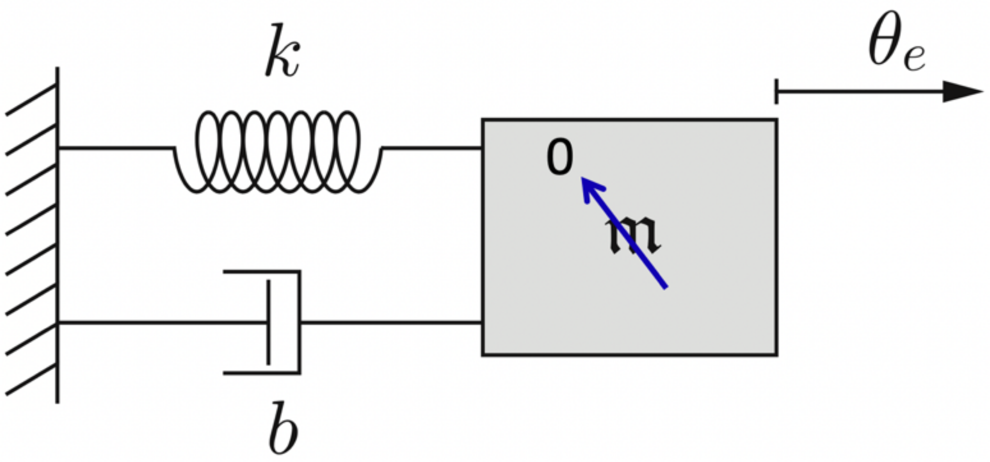
Where we are:

Chap 2	Configuration Space
Chap 3	Rigid-Body Motions
Chap 4	Forward Kinematics
Chap 5	Velocity Kinematics and Statics
Chap 6	Inverse Kinematics
Chap 8	Dynamics of Open Chains
Chap 9	Trajectory Generation
Chap 11	Robot Control
	11.1 Control System Overview
	11.2 Error Dynamics
Chap 13	Wheeled Mobile Robots

Important concepts, symbols, and equations

First-order error dynamics



0

$$\cancel{m}\ddot{\theta}_e + b\dot{\theta}_e + k\theta_e = 0$$

$$\dot{\theta}_e(t) + \frac{k}{b}\theta_e(t) = 0$$

standard first-order form

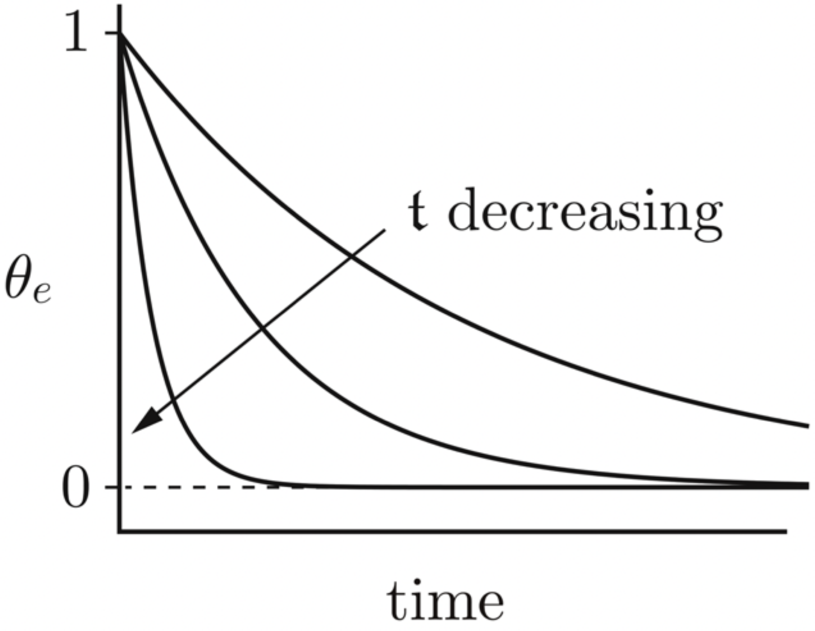
time constant

$$\mathfrak{t} = b/k$$

$$\dot{\theta}_e(t) + \frac{1}{\mathfrak{t}}\theta_e(t) = 0$$

Important concepts, symbols, and equations (cont.)

