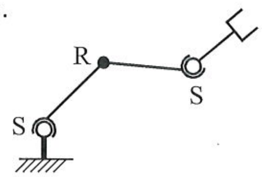
ME 449 Midterm
October 26, 2022

Name: Solutions

Always show your work or reasoning so your thought process is clear! If you need more space for your work, you can use the back side of the previous page. No electronics (phone, watch, tablet, computer, calculator, etc.).

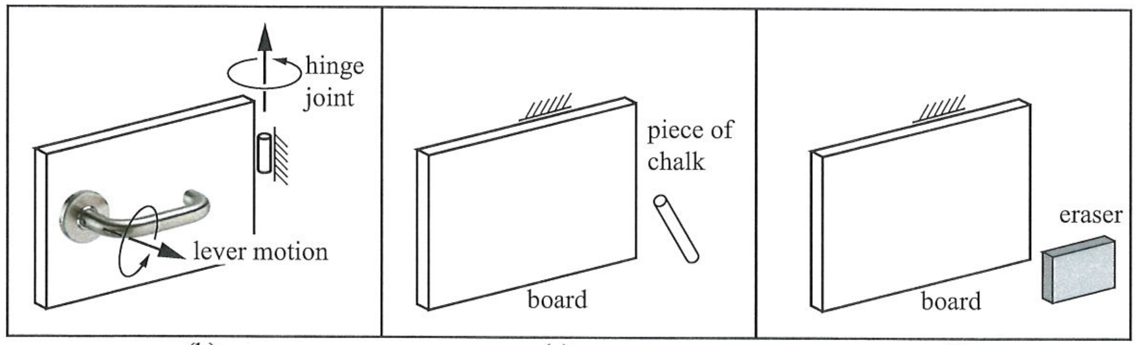
1. **(8 pts)** A grounded SRS robot arm, shown below, is used for a variety of tasks, such as opening a door, writing on a board, and erasing a board. When the robot gripper firmly grabs an object (a door lever, a piece of chalk, or an eraser), the object is fixed (stationary) relative to the last link of the robot.



(a) How many degrees of freedom does the arm have? (Do not count any dof of the gripper that is mounted at the end of the arm.)

$S + R + S \rightarrow 3 + 1 + 3 = 7$ dof

Now we will constrain the arm in various ways. See the figure below for the next three parts of the question.



(b) (c) (d)

(b) The robot firmly grabs a lever door handle. The door rotates about a hinge and the lever rotates about a pivot. After the robot firmly grabs the handle, how many degrees of freedom of motion does the robot-lever-door system have?

The handle has 2 dof, so provides 4 constraints. $7 - 4 = 3$ dof
or, using Grübler's: $N = 1$ (ground) $+ 3$ (robot + lever) $+ 1$ (door) $= 5$
$J = 3 + 1 + 1 = 5$ $\quad \Sigma f_i = 3 + 1 + 3 + 1 + 1 = 9$ $\quad$ dof $= 6(5 - 1 - 5) + 9 = 3$

(c) Now the robot grabs a piece of chalk and presses the tip against a chalkboard. While constrained to keep the chalk pressed against the blackboard, how many degrees of freedom does the robot-chalk-board system have?

distance from chalk to board $= 0$ is 1 constraint. $7 - 1 = 6$ dof
or: $N = 4$, $J = 4$, $\Sigma f_i = 12$, dof $= 6(4 - 1 - 4) + 12 = 6$

(d) Now the robot grabs an eraser and presses it flat against the chalkboard. While constrained to keep the eraser flat against the blackboard, how many degrees of freedom does the robot-eraser-board system have?

eraser-board contact gives 3 constraints: distance from board $= 0$ and no rotation about 2 axes in plane of board. $7 - 3 = 4$ dof
or: $N = 4$, $J = 4$, $\Sigma f_i = 10$, dof $= 6(4 - 1 - 4) + 10 = 4$

2. **(4 pts)** Consider a world frame {a}, a frame fixed to an object {b}, and a frame fixed to the end-effector of a robot arm {c}. The configuration of {b} relative to {a} is T_1 and the configuration of {c} relative to {a} is T_2.

   (a) Express T_{cb} in terms of T_1 and T_2.

   $$T_{cb} = T_{c\cancel{a}} T_{\cancel{a}b} = T_2^{-1} T_1$$

   (b) Give the $se(3)$ form of the twist, expressed in the {c} frame, that would move the end-effector frame {c} to the object {b} in one unit of time.

   $$[\mathcal{V}_c] = \log T_{cb} = \log(T_2^{-1} T_1)$$

3. **(6 pts)** Consider two frames, a stationary frame {a} and a frame at {b} attached to a rigid body. The configuration of {b} relative to {a} is given by T_{ab}. The rigid body moves, following a twist $\mathcal{V}_b$ for a time t. Then it moves again, following a screw axis $\mathcal{S}_a$ a distance θ. The frame attached to the rigid body is now at {b'}.

