Where we are:

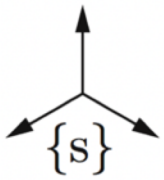
Chap 2 Configuration Space
Chap 3 Rigid-Body Motions
Chap 4 Forward Kinematics
Chap 5 Velocity Kinematics and Statics
Chap 6 Inverse Kinematics
Chap 8 Dynamics of Open Chains
Chap 9 Trajectory Generation
Chap 11 Robot Control
Chap 13 Wheeled Mobile Robots

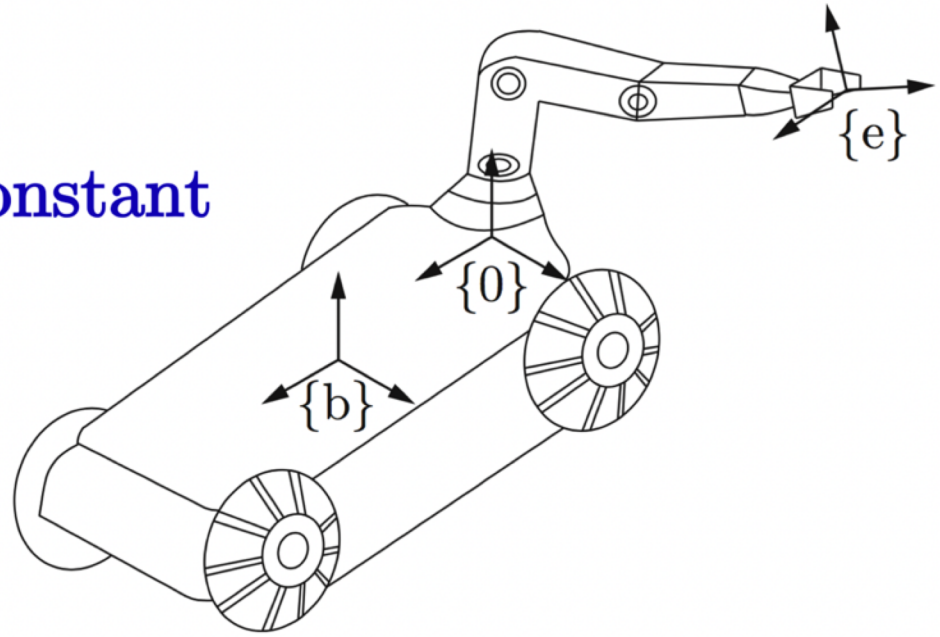
- 13.1 Types of Wheeled Mobile Robots
- 13.2 Omnidirectional Wheeled Mobile Robots
- 13.4 Odometry
- 13.5 Mobile Manipulation

Important concepts, symbols, and equations

$$T_{sb}(q) = \begin{bmatrix} \cos\phi & -\sin\phi & 0 & x \\ \sin\phi & \cos\phi & 0 & y \\ 0 & 0 & 1 & \boxed{z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

constant





$$X(q,\theta) = T_{se}(q,\theta) = T_{sb}(q)\,T_{b0}\,T_{0e}(\theta)$$

Important concepts, symbols, and equations (cont.)

Mobile manipulator Jacobian $J_e(\theta)$ (not a function of q)

$$\mathcal{V}_e = J_e(\theta)\begin{bmatrix} u \\ \dot{\theta} \end{bmatrix} = \overbrace{[J_{\text{base}}(\theta)}^{J_b(\theta)} \; J_{\text{arm}}(\theta)]\begin{bmatrix} u \\ \dot{\theta} \end{bmatrix}$$

