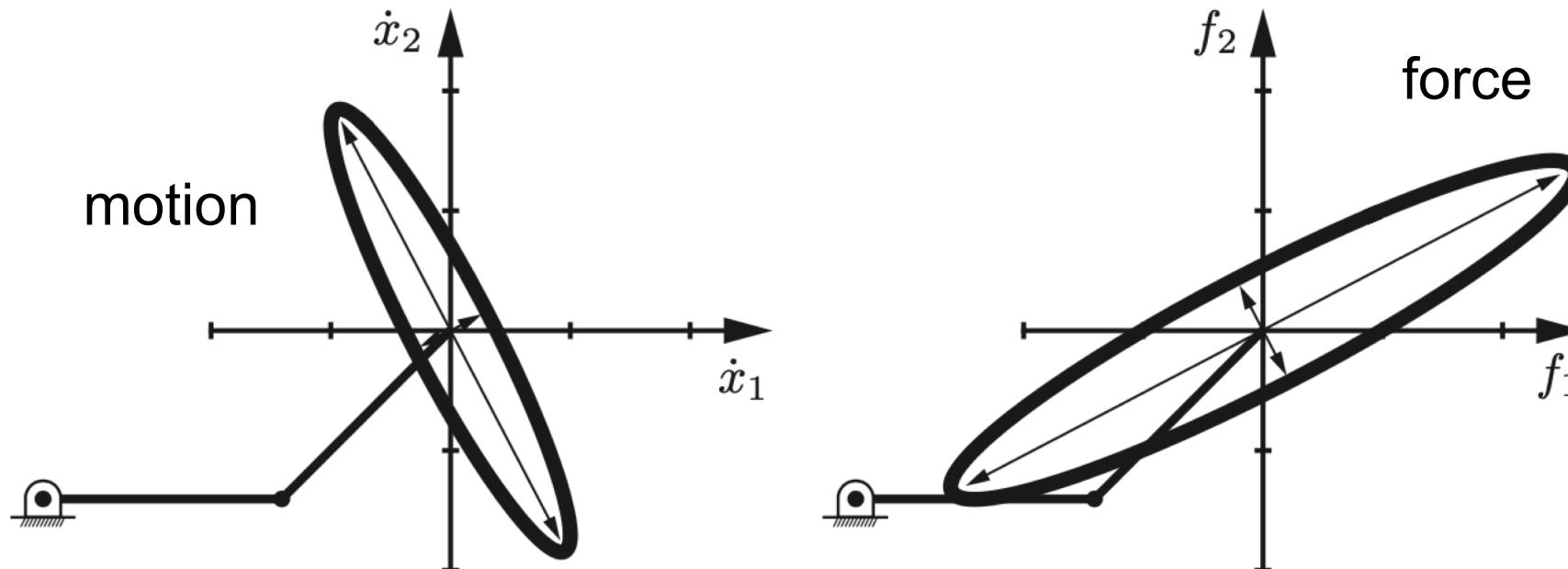
approaching a singularity...

Important concepts, symbols, and equations (cont.)

- **Space Jacobian**

$$\mathcal{V}_s = J_s(\theta)\dot{\theta} \quad \text{where} \quad \begin{cases} J_{s1} = \mathcal{S}_1 \\ J_{si}(\theta) = \mathrm{Ad}_{e^{[\mathcal{S}_1]\theta_1}\cdots e^{[\mathcal{S}_{i-1}]\theta_{i-1}}}(\mathcal{S}_i) \quad i = 2, \ldots, n \end{cases}$$

- **Body Jacobian**

$$\mathcal{V}_b = J_b(\theta)\dot{\theta} \quad \text{where} \quad \begin{cases} J_{bi}(\theta) = \mathrm{Ad}_{e^{-[\mathcal{B}_n]\theta_n}\cdots e^{-[\mathcal{B}_{i+1}]\theta_{i+1}}}(\mathcal{B}_i) \quad i = 1, \ldots, n-1 \\ J_{bn} = \mathcal{B}_n \end{cases}$$

$$J_s(\theta) = [\mathrm{Ad}_{T_{sb}}]J_b(\theta)$$
$$J_b(\theta) = [\mathrm{Ad}_{T_{bs}}]J_s(\theta)$$

Column i is the end-effector twist when $\dot{\theta}_i = 1$.

$$J_s(\theta) =$$

$$J_b(\theta) =$$