First-order error dynamics



$$\theta_e(t) = e^{-t/\mathfrak{t}}\theta_e(0)$$

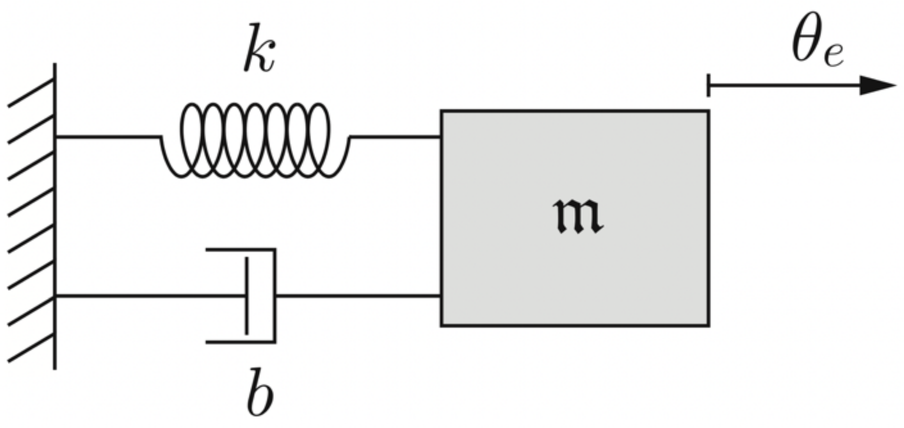
$$\theta_e(0) = 1$$

$$\frac{\theta_e(t)}{\theta_e(0)} = 0.02 = e^{-t/\mathfrak{t}}$$

$$\ln 0.02 = -t/\mathfrak{t} \quad \rightarrow \quad t = 3.91\mathfrak{t}$$

Important concepts, symbols, and equations (cont.)

Second-order error dynamics



$$\ddot{\theta}_e(t) + \frac{b}{\mathfrak{m}}\dot{\theta}_e(t) + \frac{k}{\mathfrak{m}}\theta_e(t) = 0$$

natural frequency **damping ratio**

$$\omega_n = \sqrt{k/\mathfrak{m}} \qquad \zeta = b/(2\sqrt{k\mathfrak{m}})$$

$$\ddot{\theta}_e(t) + 2\zeta\omega_n\dot{\theta}_e(t) + \omega_n^2\theta_e(t) = 0$$

standard second-order form

Important concepts, symbols, and equations (cont.)

Second-order error dynamics

$$\ddot{\theta}_e(t) + 2\zeta\omega_n\dot{\theta}_e(t) + \omega_n^2\theta_e(t) = 0$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

$\zeta > 1$: Overdamped

$\zeta = 1$: Critically damped

$\zeta < 1$: Underdamped

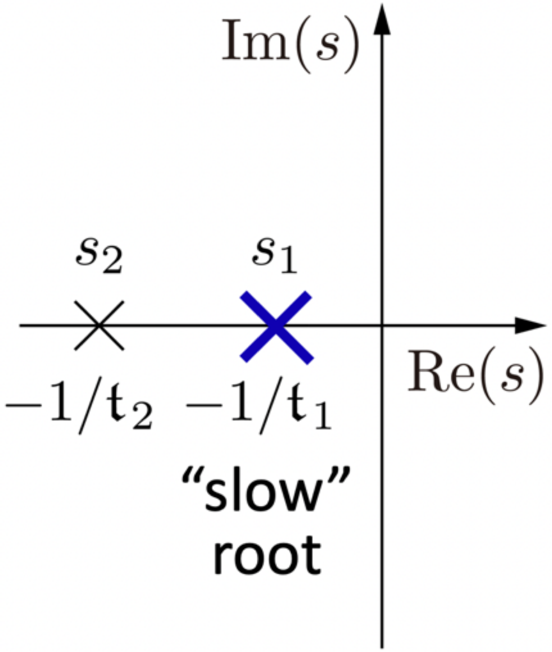
Important concepts, symbols, and equations (cont.)

