Where we are:

Important concepts, symbols, and equations

For a single joint with the joint velocity as the control:

- **Open-loop (feedforward) control**: $\dot{\theta}(t) = \dot{\theta}_d(t)$
- **Closed-loop (feedback) control**: $\dot{\theta}(t) = f(\theta_d(t), \theta(t))$
- **FF + Proportional-Integral (PI) FB control**:

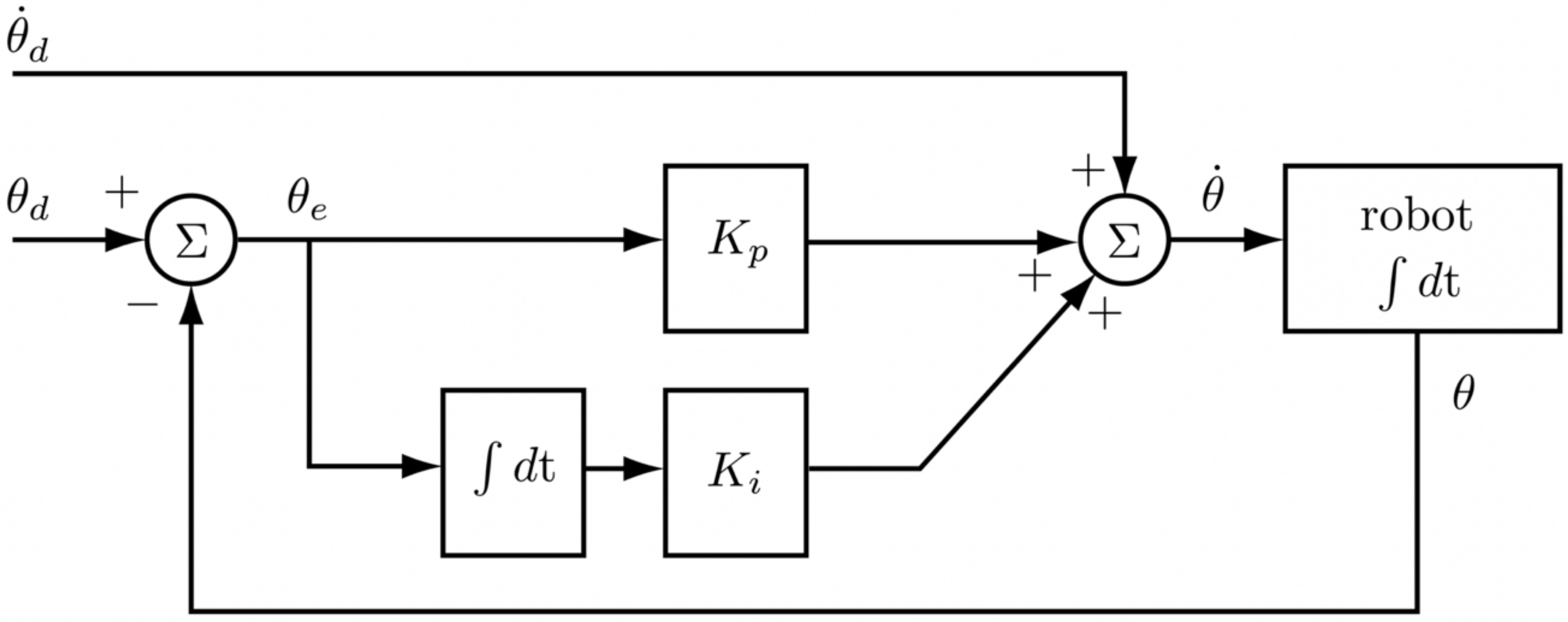
$$\dot{\theta}(t) = \dot{\theta}_d(t) + K_p \theta_e(t) + K_i \int_0^t \theta_e(\mathrm{t})\, d\mathrm{t}\ , \ K_p, K_i \geq 0$$

What is the point of FF control in this control law?

- *reduces to FF control if* $K_p, K_i = 0$
- *if no FF term:* **P control** *when* $K_i = 0$, **I control** *when* $K_p = 0$

Important concepts, symbols, and equations (cont.)

