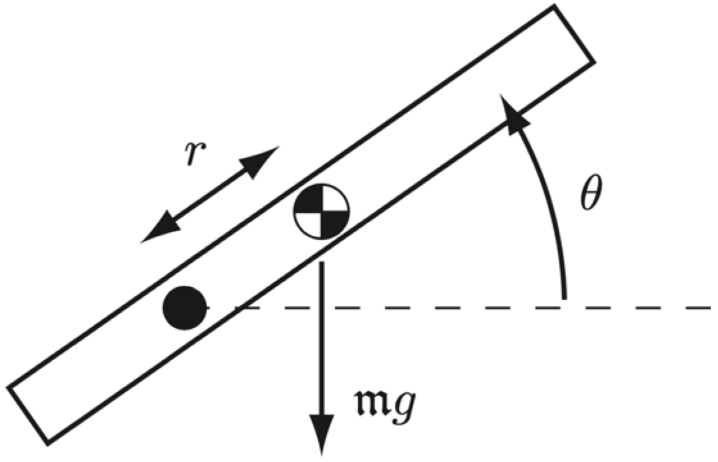
Where we are:

Chap 2 Configuration Space
Chap 3 Rigid-Body Motions
Chap 4 Forward Kinematics
Chap 5 Velocity Kinematics and Statics
Chap 6 Inverse Kinematics
Chap 8 Dynamics of Open Chains
Chap 9 Trajectory Generation
Chap 11 Robot Control
11.1 Control System Overview
11.2 Error Dynamics
11.3 Motion Control with Velocity Inputs
11.4 Motion Control with Torque or Force Inputs
Chap 13 Wheeled Mobile Robots

Important concepts, symbols, and equations



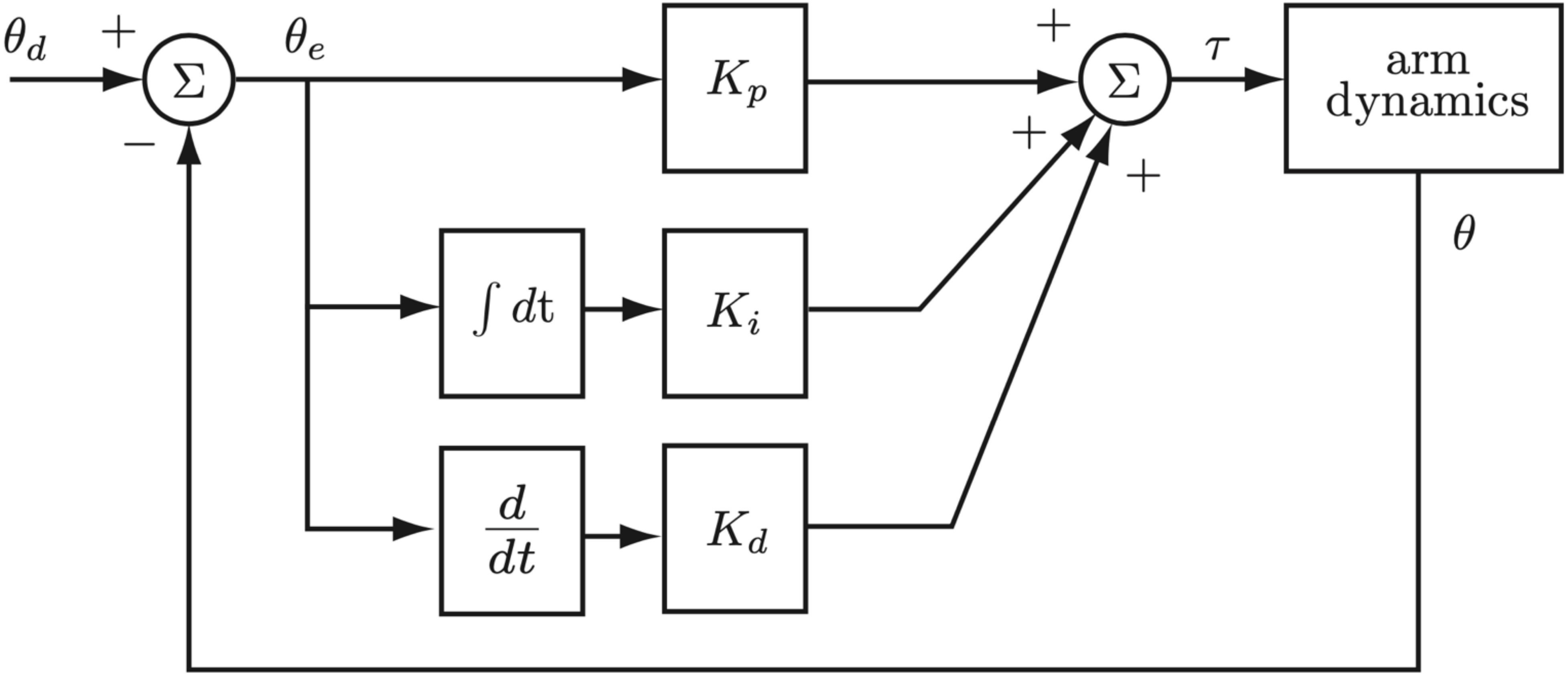
$$\tau = M\ddot{\theta} + \mathfrak{m}gr\cos\theta + b\dot{\theta}$$

