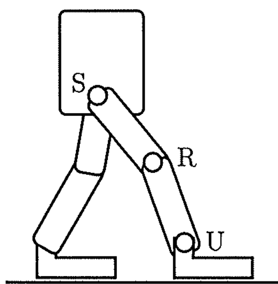
ME 449 Midterm
October 25, 2023

Name: Solutions

Always show your work or reasoning so your thought process is clear! If you need more space for your work, you can use the back side of the previous page. No electronics (phone, watch, tablet, computer, calculator, etc.).

1. **(8 pts)** The figure below shows a spatial (three-dimensional) biped walking robot with a rigid-body torso and two legs. Each leg has three joints: a hip (S) connecting to the torso, knee (R), and ankle (U). When a foot is in contact with the ground, it is "flat footed" (the entire flat bottom surface of the foot is in contact with the flat ground). The image shows two feet in flat-foot contact with the ground.



(a) If two feet are in contact with the ground, as shown in the image, and both feet are fixed to the ground (cannot move relative to the ground), how many degrees of freedom does the system have?

$N=6$: ground/feet, $2\times$(2 leg links), torso $J=6$ joints)

$$\text{dof} = 6(N-1-J) + \sum f_i = 6(6-1-6) + 2\times(2+1+3)$$
$$= -6+12 = 6 \text{ dof}$$

(b) Now assume one foot is fixed to the ground, but the other foot is allowed to slide relative to the ground (but it still must remain flat in contact with the ground). How many degrees of freedom does the system have?

$N=7$, $J=7$ (add 3 dof joint at sliding foot)

$$\text{dof} = 6(7-1-7) + 3 + 12 = 9 \text{ dof}$$

(c) Now one foot is fixed to the ground and the other is lifted off the ground as the robot takes a step. How many degrees of freedom does the system have?

$N=7$, $J=6$

$$\text{dof} = 6(7-1-6) + 12 = 12 \text{ dof}$$

(d) The robot has jumped into the air, so both feet are in free space. How many degrees of freedom does the system have?

$N=8$, $J=6$ $\quad \text{dof} = 6(8-1-6)+12 = 18 \text{ dof}$

2. **(6 pts)** Four frames are defined: a world frame {s} fixed to the floor of a room; a frame {0} attached to the chassis of a wheeled robot that rolls around the room; a frame {b} at the end-effector of a robot arm mounted to the wheeled robot; and a frame {d} attached to an object that the mobile robot wants to grasp. The transformation matrix T_1 defines the initial configuration of {0} in the {s} frame, T_2 represents the initial configuration of {b} in the {0} frame, and T_3 represents the configuration of {d} in the {s} frame.

First the chassis of the wheeled robot follows a twist $\mathcal{V}_0$ expressed in the {0} frame for t_1 seconds. (The robot's arm does not move relative to the chassis during this time.) Then the wheeled base stops, and the robot arm reaches to align {b} with {d} by moving the arm along a twist $\mathcal{V}_b$ (expressed in the {b} frame) for t_2 seconds.

Write $[\mathcal{V}_b]$ in terms of T_1, T_2, T_3, t_1, t_2, and $\mathcal{V}_0$.

$$T_1 = T_{so_1} \qquad T_2 = T_{ob_1}, \qquad T_3 = T_{sd}$$

$$T_3 = T_1 \exp([\mathcal{V}_0 t_1])\, T_2 \exp([\mathcal{V}_b t_2])$$

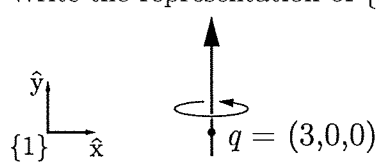
$$(T_1 \exp([\mathcal{V}_0 t_1])\, T_2)^{-1}\, T_3 = \exp([\mathcal{V}_b t_2])$$

$$\frac{1}{t_2} \log\left((T_1 \exp([\mathcal{V}_0 t_1])\, T_2)^{-1}\, T_3\right) = [\mathcal{V}_b]$$

$$\text{or} \quad \frac{1}{t_2} \log\left(T_2^{-1} \exp([-\mathcal{V}_0 t_1])\, T_1^{-1}\, T_3\right) = [\mathcal{V}_b]$$

