**Where we are:**

Chap 2 Configuration Space
Chap 3 Rigid-Body Motions
Chap 4 Forward Kinematics
- 4.1 Product of Exponentials Formula
- 4.2 The Universal Robot Description Format (URDF)

Chap 5 Velocity Kinematics and Statics
Chap 6 Inverse Kinematics
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Chap 9 Trajectory Generation
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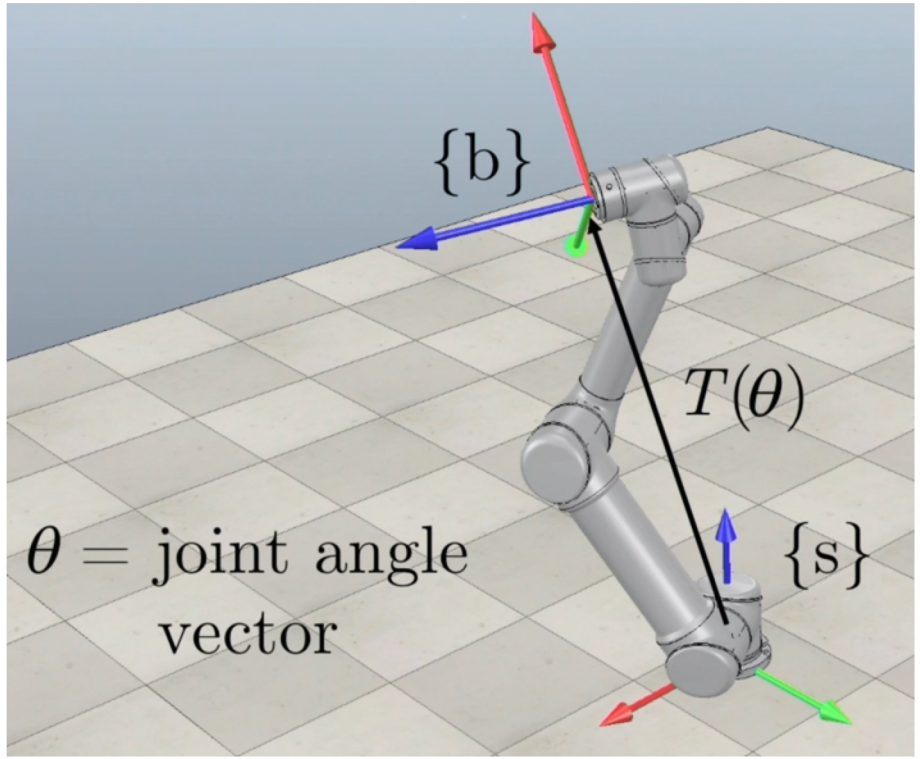
# Important concepts, symbols, and equations

**Forward kinematics** of a serial chain:

Given

- $M = T_{sb}(0) \in SE(3)$, the configuration of the end-effector frame $\{\mathrm{b}\}$ at the home configuration $\theta = 0$,
- the screw axes for each joint at $\theta = 0$, and
- the joint vector $\theta$,

find $T_{sb}(\theta) \in SE(3)$.



## Important concepts, symbols, and equations (cont.)

- For screw axes $\mathcal{S}$ expressed in $\{\mathrm{s}\}$, the **product of exponentials** (PoE) in the space frame is

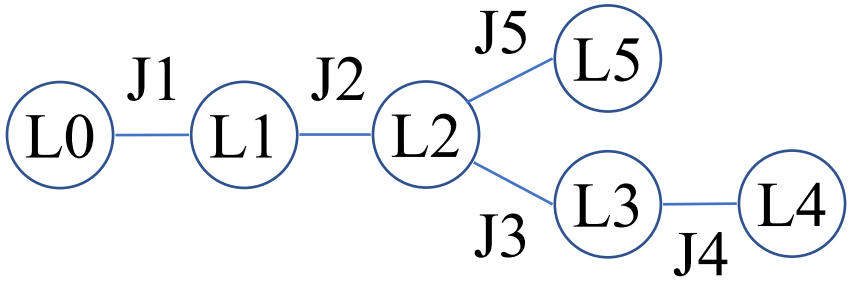
$$T(\theta) = e^{[\mathcal{S}_1]\theta_1} \cdots e^{[\mathcal{S}_n]\theta_n} M$$

- For screw axes $\mathcal{B}$ expressed in $\{\mathrm{b}\}$, the PoE is

$$T(\theta) = M e^{[\mathcal{B}_1]\theta_1} \cdots e^{[\mathcal{B}_n]\theta_n}$$

# Important concepts, symbols, and equations (cont.)

The **Universal Robot Description Format (URDF)** is an XML file describing the kinematics, inertial properties, and geometry of a tree-structured robot.