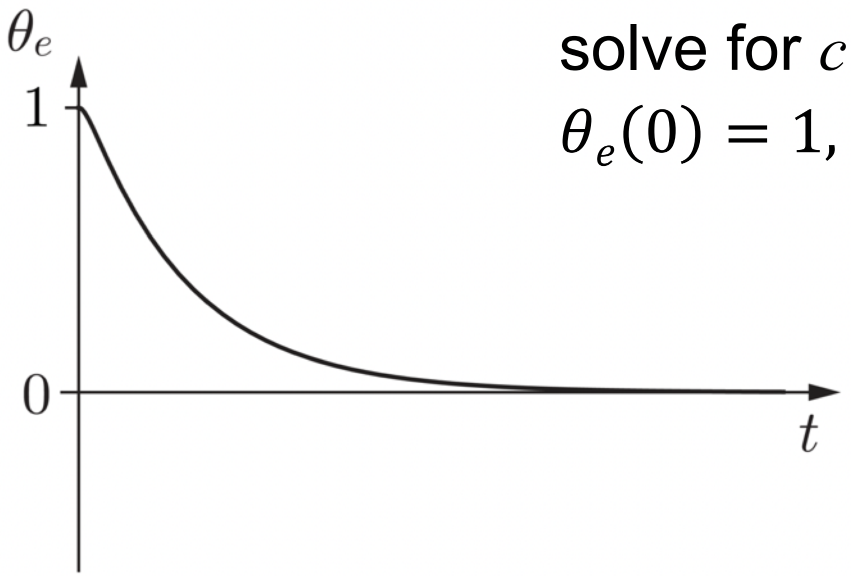
$\zeta > 1$: Overdamped

$$\theta_e(t) = c_1 e^{s_1 t} + c_2 e^{s_2 t}$$

$$s_1 = -\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1}$$

$$s_2 = -\zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1}$$



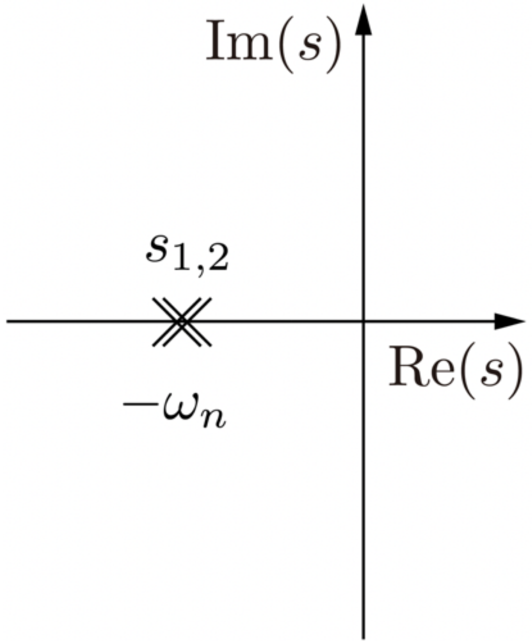


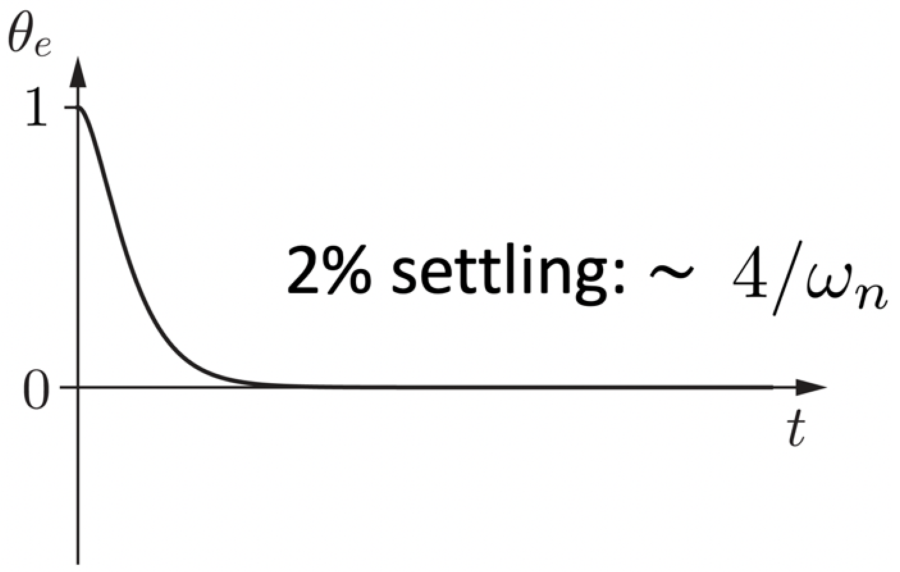
solve for c_1 and c_2 using $\theta_e(0) = 1, \dot{\theta}_e(0) = 0$

Important concepts, symbols, and equations (cont.)

$\zeta = 1$: Critically damped

$$\theta_e(t) = (c_1 + c_2 t)e^{-\omega_n t} \qquad s_{1,2} = -\omega_n$$





Important concepts, symbols, and equations (cont.)