$$\tau = M\ddot{\theta} + h(\theta, \dot{\theta})$$

Proportional-Integral-Derivative (PID) control

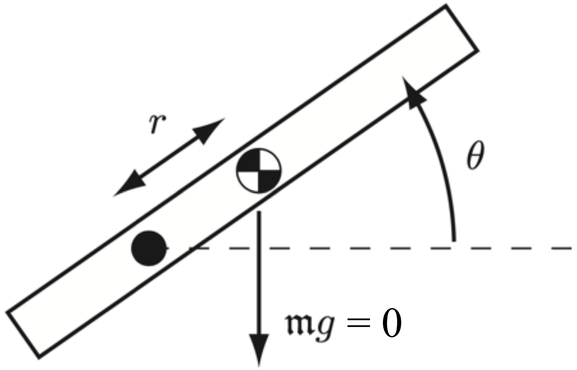
$$\tau = K_p\theta_e + K_i \int \theta_e(\mathrm{t})dt + K_d\dot{\theta}_e$$

Important concepts, symbols, and equations (cont.)



Important concepts, symbols, and equations (cont.)

Setpoint PD control, $g = 0$



$$\tau = K_p\theta_e + \cancel{K_i}^{\,0} \int \theta_e(\mathrm{t})dt + K_d\dot{\theta}_e$$

$$\tau = M\ddot{\theta} + \cancel{\mathfrak{m}g}^{\,0} r\cos\theta + b\dot{\theta}$$

$$\dot{\theta}_d = \ddot{\theta}_d = 0$$

$$M\ddot{\theta} + b\dot{\theta} = K_p\theta_e + K_d\dot{\theta}_e$$

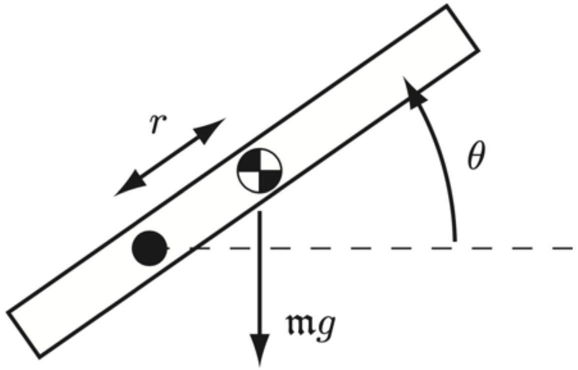
$$\ddot{\theta}_e + \frac{b + K_d}{M}\dot{\theta}_e + \frac{K_p}{M}\theta_e = 0$$

$$\zeta = \frac{b + K_d}{2\sqrt{K_p M}} \qquad \omega_n = \sqrt{\frac{K_p}{M}}$$

For what gains are the error dynamics stable?