   (a) Using matrix logs and exponentials as appropriate, give the $se(3)$ form of the twist $[\mathcal{V}_a]$ (i.e., represented in the {a} frame) that moves {b} to {b'} in T units of time.

   $$(*) \quad T_{ab'} = \underbrace{\exp([S_a]\theta)}_{\text{motion 2}} T_{ab} \underbrace{\exp([V_b]t)}_{\text{motion 1}}$$

   also, $T_{ab'} = \exp([V_a]T)\, T_{ab}$, where we are trying to solve for $[V_a]$

   $$T_{ab'} T_{ab}^{-1} = \exp([V_a]T)$$

   $\frac{1}{T} \log(T_{ab'} T_{ab}^{-1}) = [V_a] \longrightarrow$ and plug in $T_{ab'}$ from $(*)$

   (b) If $[\mathcal{V}_a]$ is represented as

   $$[\mathcal{V}_a] = \begin{bmatrix} 0 & d & e & a \\ -d & 0 & f & b \\ -e & -f & 0 & c \\ 0 & 0 & 0 & 0 \end{bmatrix} \in se(3),$$

   write the corresponding screw axis $\mathcal{S}_a \in \mathbb{R}^6$.

   $$\mathcal{V}_a = \begin{bmatrix} -f \\ e \\ -d \\ a \\ b \\ c \end{bmatrix} \qquad \text{If } \begin{bmatrix} f \\ e \\ d \end{bmatrix} \neq 0, \text{ then } S_a = \frac{1}{\sqrt{d^2+e^2+f^2}} \begin{bmatrix} -f \\ e \\ -d \\ a \\ b \\ c \end{bmatrix}$$

   $$\text{else } S_a = \frac{1}{\sqrt{a^2+b^2+c^2}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ a \\ b \\ c \end{bmatrix}$$

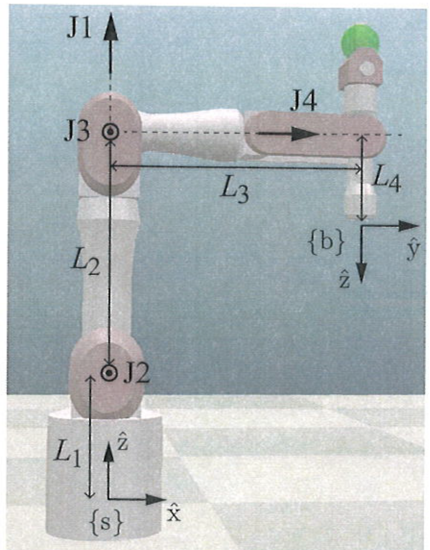
4. **(4 pts)** A robot's end-effector is moving with a twist $\mathcal{V}_c$ (expressed in a frame {c}) while applying a wrench $\mathcal{F}_d$ (expressed in a frame {d}). What is the power generated (or absorbed) by the end-effector?

   $$\mathcal{V}_d = [Ad_{T_{dc}}]\, \mathcal{V}_c \qquad \text{power} = \mathcal{F}_d^T \mathcal{V}_d = \mathcal{V}_d^T \mathcal{F}_d = \mathcal{F}_c^T \mathcal{V}_c = \mathcal{V}_c^T \mathcal{F}_c$$

   $$\mathcal{F}_c = [Ad_{T_{dc}}]^T \mathcal{F}_d$$

5. (**10 pts**) Consider the 4R robot arm below, shown at its home configuration, $\theta = 0$. The axes of the four revolute joints, J1–J4, are illustrated. J1 points up on the page, J2 and J3 are out of the page, and J4 points to the right.

$\hat{y}$-axis into page



$\hat{x}$-axis into page

(a) Give $T_{sb}(0)$.

$$T_{sb}(0) = \begin{bmatrix} 0 & 1 & 0 & L_3 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & L_1+L_2-L_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) Write the body Jacobian $J_b(0)$.

$$J_b(0) = \begin{bmatrix} 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ L_3 & 0 & 0 & L_4 \\ 0 & L_4-L_2 & L_4 & 0 \\ 0 & -L_3 & -L_3 & 0 \end{bmatrix}$$

(c) At this configuration, what is the dimension of the space of wrenches applied to the end-effector that require no torques by the robot joints to resist?

rank$(J_b(0)) = 4$, so there are 2 directions the robot cannot move, so wrenches can be applied in 2 directions without requiring torques to resist.

(d) If a wrench $\mathcal{F}_b = (m_x, m_y, m_z, f_x, f_y, f_z)$ is applied to the end-effector, and the robot applies joint torques to resist it, what torque τ_1 is being applied by the motor at joint 1?

$\tau = J_b^T(0)(-\mathcal{F}_b)$ → only need to look at first row of $J_b^T(0)$

$$\tau_1 = [0, 0, -1, L_3, 0, 0] \begin{bmatrix} -m_x \\ -m_y \\ -m_z \\ -f_x \\ -f_y \\ -f_z \end{bmatrix} = m_z - L_3 f_x$$