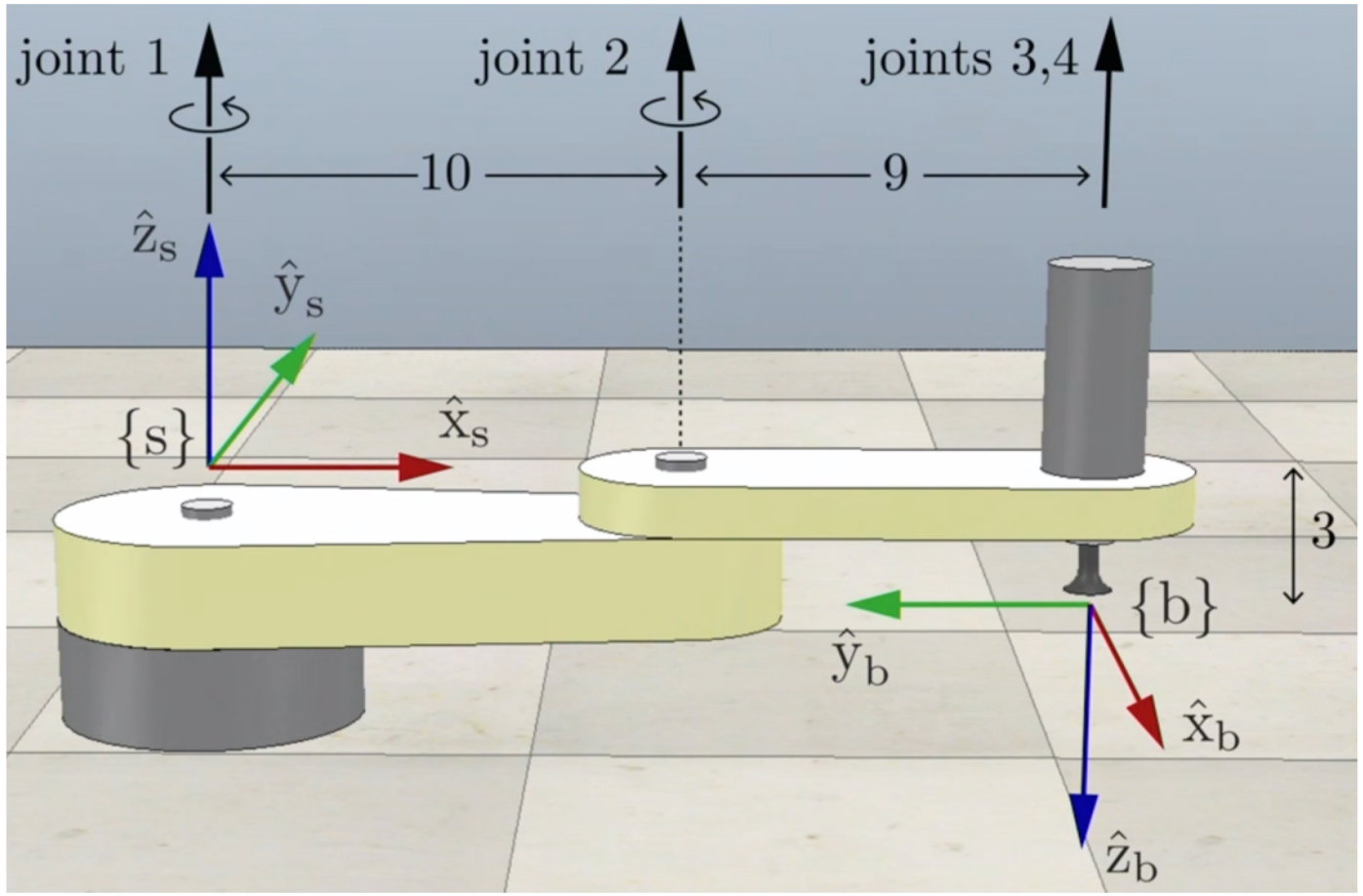



the **joint** element (kinematics)