Important concepts, symbols, and equations (cont.)

Setpoint PD control, $g \neq 0$



$$\tau = K_p\theta_e + \overset{0}{\cancel{K_i}} \int \theta_e(\mathrm{t})dt + K_d\dot{\theta}_e$$

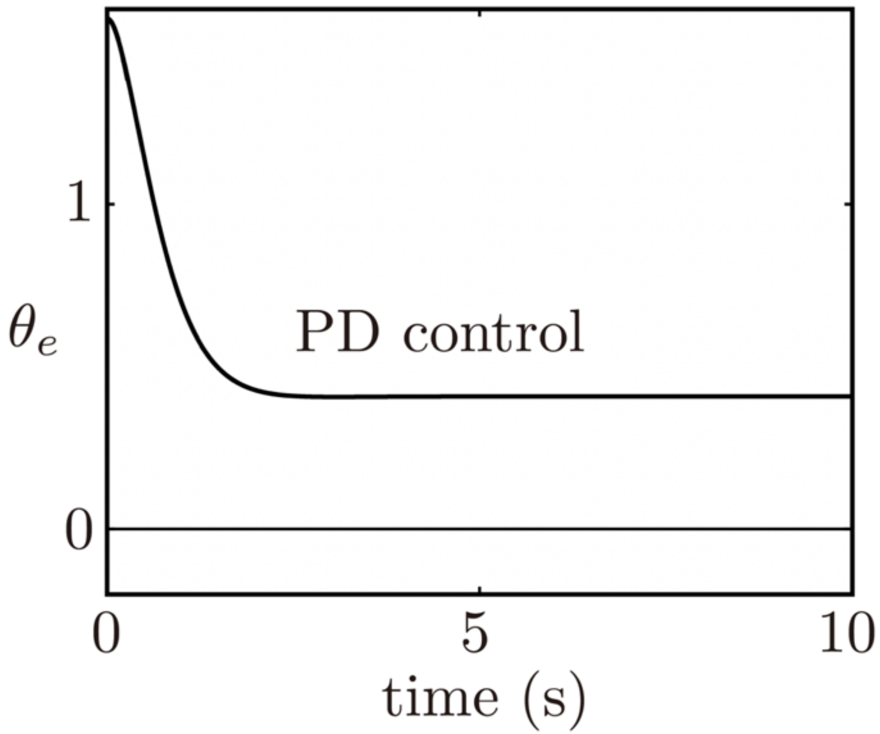
$$\tau = M\ddot{\theta} + \mathfrak{m}gr\cos\theta + b\dot{\theta}$$

$$\dot{\theta}_d = \ddot{\theta}_d = 0$$

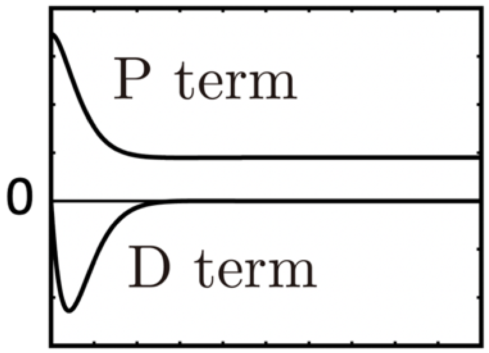
$$M\ddot{\theta}_e + (b + K_d)\dot{\theta}_e + K_p\theta_e = \mathfrak{m}gr\cos\theta$$

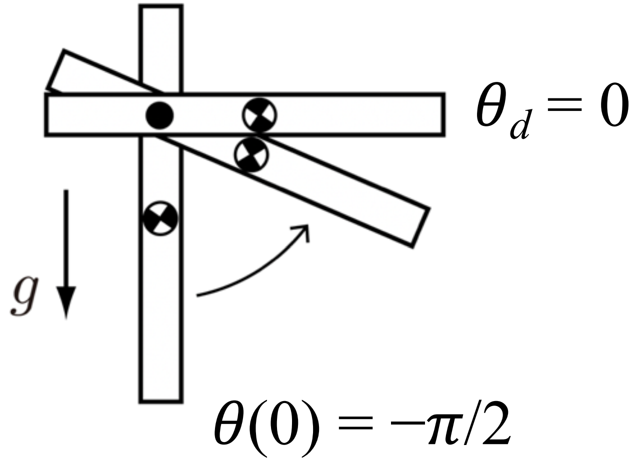
Nonhomogeneous. What is the steady-state error?