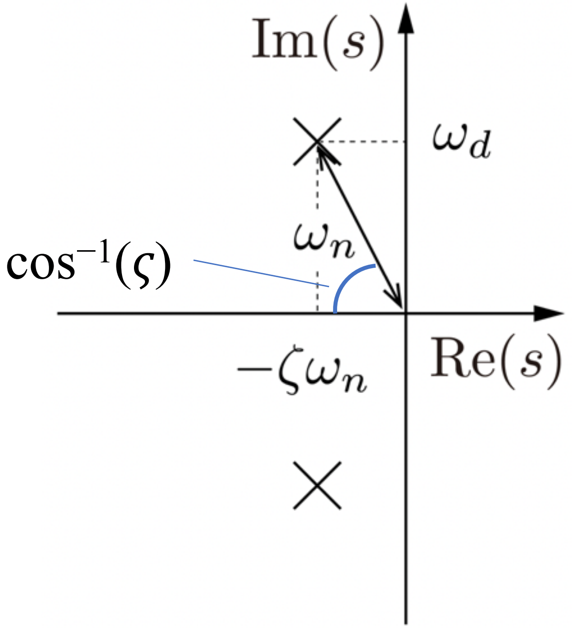
$\zeta < 1$: Underdamped

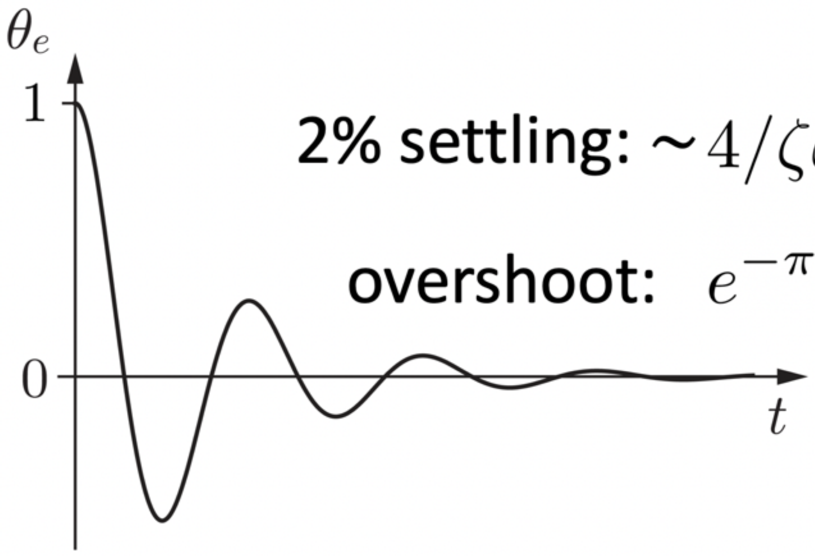
$$\theta_e(t) = (c_1 \cos \omega_d t + c_2 \sin \omega_d t)\, e^{-\zeta \omega_n t}$$