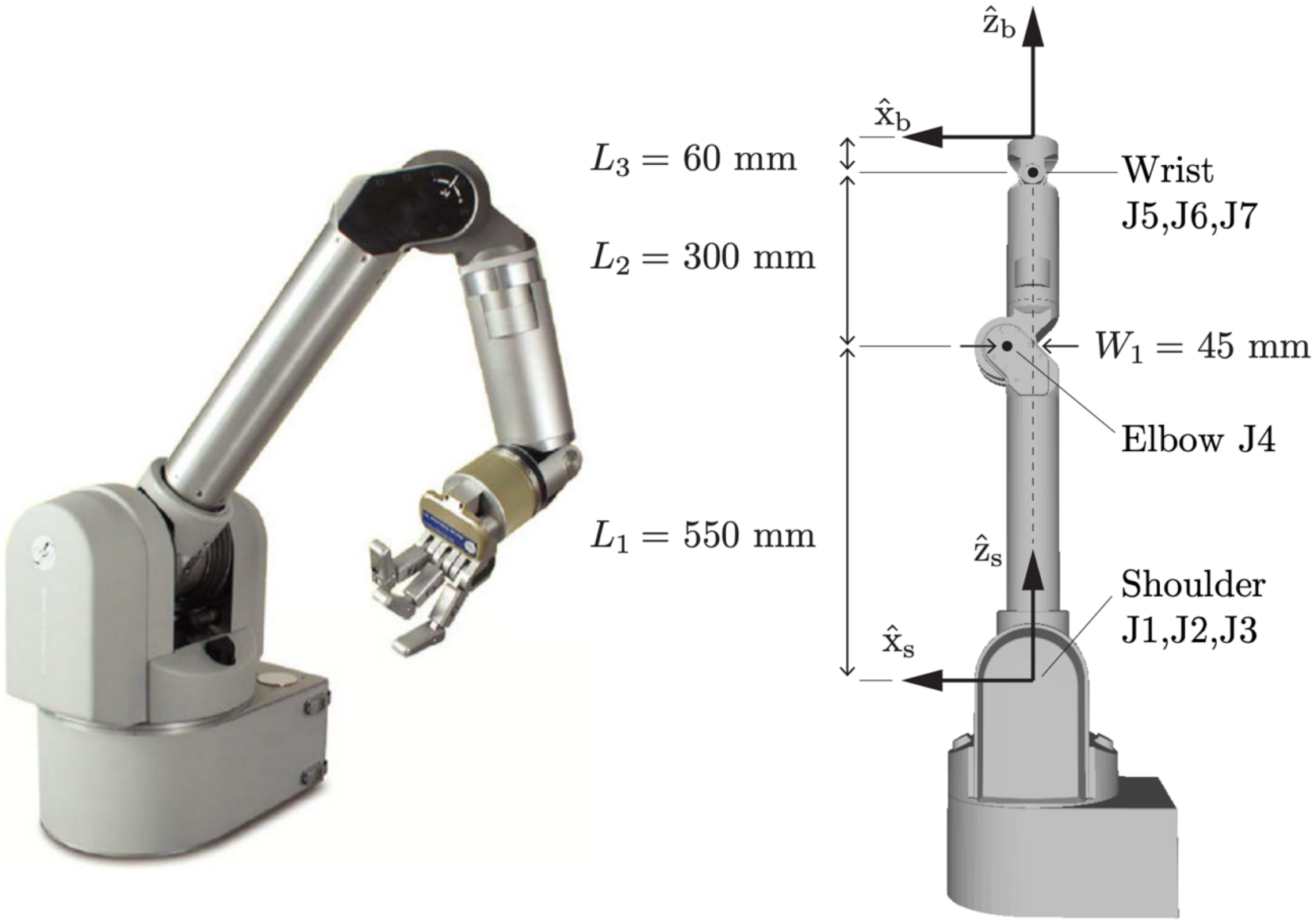
```
<joint name="joint2" type="revolute">
  <parent link="link1"/>
  <child link="link2"/>
  <origin rpy="0 1.5708 0" xyz="0 0.5 0">
  <axis xyz="0 1 0">
  <limit lower="-3.0" upper="3.0">
</joint>
```

the **link** element (inertial properties)