Important concepts, symbols, and equations (cont.)



control terms





Important concepts, symbols, and equations (cont.)

Setpoint PID control, $g \neq 0$

$$\tau = K_p\theta_e + K_i \int \theta_e(t)dt + K_d\dot{\theta}_e$$

$$\tau = M\ddot{\theta} + \underbrace{\mathfrak{m}gr\cos\theta}_{\tau_{\text{dist}}} + b\dot{\theta}$$

$$\dot{\theta}_d = \ddot{\theta}_d = 0$$

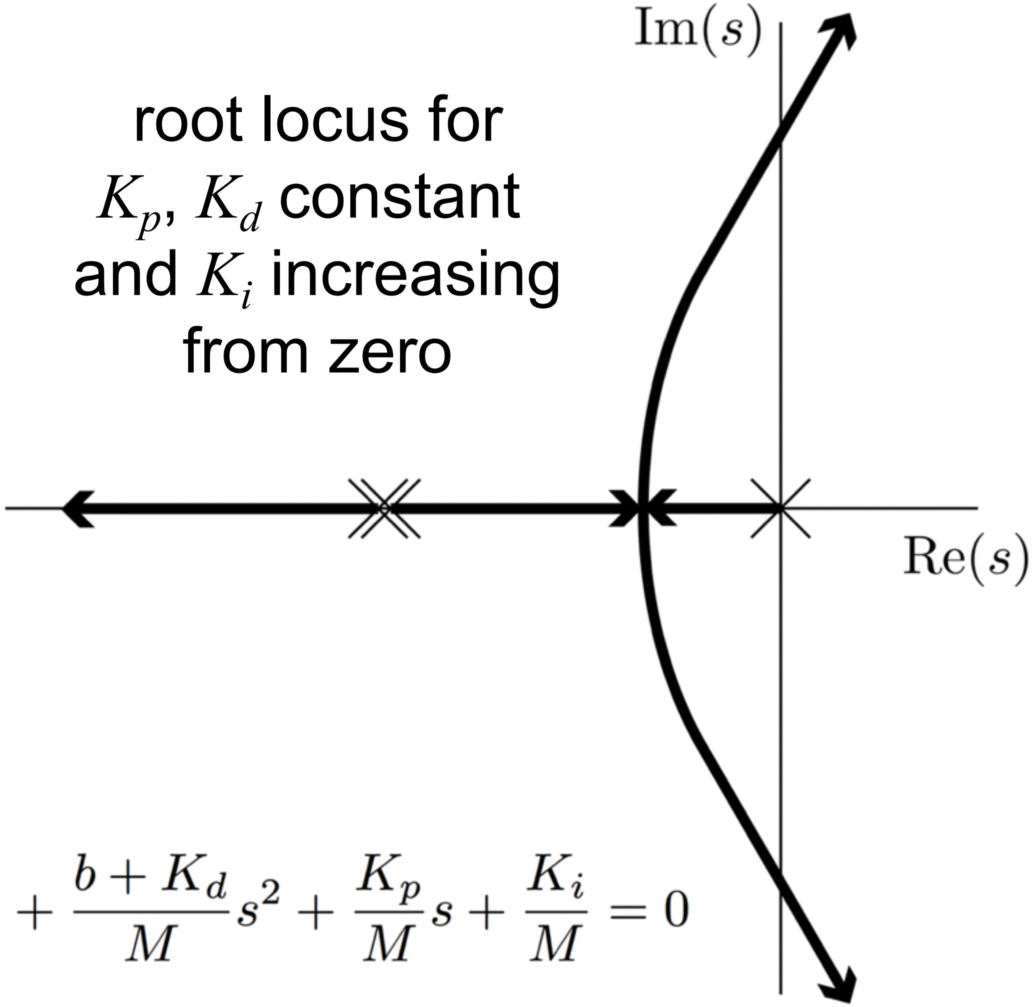
$$M\ddot{\theta}_e + (b + K_d)\dot{\theta}_e + K_p\theta_e + K_i \int \theta_e(t)dt = \tau_{\text{dist}}$$