Block diagram



$$\dot{\theta}(t) = \dot{\theta}_d(t) + K_p\theta_e(t) + K_i \int_0^t \theta_e(\mathrm{t})\, dt$$

Important concepts, symbols, and equations (cont.)

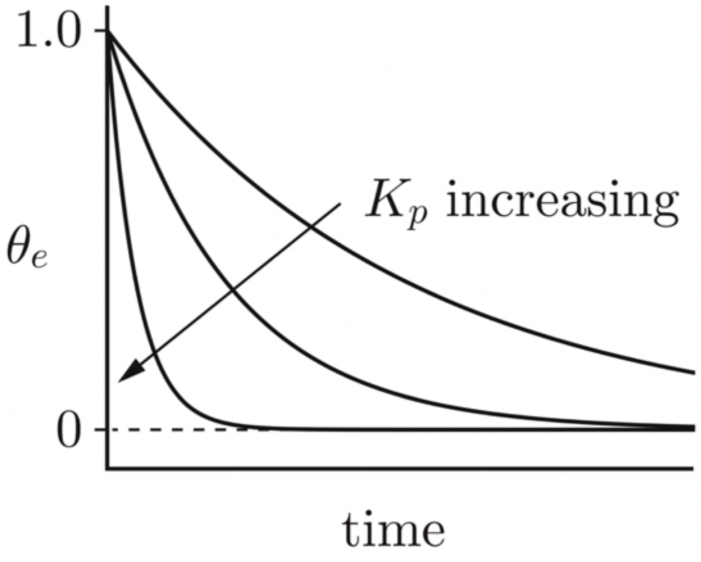
Setpoint control, $\theta_d(t) = c$, with a P controller

$$\dot{\theta}_e(t) = \dot{\theta}_d(t) - \dot{\theta}(t), \quad \dot{\theta}_d(t) \to 0$$

$$\dot{\theta}_e(t) = -K_p\theta_e(t) \quad \rightarrow \quad \dot{\theta}_e(t) + K_p\theta_e(t) = 0$$

$$\mathfrak{t} = 1/K_p$$

$$\theta_e(t) = e^{-t/\mathfrak{t}}\theta_e(0)$$



Important concepts, symbols, and equations (cont.)

Constant velocity control, $\theta_d(t) = ct + a$

P control

$$\dot{\theta}_e(t) = \dot{\theta}_d(t) - \dot{\theta}(t) = c - K_p\theta_e(t)$$

$$\dot{\theta}_e(t) + K_p\theta_e(t) = c$$

$$\theta_e(t) = \frac{c}{K_p} + \left(\theta_e(0) - \frac{c}{K_p}\right) e^{-K_p t}$$

PI control

$$\dot{\theta}(t) = K_p\theta_e(t) + K_i \int_0^t \theta_e(\mathrm{t})\, d\mathrm{t}$$

$$\dot{\theta}_e(t) + K_p\theta_e(t) + K_i \int_0^t \theta_e(\mathrm{t})\, d\mathrm{t} = c$$

$$\ddot{\theta}_e(t) + K_p\dot{\theta}_e(t) + K_i\theta_e(t) = 0$$

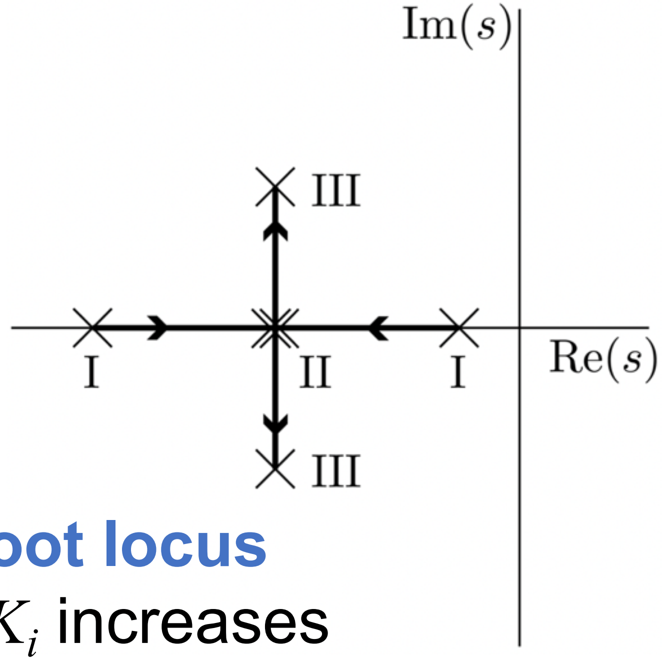
$$\zeta = K_p/(2\sqrt{K_i})$$

$$\omega_n = \sqrt{K_i}$$

Important concepts, symbols, and equations (cont.)

