

## Where we are:

Chap 2	Configuration Space
Chap 3	Rigid-Body Motions
Chap 4	Forward Kinematics
Chap 5	Velocity Kinematics and Statics
Chap 6	Inverse Kinematics
Chap 8	Dynamics of Open Chains
	8.1 Lagrangian Formulation
Chap 9	Trajectory Generation
Chap 11	Robot Control
Chap 13	Wheeled Mobile Robots

## Important concepts, symbols, and equations

- **forward dynamics** (for simulation):  $\theta, \dot{\theta}, \tau \rightarrow \ddot{\theta}$
- **inverse dynamics** (for control):  $\theta, \dot{\theta}, \ddot{\theta} \rightarrow \tau$
- two equivalent approaches to computing the dynamics:
  - Lagrangian (variational, based on energy)
  - Newton-Euler (“ $f = ma$ ” for the rigid bodies)

## Important concepts, symbols, and equations (cont.)

Lagrangian approach:

**Lagrangian**

$$\mathcal{L}(\theta, \dot{\theta}) = \mathcal{K}(\theta, \dot{\theta}) - \mathcal{P}(\theta)$$

kinetic minus potential energy

eqs of motion

$$\tau = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\theta}} - \frac{\partial \mathcal{L}}{\partial \theta} \in \mathbb{R}^n$$

$$\tau_i = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\theta}_i} - \frac{\partial \mathcal{L}}{\partial \theta_i}$$

in components

## Important concepts, symbols, and equations (cont.)

Standard forms of dynamic equations:

$$\begin{aligned}\tau &= M(\theta)\ddot{\theta} + h(\theta, \dot{\theta}) \\ &= M(\theta)\ddot{\theta} + c(\theta, \dot{\theta}) + g(\theta) \\ &= M(\theta)\ddot{\theta} + \dot{\theta}^T \Gamma(\theta) \dot{\theta} + g(\theta) \\ &= M(\theta)\ddot{\theta} + C(\theta, \dot{\theta}) \dot{\theta} + g(\theta)\end{aligned} \quad + J^T(\theta) \mathcal{F}_{\text{tip}}$$

$M(\theta)$

$n \times n$  symmetric positive definite **mass matrix**

$g(\theta)$

gravity (potential) terms

$c(\theta, \dot{\theta})$

velocity-product terms

$\Gamma(\theta)$

$n \times n \times n$  tensor of **Christoffel symbols** (due to nonzero  $\partial M / \partial \theta$ )

$C(\theta, \dot{\theta})$

**Coriolis matrix**

## Important concepts, symbols, and equations (cont.)

Velocity-product terms ( $c(\theta, \dot{\theta})$ ,  $C(\theta, \dot{\theta})\dot{\theta}$ ,  $\dot{\theta}^T \Gamma(\theta)\dot{\theta}$ ) consist of

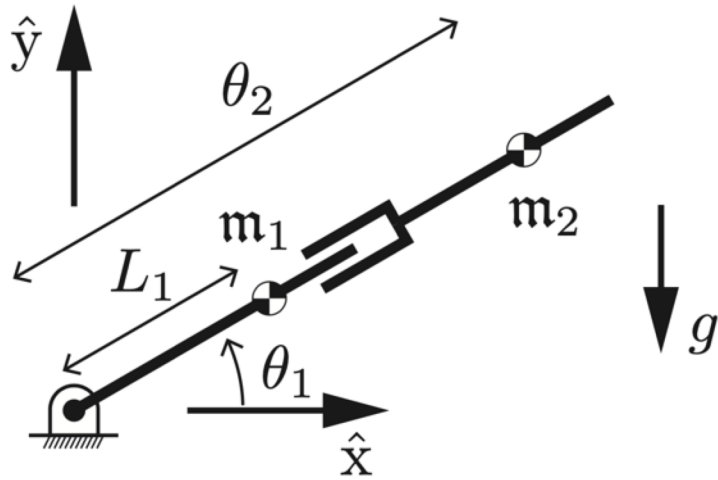
- **centripetal** terms, e.g.,

$$m_2 L_1 L_2 \dot{\theta}_1^2 \sin \theta_2$$

- **Coriolis** terms, e.g.,

$$-m_2 L_1 L_2 \sin \theta_2 (2\dot{\theta}_1 \dot{\theta}_2)$$

RP in gravity



$$\mathcal{P}_1 = m_1 g y_1 = m_1 g L_1 \sin \theta_1$$

$$\mathcal{P}_2 = m_2 g y_2 = m_2 g \theta_2 \sin \theta_1$$

$$\mathcal{K}_1 = \frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2} \mathcal{I}_1 \dot{\theta}_1^2 = \frac{1}{2} (\mathcal{I}_1 + m_1 L_1^2) \dot{\theta}_1^2$$

$$\mathcal{K}_2 = \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2) + \frac{1}{2} \mathcal{I}_2 \dot{\theta}_2^2 = \frac{1}{2} \left( (\mathcal{I}_2 + m_2 \theta_2^2) \dot{\theta}_1^2 + m_2 \dot{\theta}_2^2 \right)$$