$$M\theta_e^{(3)} + (b + K_d)\ddot{\theta}_e + K_p\dot{\theta}_e + K_i\theta_e = 0$$

$$s^3 + \frac{b + K_d}{M}s^2 + \frac{K_p}{M}s + \frac{K_i}{M} = 0$$

Important concepts, symbols, and equations (cont.)

root locus for
K_p, K_d constant
and K_i increasing
from zero

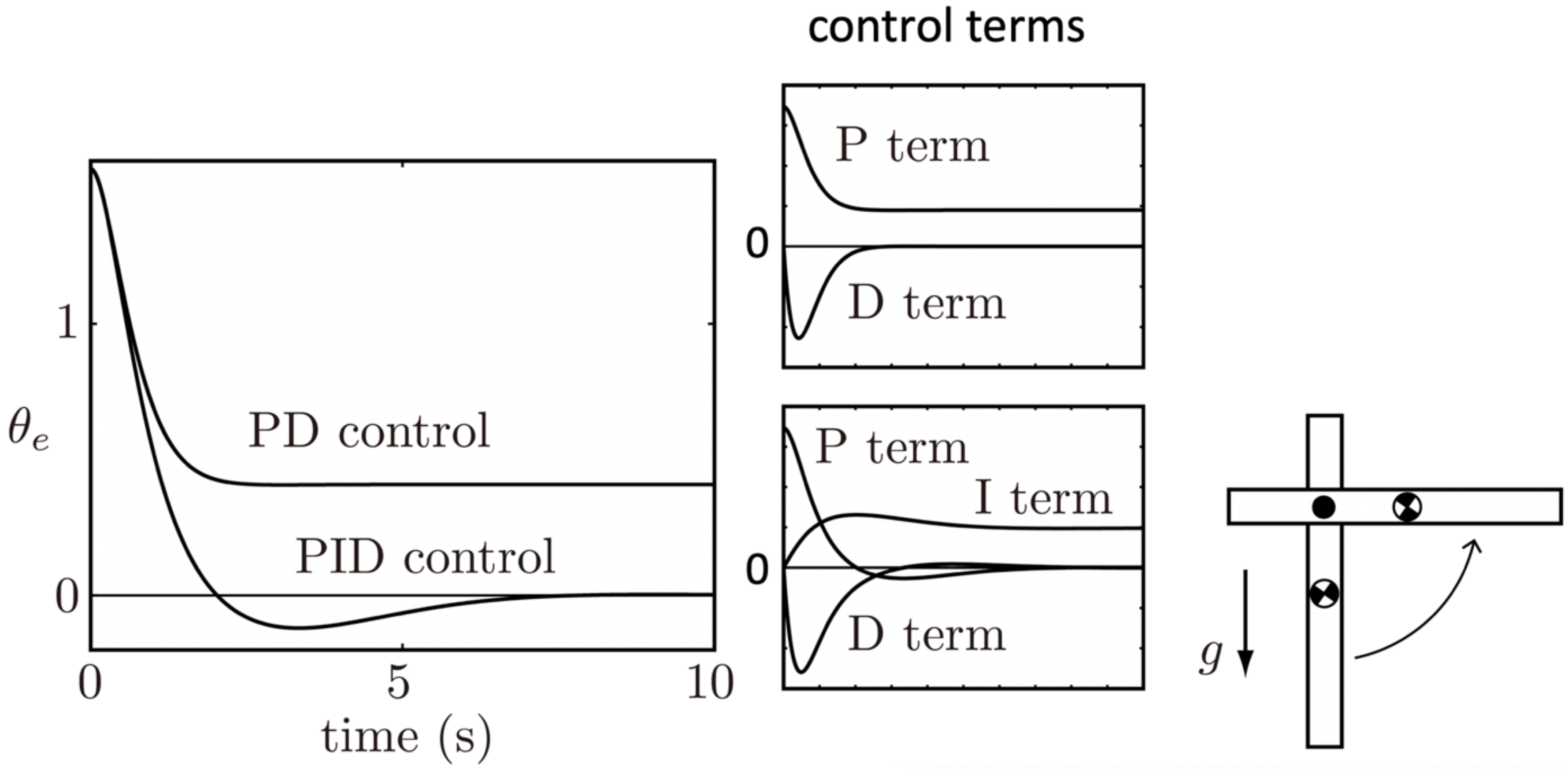


$$s^3 + \frac{b + K_d}{M} s^2 + \frac{K_p}{M} s + \frac{K_i}{M} = 0$$

$$\begin{aligned} K_d &> -b \\ K_p &> 0 \\ \frac{(b+K_d)K_p}{M} > K_i &> 0 \end{aligned}$$

K_i improves steady-state response but can worsen the transient response.

Important concepts, symbols, and equations (cont.)