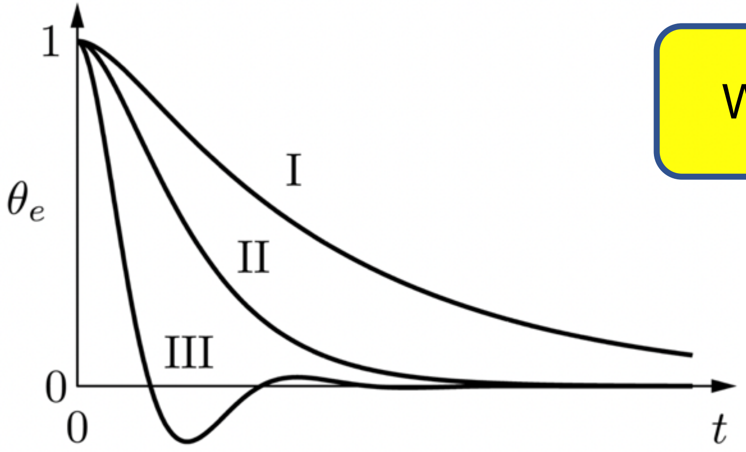
characteristic equation of PI error dynamics: $s^2 + K_p s + K_i = 0$

$$s_{1,2} = -\frac{K_p}{2} \pm \sqrt{\frac{K_p^2}{4} - K_i}$$