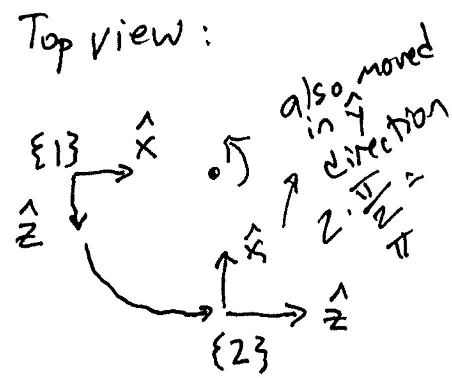
3. **(6 pts)** The figure below shows a frame {1} and a screw axis with pitch $h = 2$. The screw axis is aligned with the $\hat{y}$-axis of {1} and passes through a point $q = (3, 0, 0)$ in the {1} frame. Imagine a frame initially coincident with {1} that then follows the screw a distance $\pi/2$, ending up at a new frame location {2}.

(a) Write the representation of {2} relative to {1} as $T_{12} \in SE(3)$.



$$T_{12} = \begin{bmatrix} 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & \pi \\ -1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

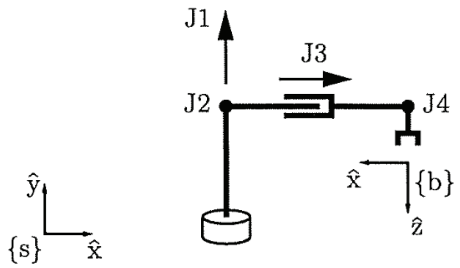
Top view :



(b) Give the exponential coordinate representation of T_{12}.

$$\mathcal{S}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 2 \\ 3 \end{bmatrix} \qquad \theta = \frac{\pi}{2} \qquad \mathcal{S}_1\theta = \begin{bmatrix} 0 \\ \pi/2 \\ 0 \\ 0 \\ \pi \\ 3\pi/2 \end{bmatrix}$$

4. **(12 pts)** Consider the RRPR robot arm below, shown at its home configuration $\theta = 0$. The J1 axis points up on the page, the J3 axis points to the right, and the J2 and J4 axes point out of the page. At the home configuration, the $\hat{x}$-$\hat{y}$ plane of the {s} frame is coincident with the $\hat{x}$-$\hat{z}$ plane of the {b} frame; the origin of {b} is at $(6, 1, 0)$ in {s}; the J1 and J2 axes pass through $(3, 2, 0)$ in {s}; and the J4 axis passes through $(6, 2, 0)$ in {s}.



(a) Give $M = T_{sb}(0)$.

$$M = \begin{bmatrix} -1 & 0 & 0 & 6 \\ 0 & 0 & -1 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) Write the numerical space Jacobian $J_s(0)$.

$$J_s(0) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 2 & 1 & 2 \\ 0 & -3 & 0 & -6 \\ 3 & 0 & 0 & 0 \end{bmatrix}$$

(c) A wrench $\mathcal{F}_s = (0, 0, 0, 5, 2, 0)^{\mathrm{T}}$ (expressed in the {s} frame) is applied to the end-effector at $\theta = 0$. What τ (joint forces/torques) must be applied by the robot joints to resist motion?

$$\tau = -J_s^{\mathrm{T}} \mathcal{F}_s = -\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & 2 & -3 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & -6 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 5 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -4 \\ -5 \\ 2 \end{bmatrix}$$

(d) Assuming no joint limits, either describe a joint configuration θ where $J_s(\theta)$ is singular, or indicate that there are no singularities for this robot. Explain your answer. If you give a singularity, give the rank of the Jacobian at that configuration.

Choose $\theta_3 = -3$ where J_2 & J_4 are the same. Screws provided by J_1, $\{J_2, J_4\}$, and J_3 are independent, so rank = 3.

(e) For the task of making the origin of the {b} frame follow paths in three-dimensional space (expressed as (x_s, y_s, z_s) as a function of time), is the robot kinematically deficient, redundant, or neither? Explain.

Redundant. Only 3 dof for the task, which can be accomplished by J1, J2, J3 (for example) keeping J4 fixed at any angle.