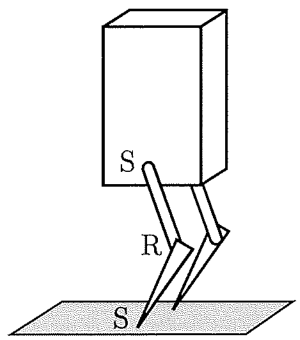
ME 449 Quiz 1
October 18, 2019

Name: Solutions

Always show your work or reasoning so your thought process is clear! If you need more space for your work, you can use the back side of the previous page.

1. A spatial bipedal humanoid robot consists of a rigid-body torso and two legs. Each leg has two links: an upper link connected to the torso by a spherical joint and a lower link connected to the upper link by a revolute joint. Each "foot" is a point. When a foot is in contact with the ground, the point remains stationary (it doesn't slide), so the contact can be modeled as a spherical joint.



Using Grübler's formula, determine:

(a) The number of degrees of freedom of the humanoid when two feet are touching the ground.

$N = 6$ (torso, 4 leg links, ground)

$J = 6$ (4 S, 2 R)

$\Sigma f_i = 14$ $(4 \times 3 + 2 \times 1)$

$dof = m(N-1-J) + \Sigma f_i = 6(6-1-6) + 14 = 8$

(b) The number of degrees of freedom when one foot is touching the ground and the other is in the air.

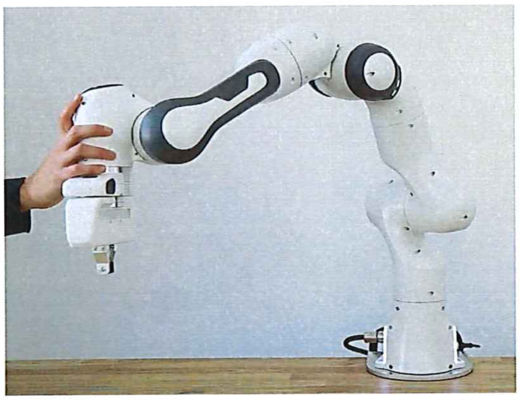
$N = 6$

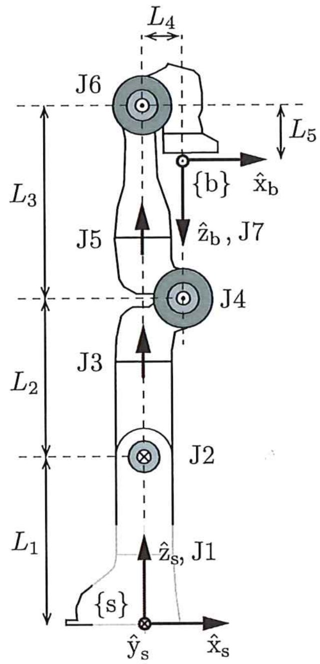
$J = 5$

$\Sigma f_i = 11$

$dof = m(N-1-J) + \Sigma f_i = 6(6-1-5) + 11 = 11$

2. The 7R Panda robot arm, made by Franka Emika, is an increasingly popular lightweight robot arm appropriate for human interaction. The figure on the right below shows the Panda at its home configuration. As shown in the figure, at the home configuration the origins of the {s} and {b} frames are both in the plane of the page. The axes of positive rotation of joints 1, 3, and 5 (J1, J3, and J5) are all coincident, pointing in the $+\hat{z}_s$ direction. (Positive rotation about all axes is by the right-hand rule, as always.) The axis of positive rotation of J7 is in the $+\hat{z}_b$ direction. The axis of positive rotation of J2 is into the page, while the axes of positive rotation of J4 and J6 are out of the page. The figure shows the five distances $(L_1, L_2, L_3, L_4, L_5)$ needed to determine the kinematics of the Panda.





