Where we are:

Chap 2	Configuration Space
Chap 3	Rigid-Body Motions
Chap 4	Forward Kinematics
Chap 5	Velocity Kinematics and Statics
Chap 6	Inverse Kinematics
Chap 8	Dynamics of Open Chains
Chap 9	Trajectory Generation
Chap 11	Robot Control
Chap 13	Wheeled Mobile Robots
	13.1 Types of Wheeled Mobile Robots
	13.2 Omnidirectional Wheeled Mobile Robots

Important concepts, symbols, and equations

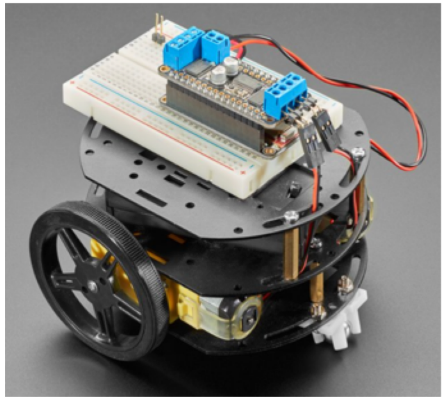
Types of wheels

conventional

omniwheel

mecanum wheel

Kinematic wheeled
mobile robots
(no slipping)

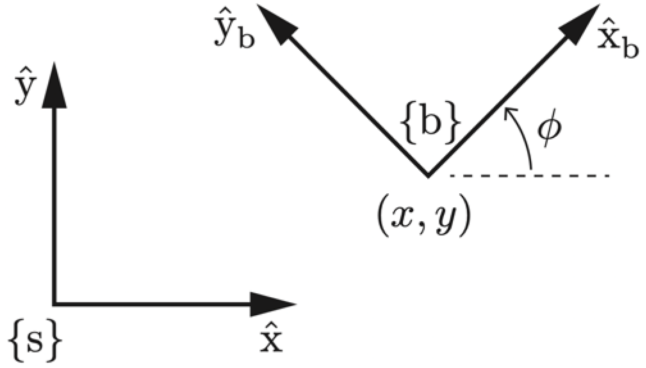


differential drive

car-like

omnidirectional

Important concepts, symbols, and equations (cont.)



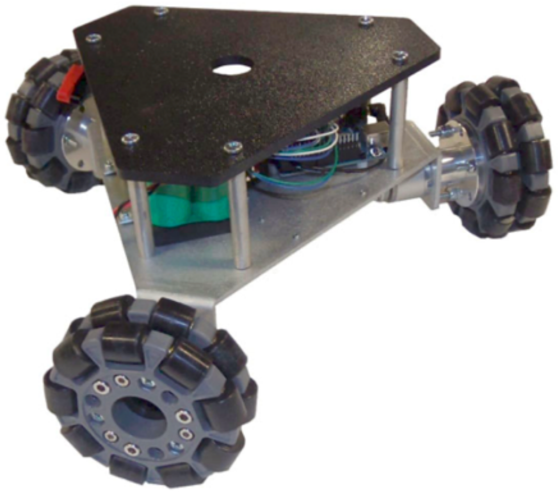
Configuration of the mobile base

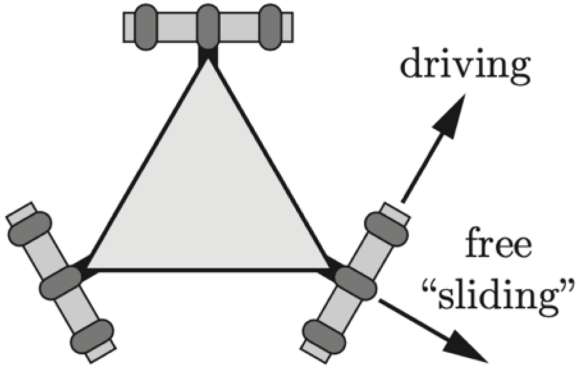
$$T_{sb} = \begin{bmatrix} r_{11} & r_{12} & \cancel{r_{13}} & p_1 \\ r_{21} & r_{22} & \cancel{r_{23}} & p_2 \\ \cancel{r_{31}} & \cancel{r_{32}} & \cancel{r_{33}} & \cancel{p_3} \\ 0 & 0 & \cancel{0} & 1 \end{bmatrix} \in SE(2) \text{ or } q = (\phi, x, y) \in \mathbb{R}^3$$

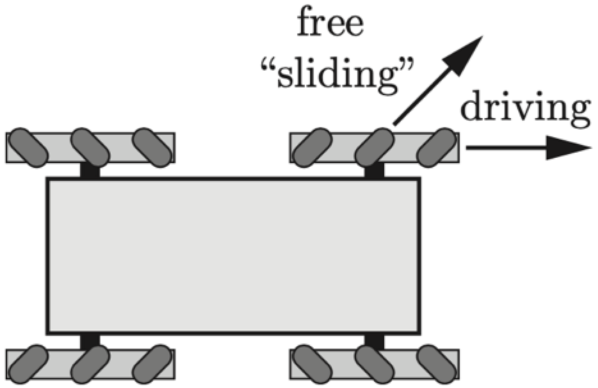
Velocity of the mobile base: $\mathcal{V}_b = (\cancel{\omega_{bx}}, \cancel{\omega_{by}}, \omega_{bz}, v_{bx}, v_{by}, \cancel{v_{bz}})$ or $\dot{q} \in \mathbb{R}^3$

Important concepts, symbols, and equations (cont.)