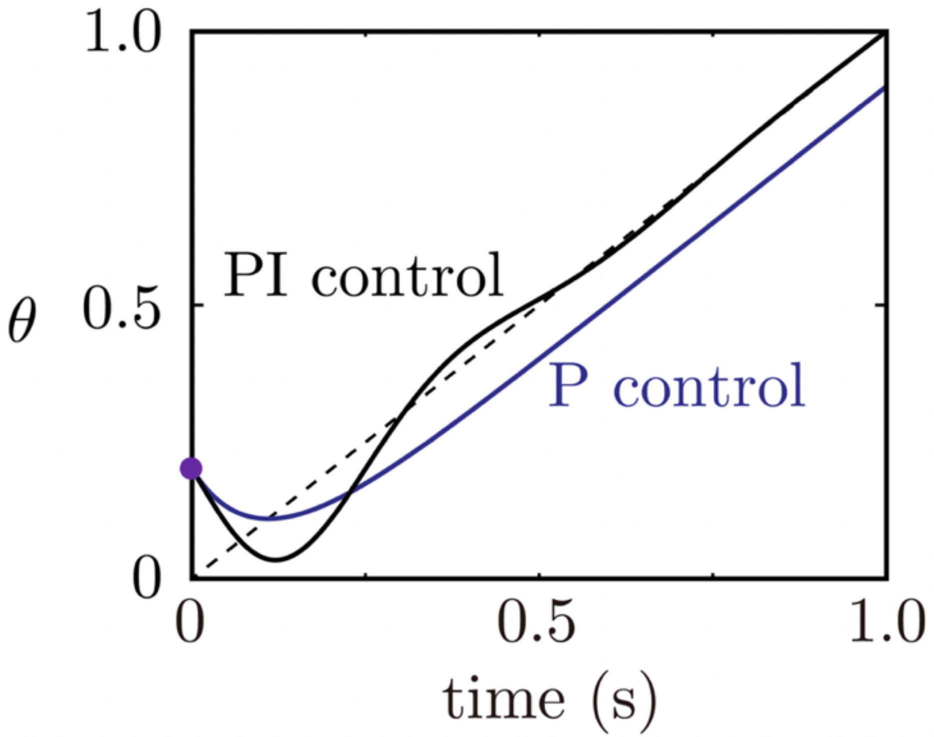
root locus

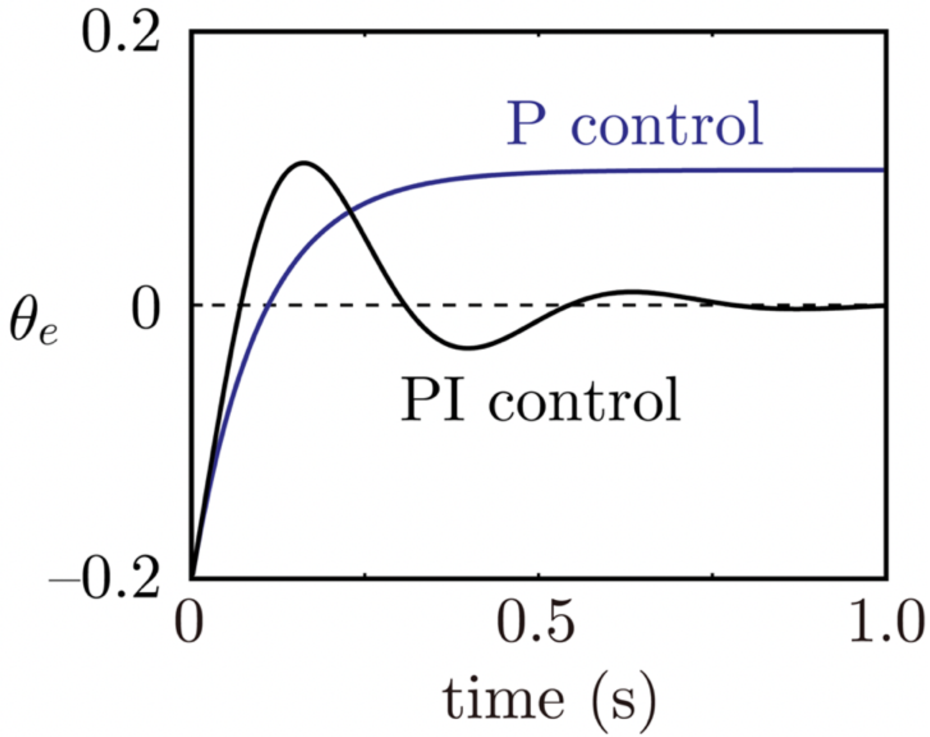
as K_i increases



What if $K_i = 0$?