$6\times(m+n)$ $\quad$ $6\times m$ $\quad$ $6\times n$

$$\mathcal{V}_b = Fu, \; F \in \mathbb{R}^{3\times m} \quad \text{(from odometry)}$$

$$F_6 = \begin{bmatrix} 0_m \\ 0_m \\ F \\ 0_m \end{bmatrix} \in \mathbb{R}^{6\times m}$$

$$\mathcal{V}_{b6} = F_6 u$$

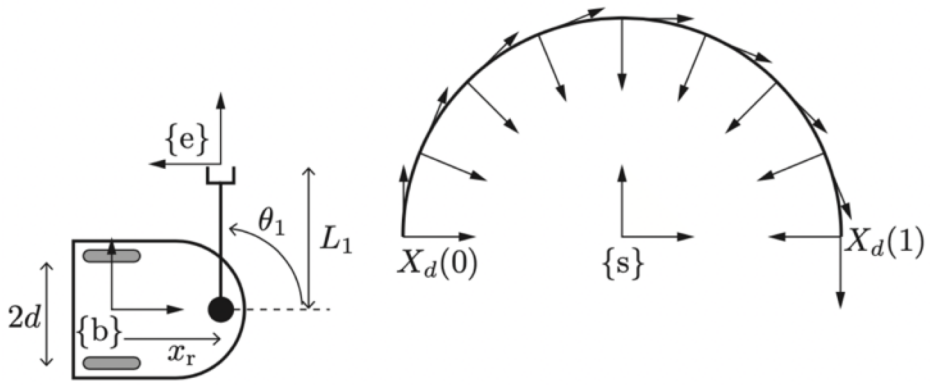
$$[\text{Ad}_{T_{eb}(\theta)}]\mathcal{V}_{b6} = \overbrace{[\text{Ad}_{T_{0e}^{-1}(\theta)T_{b0}^{-1}}]F_6}^{J_{\text{base}}(\theta)}u$$

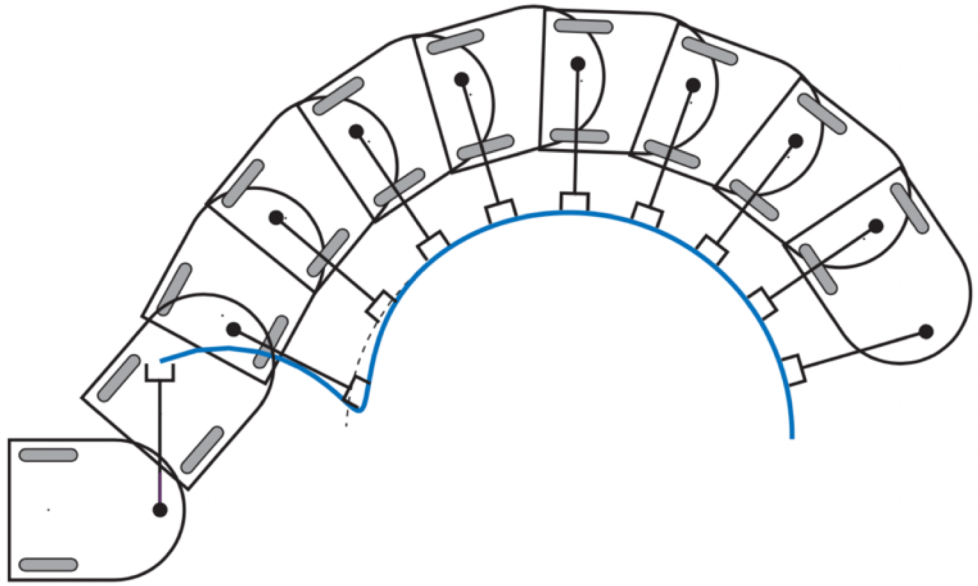
Important concepts, symbols, and equations (cont.)