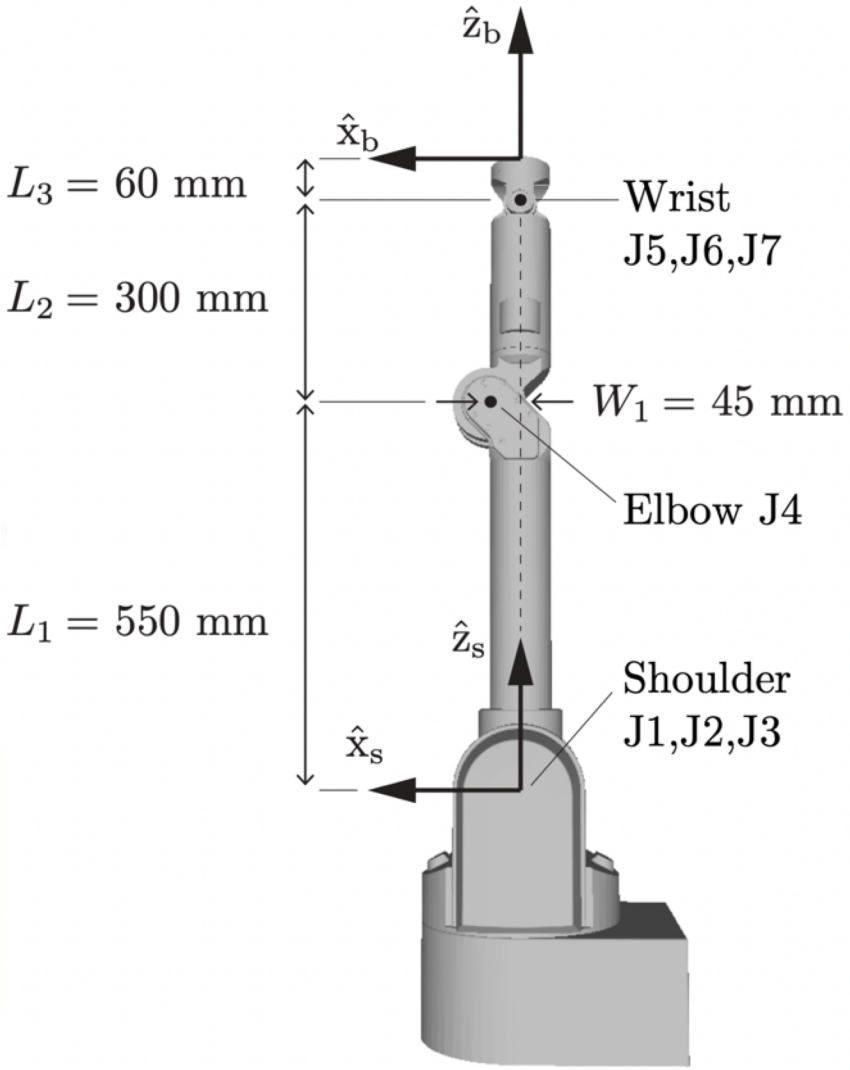
```
<link name="link2">
  <inertial>
    <origin rpy="0 0 0" xyz="0 0 0.5">
    <mass value="2.25">
    <inertia ixx="1.0" ixy="0" ixz="0"
             iyy="2.0" iyz="0" izz="3.0"/>
  </inertial>
</link>
```


joint 1
joint 2
joints 3,4
10
9
$\hat{z}_s$
$\hat{y}_s$
$\hat{x}_s$
{s}
3
{b}
$\hat{y}_b$
$\hat{x}_b$
$\hat{z}_b$