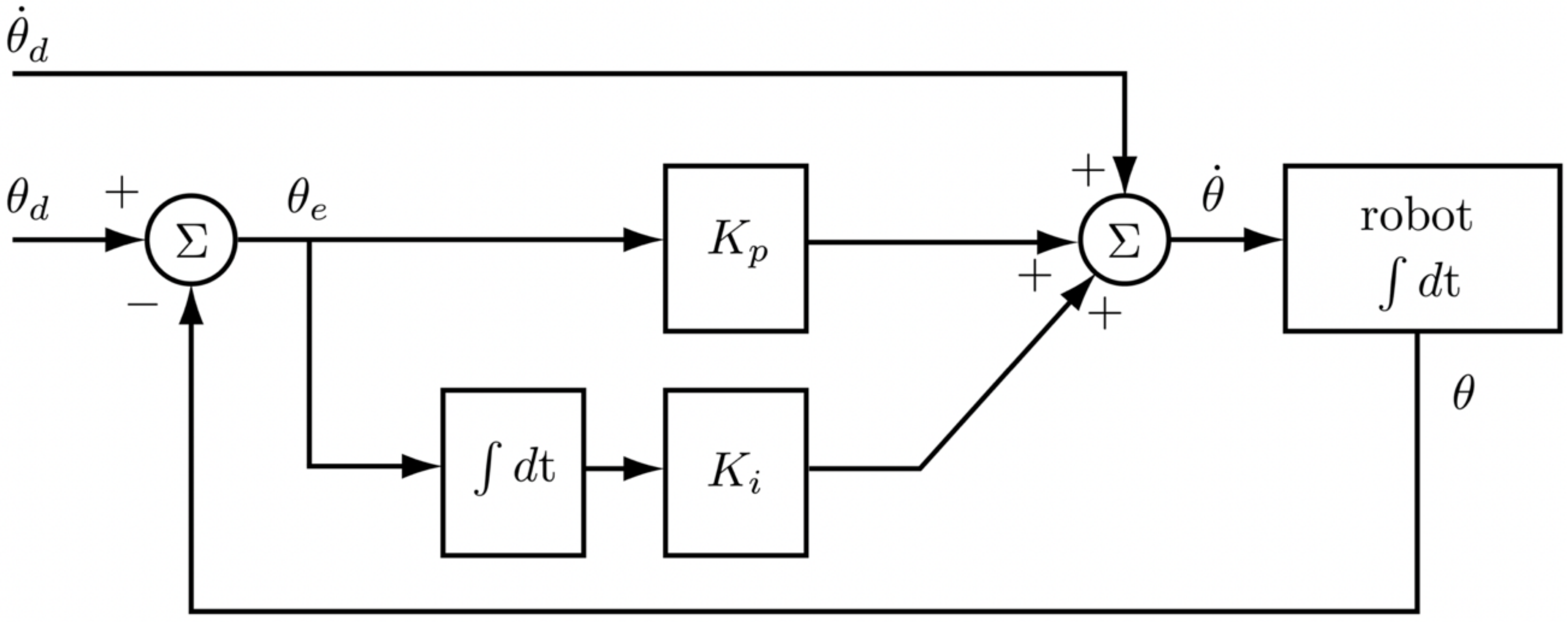
Important concepts, symbols, and equations (cont.)





Important concepts, symbols, and equations (cont.)

Multi-joint control: θ, θ_d, and θ_e are vectors and $K_p = k_p I$, $K_i = k_i I$



$$\dot{\theta}(t) = \dot{\theta}_d(t) + K_p\theta_e(t) + K_i \int_0^t \theta_e(\mathrm{t})\, dt$$

Important concepts, symbols, and equations (cont.)

$$\dot{\theta}(t) = \dot{\theta}_d(t) + K_p\theta_e(t) + K_i \int_0^t \theta_e(\mathrm{t})\ dt$$

Task-space motion control

desired motion

$$X_d \in SE(3)$$

$$[\mathcal{V}_d] = X_d^{-1}\dot{X}_d$$

actual motion

$$X \in SE(3)$$

$$[\mathcal{V}_b] = X^{-1}\dot{X}$$

error: $[X_e] = \log X_{bd} = \log(X^{-1}X_d)$

$$\mathcal{V}_b(t) = [\mathrm{Ad}_{X_{bd}}]\mathcal{V}_d(t) + K_pX_e(t) + K_i \int_0^t X_e(\mathrm{t})dt$$

$$\dot{\theta} = J_b^{\dagger}(\theta)\mathcal{V}_b$$

Important concepts, symbols, and equations (cont.)