Task-space feedforward + PI feedback control

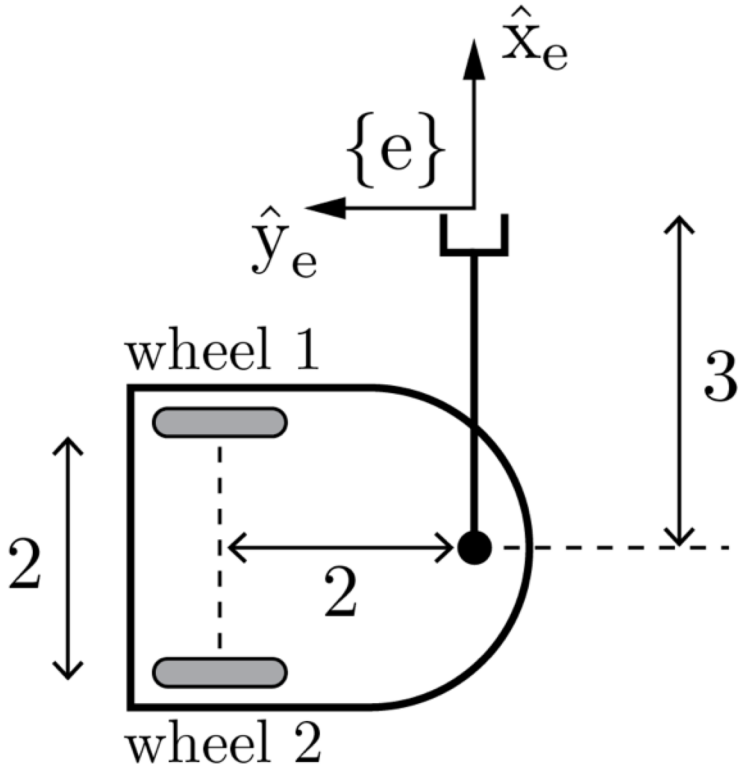
$$\mathcal{V}_e(t) = [\mathrm{Ad}_{X^{-1}X_d}]\mathcal{V}_d(t) + K_p X_{\mathrm{err}}(t) + K_i \int_0^t X_{\mathrm{err}}(\mathrm{t})\, d\mathrm{t}$$

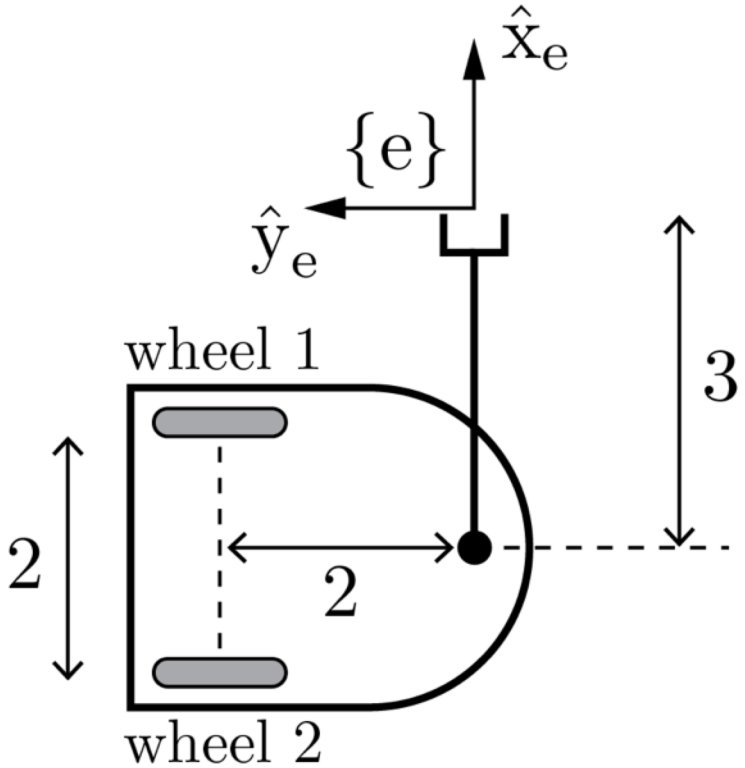
$$\begin{bmatrix} u \\ \dot{\theta} \end{bmatrix} = J_e^{\dagger}(\theta)\mathcal{V}_e \qquad [X_{\mathrm{err}}] = \log(X^{-1}X_d)$$





The planar mobile manipulator below has a diff-drive mobile base and a 1R robot arm. Each wheel radius is 0.5. The positive driving direction for each wheel moves the robot forward, and positive rotation for the arm joint is about an axis out of the page. At the configuration below, give the Jacobian J_e.




$\hat{x}_e$
{e}
$\hat{y}_e$
wheel 1
3
2
2
wheel 2

Why does the arm stretch out before the wheels move?
Would anything change if the wheels had a larger radius?

$$\begin{bmatrix} u \\ \dot{\theta} \end{bmatrix} = J_e^{\dagger}(\theta)\mathcal{V}_e$$