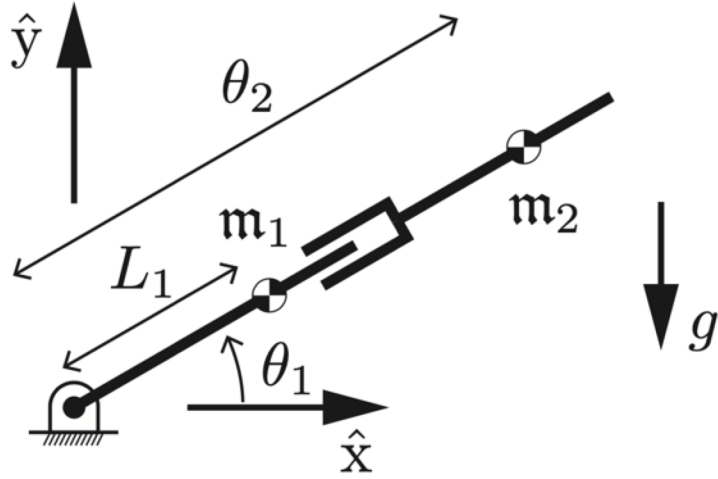
$$\mathcal{L} = \mathcal{K}_1 + \mathcal{K}_2 - \mathcal{P}_1 - \mathcal{P}_2 = \frac{1}{2} m_2 \theta_2^2 \dot{\theta}_1^2 + \dots$$

$$= \mathcal{L}_1 + \dots$$

product rule:  $(fg)' = f'g + fg'$

chain rule:  $\frac{df(g(t))}{dt} = \left( \frac{\partial f}{\partial g} \right) \frac{dg}{dt}$

RP in gravity

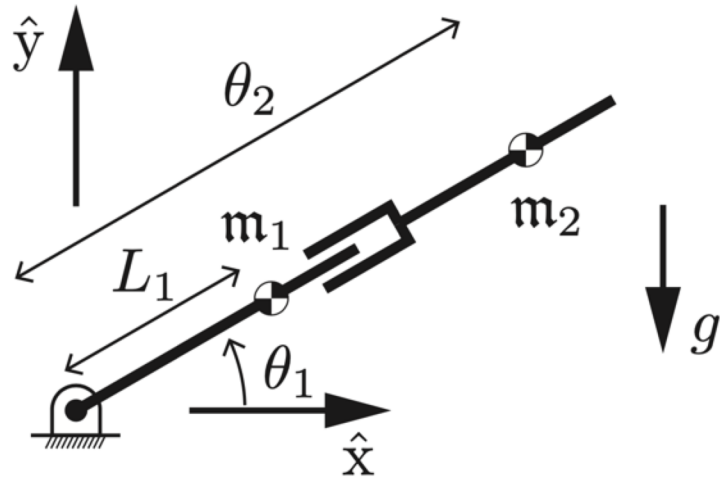


$$\mathcal{L}_1 = \frac{1}{2} m_2 \theta_2^2 \dot{\theta}_1^2$$

Contribution to  $\tau_2$ :

Contribution to  $\tau_1$ :

## RP in gravity



$$\tau = M(\theta)\ddot{\theta} + c(\theta, \dot{\theta}) + g(\theta)$$

$$M(\theta) = \begin{bmatrix} \mathcal{I}_1 + \mathcal{I}_2 + m_1 L_1^2 + m_2 \theta_2^2 & 0 \\ 0 & m_2 \end{bmatrix}$$

$$c(\theta, \dot{\theta}) = \begin{bmatrix} 2m_2 \theta_2 \dot{\theta}_1 \dot{\theta}_2 \\ -m_2 \theta_2 \dot{\theta}_1^2 \end{bmatrix}$$

$$g(\theta) = \begin{bmatrix} (m_1 L_1 + m_2 \theta_2) g \cos \theta_1 \\ m_2 g \sin \theta_1 \end{bmatrix}$$

Explain the velocity-product terms.