Important concepts, symbols, and equations (cont.)

What about tracking general trajectories, not just setpoint control?

$$\tau = M\Big(\overbrace{\ddot{\theta}_d + K_p\theta_e + K_i\int \theta_e \mathrm{d}t + K_d\dot{\theta}_e}^{\text{commanded } \ddot{\theta}}\Big) + h(\theta,\dot{\theta})$$

$$\ddot{\theta}_e = \ddot{\theta}_d - \ddot{\theta}$$

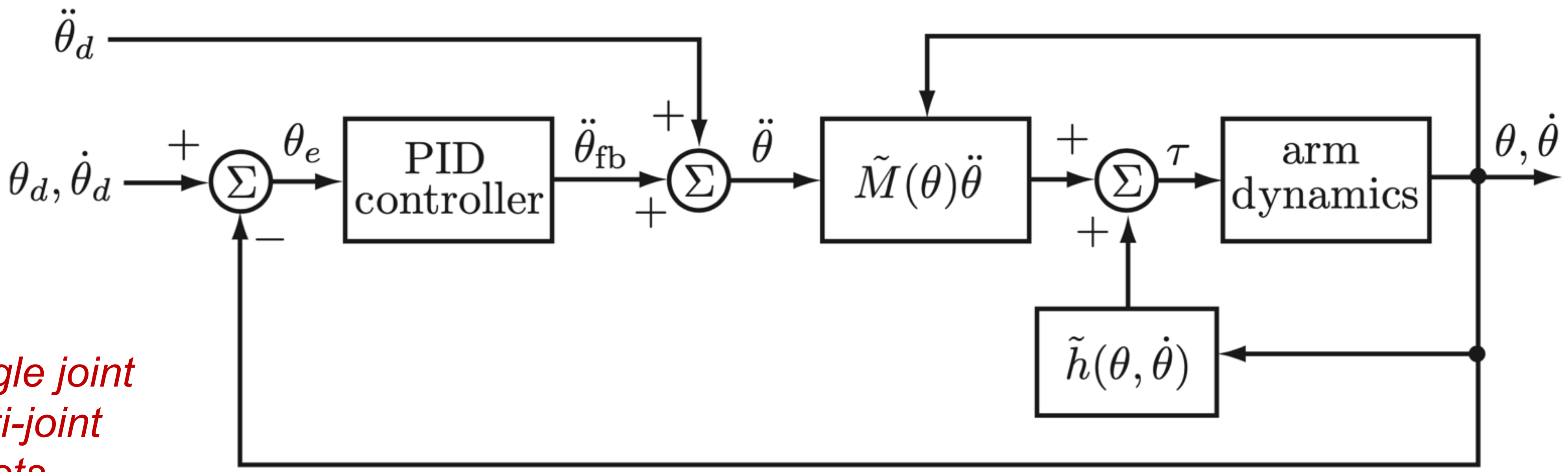
$$\ddot{\theta}_e = -K_d\dot{\theta}_e - K_p\theta_e - K_i\int \theta_e dt$$

$$\theta_e^{(3)} + K_d\ddot{\theta}_e + K_p\dot{\theta}_e + K_i\theta_e = 0$$

Important concepts, symbols, and equations (cont.)