damped natural frequency

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

$$s_{1,2} = -\zeta\omega_n \pm j\omega_d$$

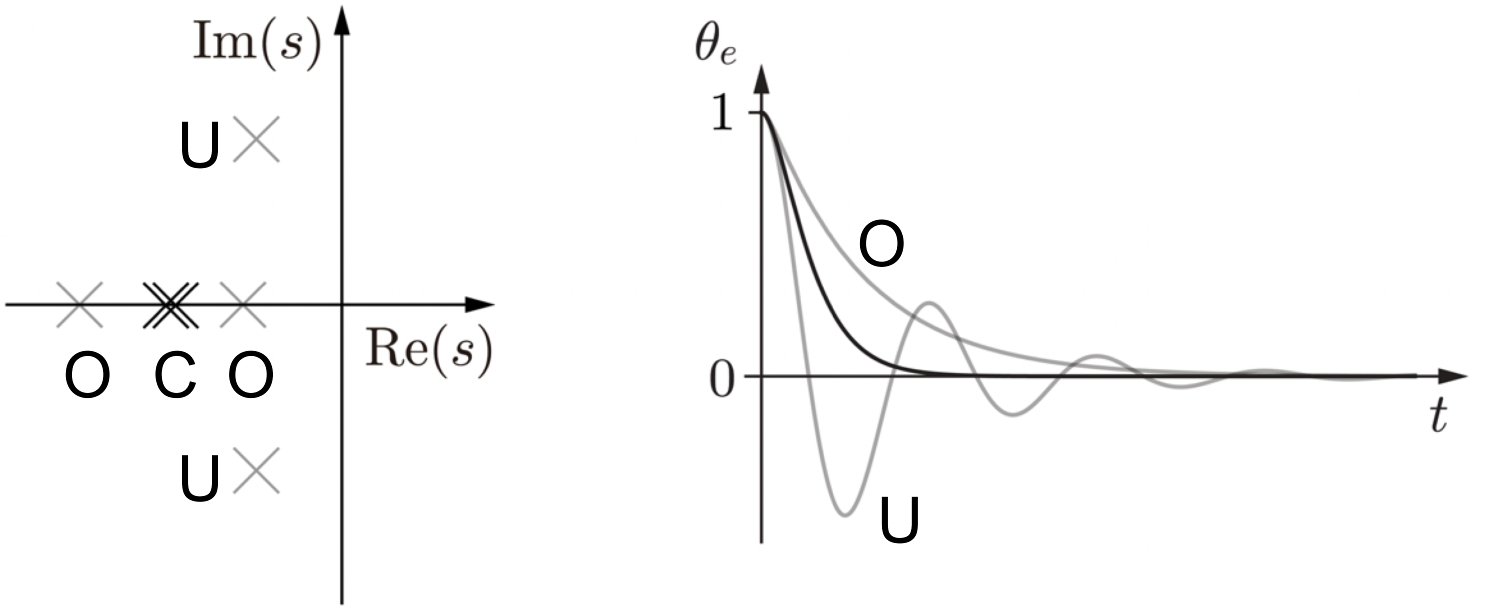




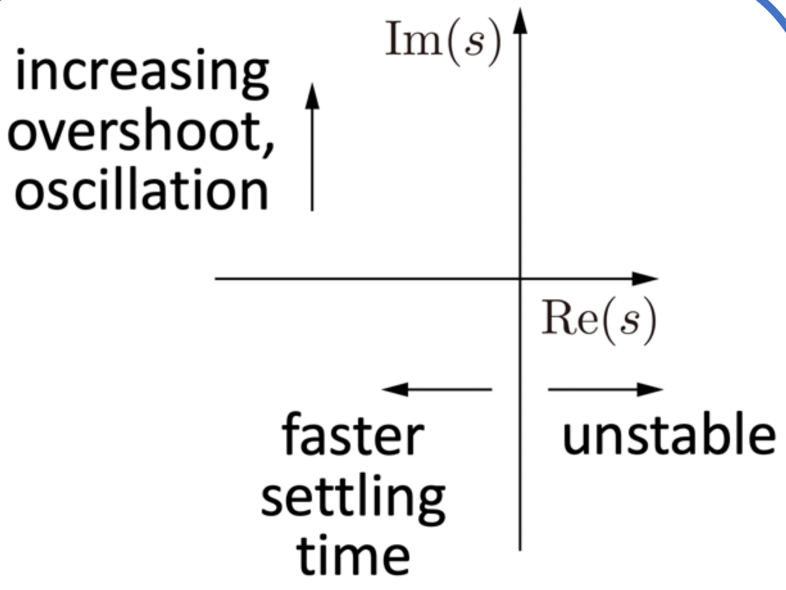
2% settling: $\sim 4/\zeta\omega_n$

overshoot: $e^{-\pi\zeta/\sqrt{1-\zeta^2}} \times 100\%$

Important concepts, symbols, and equations (cont.)



second-order systems



time response relative to root locations for general systems

$\omega_n = \sqrt{k/\mathfrak{m}}$ $\zeta = b/(2\sqrt{k\mathfrak{m}})$ 2% settling: $\sim 4/\zeta\omega_n$

$\ddot{\theta}_e(t) + 2\zeta\omega_n\dot{\theta}_e(t) + \omega_n^2\theta_e(t) = 0$ overshoot: $e^{-\pi\zeta/\sqrt{1-\zeta^2}} \times 100\%$

When controlling a robot joint, what do b, k, and $\mathfrak{m}$ usually correspond to?

How do you change $\mathfrak{m}$ to decrease settling time? k, b?

How do you change $\mathfrak{m}$ to decrease overshoot? k, b?

Shown are one of the roots of five different second-order systems, A, B, C, D, and E.
List them in the following orders:

1. Natural frequency, highest to lowest.
2. Damped natural frequency, highest to lowest.
3. Damping ratio, highest to lowest.
4. Overshoot in unit step error response, highest to lowest.
5. Settling time, longest to shortest.

Which has the “best” transient response?