Decoupled task-space motion control

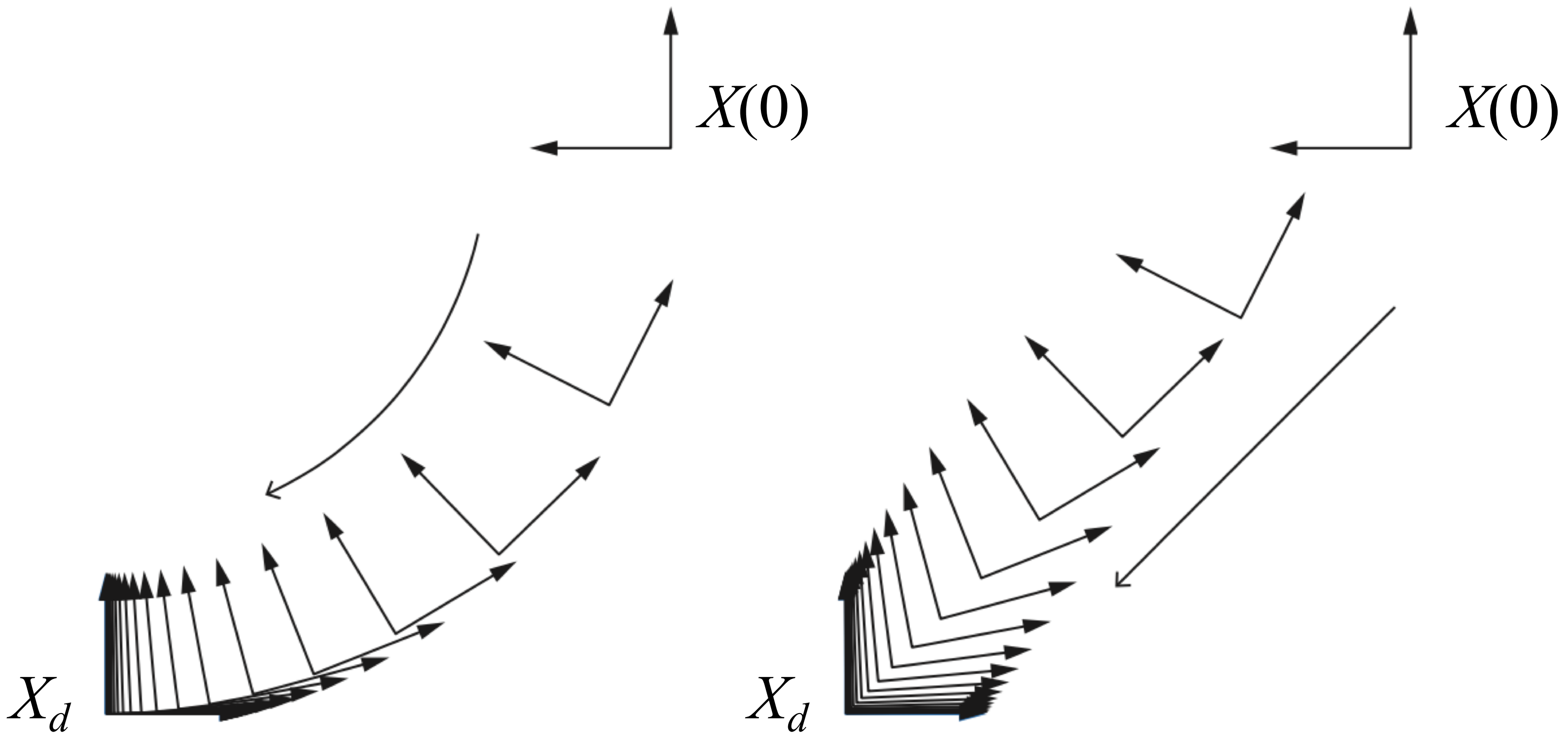
$$X_e(t) = \begin{bmatrix} \omega_e(t) \\ p_d(t) - p(t) \end{bmatrix} \qquad [\omega_e] = \log(R^{\mathrm{T}} R_d)$$

$$\begin{bmatrix} \omega_b(t) \\ \dot{p}(t) \end{bmatrix} = \begin{bmatrix} R^{\mathrm{T}}(t) R_d(t) & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \omega_d(t) \\ \dot{p}_d(t) \end{bmatrix} + K_p X_e(t) + K_i \int_0^t X_e(\mathrm{t})\, dt$$

$$\dot{\theta} = J^{\dagger}(\theta) \begin{bmatrix} \omega_b(t) \\ \dot{p}(t) \end{bmatrix}$$

What is this Jacobian?

Important concepts, symbols, and equations (cont.)



task-space P controller

decoupled task-space P controller

You are designing a P controller to track a joint reference trajectory that is moving at a constant rate of 3 radians/s. What is the smallest gain K_p that ensures a steady-state position error of no more than 0.1 radians? Give units.

To eliminate steady-state error, you decide to use a PI controller. What gains K_p and K_i should you choose to achieve critical damping and a settling time of 0.1 s? Give units.

Explain how to estimate the error integral if the controller's frequency is $1/T$.

Why not choose arbitrarily large K_p and K_i to achieve arbitrarily fast settling?

How well would a PI controller track a quadratic joint trajectory? (Not a ramp.)