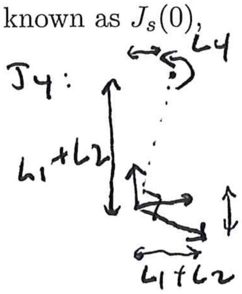
(a) Write the matrix $M = T_{sb}(0)$ describing the configuration of {b} relative to {s} when the arm is at its home configuration.

$$M = \begin{bmatrix} 1 & 0 & 0 & L_4 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & L_1+L_2+L_3-L_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) Write the 6×7 matrix whose columns are $\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_7$, the joint screw axes in the {s} frame when the robot is at its home configuration. (This matrix is also known as $J_s(0)$, the space Jacobian when the robot is at its home configuration.)

$$\begin{array}{c} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & -L_1 & 0 & L_1+L_2 & 0 & L_1+L_2+L_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & L_4 \\ 0 & 0 & 0 & -L_4 & 0 & 0 & 0 \end{bmatrix} \\ \begin{array}{ccccccc} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 \end{array} \end{array}$$

for J_4:



v or $-\omega \times q = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \times \begin{bmatrix} L_4 \\ 0 \\ L_1+L_2 \end{bmatrix} = \begin{bmatrix} L_1+L_2 \\ 0 \\ -L_4 \end{bmatrix}$

3. A rigid body has a frame {2} attached to it, which is initially coincident with a stationary frame {1}, i.e., $T_{12} = I$. The rigid body then follows a twist $\mathcal{V}_1$ for time $t = 0.5$ seconds, so that its frame moves to {2'}, represented as $T_{12'} \neq I$. If

$$[\mathcal{V}_1] = \begin{bmatrix} 0 & -2 & 0 & -4 \\ 2 & 0 & 2 & 8 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \in se(3),$$

what are the exponential coordinates corresponding to $T_{12'}$? Your answer should be a numerical six-vector.

$$[\mathcal{S}\theta] = [\mathcal{V}_1 t] = \begin{bmatrix} 0 & -1 & 0 & -2 \\ 1 & 0 & 1 & 4 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow[\omega_x, \omega_y, \omega_z, v_x, v_y, v_z]{\text{extract}} \begin{bmatrix} -1 \\ 0 \\ 1 \\ -2 \\ 4 \\ 0 \end{bmatrix} = \mathcal{S}\theta$$

4. You are given three known frames, {1}, {2}, and {3}. If frame {2} follows a twist V for t seconds, it moves to coincide with frame {3}. (The twist V could be represented in any of the three frames, as $\mathcal{V}_1$, $\mathcal{V}_2$, or $\mathcal{V}_3$.) If the frame {2} instead follows this same twist V for one second, it ends up at {2'}. Write $T_{12'}$. Your answer can use any of the known transformation matrices T_{ab} (where a and b can each be 1, 2, or 3) and any appropriate mathematical operations on them.

$$[\mathcal{V}_2 t] = \log T_{23} \rightarrow [\mathcal{V}_2] = \frac{1}{t} \log T_{23}$$

so $\exp[\mathcal{V}_2] = \exp\left(\frac{1}{t} \log T_{23}\right)$ is the configuration $T_{22'}$

$$T_{12'} = T_{12} T_{22'} = T_{12} \exp\left(\frac{1}{t} \log T_{23}\right)$$

------------- or -------

$$T_{13} = \exp([\mathcal{V}_1 t]) T_{12} \rightarrow T_{13} T_{12}^{-1} = \exp[\mathcal{V}_1 t] \rightarrow \log(T_{13} T_{12}^{-1}) = [\mathcal{V}_1 t] \rightarrow [\mathcal{V}_1] = \frac{1}{t} \log(T_{13} T_{12}^{-1})$$

$$T_{12'} = e^{[\mathcal{V}_1]} T_{12}$$

5. A twist is written as $\mathcal{V} = (\omega_x, \omega_y, \omega_z, v_x, v_y, v_z) = (0.5, 0, 0, 3, -2, 4)$. What is the corresponding screw axis $\mathcal{S}$? Your answer should be a numerical six-vector.

Normalize so $\|\omega\| = 1$ by multiplying by 2 (or dividing by 0.5)

$$\mathcal{S} = \frac{\mathcal{V}}{\|\omega\|} = \frac{\mathcal{V}}{0.5} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 6 \\ -4 \\ 8 \end{bmatrix}$$