RRRP robot




Barrett Technology's WAM 7R robot

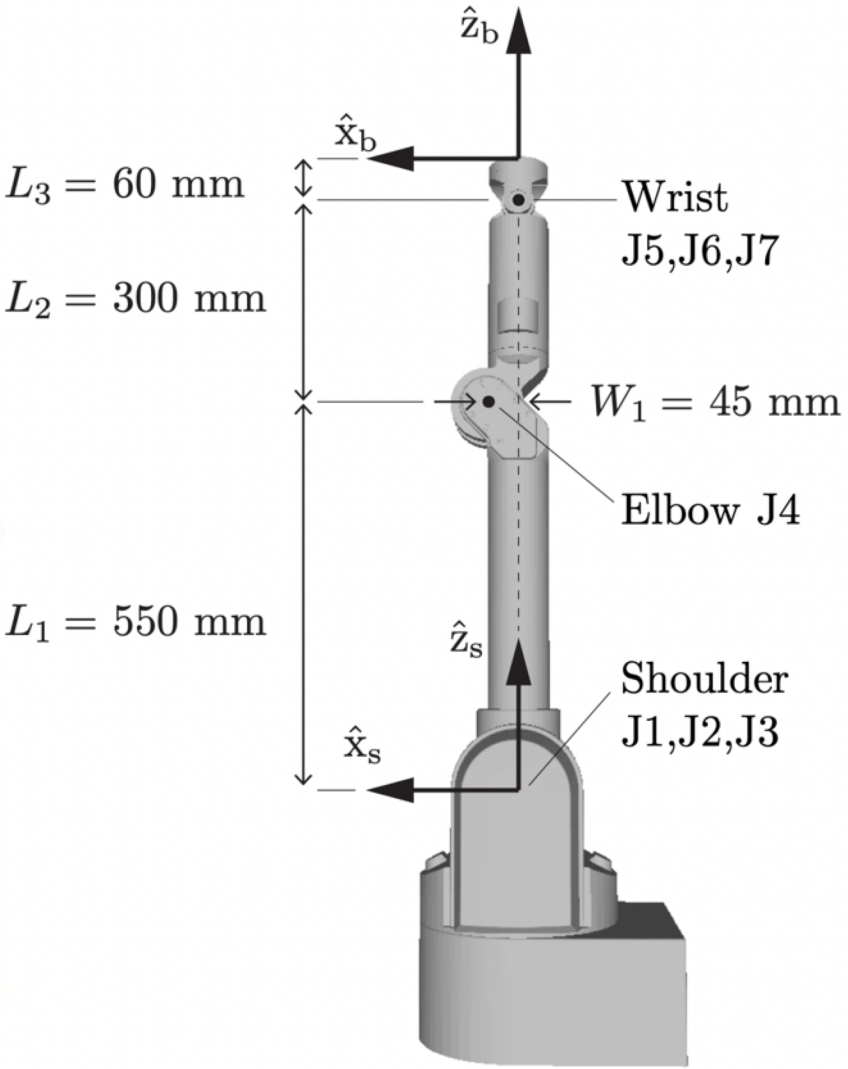



Joint axes 1, 3, 5, and 7 are aligned with $\hat{z}_s$.
Joint axes 2, 4, and 6 are out of the page.

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_1 + L_2 + L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

screw axes $\mathcal{B}_i$

| $i$ | $\omega_i$ | $v_i$ |
|---|---|---|
| 1 | $(0, 0, 1)$ | $(0, 0, 0)$ |
| 2 | $(0, 1, 0)$ | $(L_1 + L_2 + L_3, 0, 0)$ |
| 3 | $(0, 0, 1)$ | $(0, 0, 0)$ |
| 4 | $(0, 1, 0)$ | $(L_2 + L_3, 0, W_1)$ |
| 5 | $(0, 0, 1)$ | $(0, 0, 0)$ |
| 6 | $(0, 1, 0)$ | $(L_3, 0, 0)$ |
| 7 | $(0, 0, 1)$ | $(0, 0, 0)$ |




screw axes $S_i$

Joint axes 1, 3, 5, and 7 are aligned with $\hat{z}_s$.
Joint axes 2, 4, and 6 are out of the page.