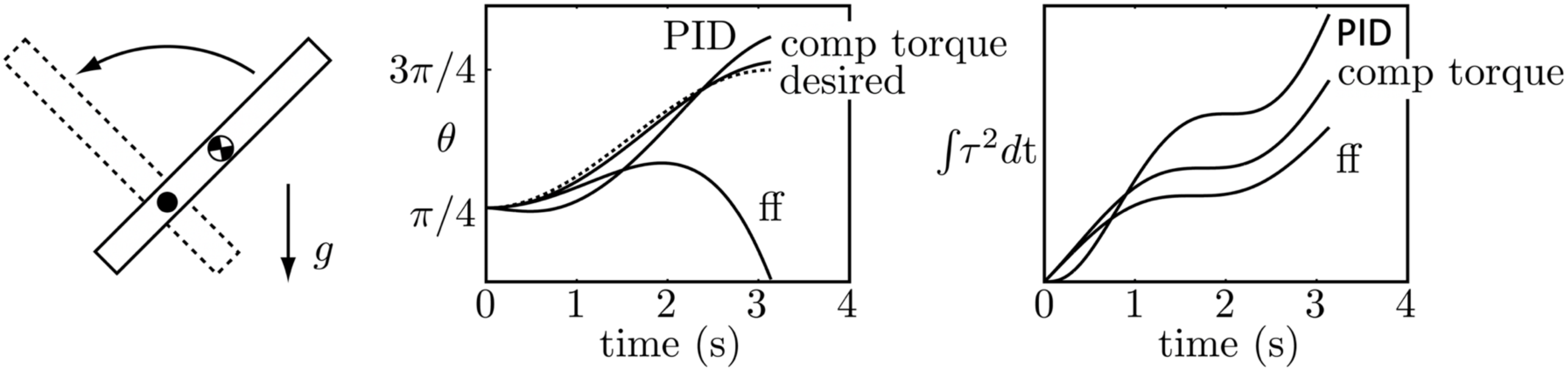
Computed torque control (feedback linearization)

$$\tau = \tilde{M}(\theta)\left(\ddot{\theta}_d + K_p\theta_e + K_i\int\theta_e(\mathrm{t})dt + K_d\dot{\theta}_e\right) + \tilde{h}(\theta,\dot{\theta})$$



for a single joint or multi-joint robots

Important concepts, symbols, and equations (cont.)



Important concepts, symbols, and equations (cont.)

Task-space computed torque control

What if your dynamic model is poor?

$$\mathcal{F}_b = \Lambda(\theta)\dot{\mathcal{V}}_b + \eta(\theta, \mathcal{V}_b)$$

dynamic model: $\{\tilde{\Lambda}, \tilde{\eta}\}$

$$\tau = \tilde{M}(\theta)\left(\ddot{\theta}_d + K_p\theta_e + K_i\int \theta_e \mathrm{d}t + K_d\dot{\theta}_e\right) + \tilde{h}(\theta, \dot{\theta})$$

$$\mathcal{F}_b = \tilde{\Lambda}(\theta)\left(\dot{\mathcal{V}}_d + K_pX_e + K_i\int X_e \mathrm{d}t + K_d\mathcal{V}_e\right) + \tilde{\eta}(\theta, \mathcal{V}_b)$$

$$[X_e] = \log(X^{-1}X_d)$$

$$\mathcal{V}_e = [\mathrm{Ad}_{X^{-1}X_d}]\mathcal{V}_d - \mathcal{V}_b$$

$$\tau = J_b^{\mathrm{T}}(\theta)\mathcal{F}_b$$

The characteristic equation of the error dynamics are

$$s^5 + 2s^4 + s^3 + 2s^2 + 4s + 2 = 0$$

Write the error dynamics in the form $\dot{x} = Ax$. Determine if the system is stable. (Use any software you want.)