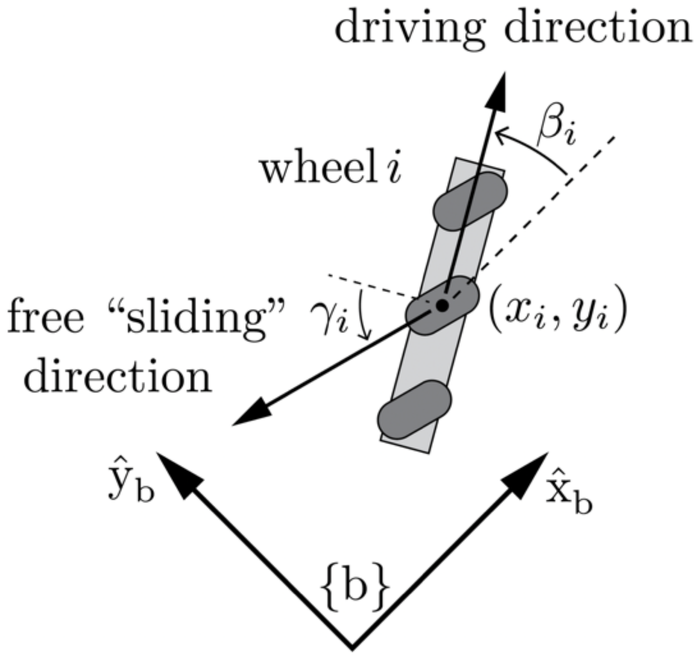
Examples of omnidirectional wheeled mobile robots

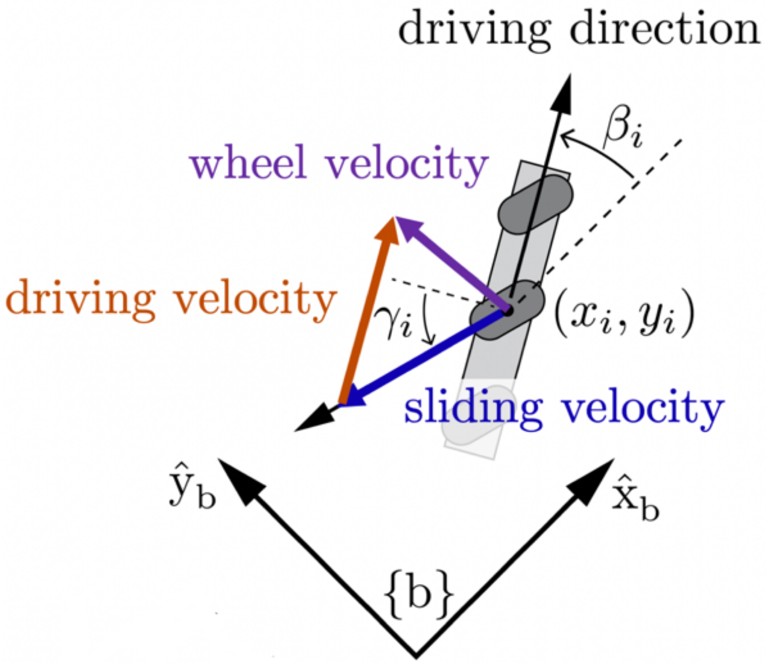






Important concepts, symbols, and equations (cont.)





$$u_i = h_i(0)\mathcal{V}_b$$

$$H(0) = \begin{bmatrix} h_1(0) \\ \vdots \\ h_m(0) \end{bmatrix} \in \mathbb{R}^{m\times 3}$$

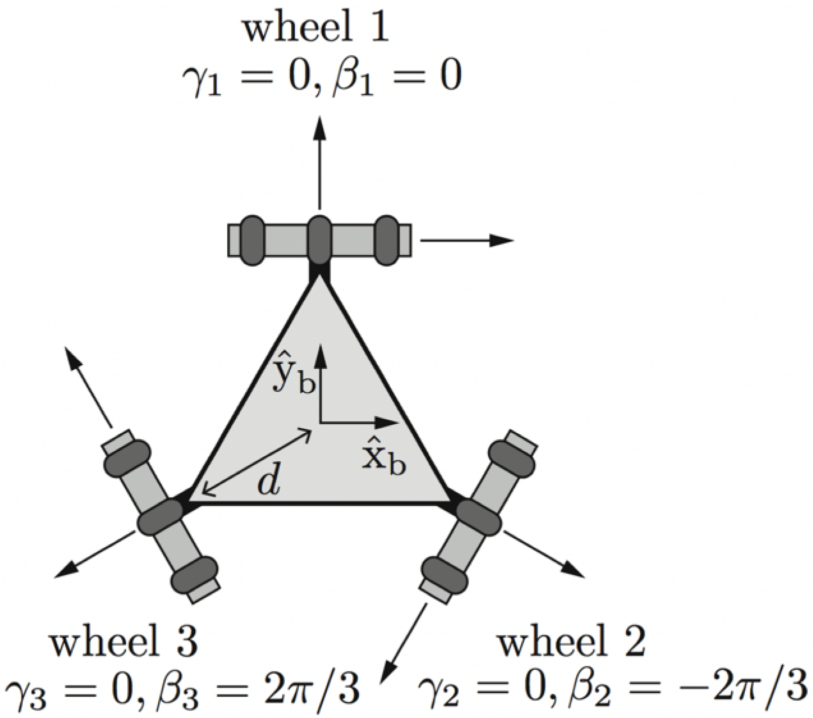
$$u = H(0)\mathcal{V}_b$$

wheel driving speed:

$$u_i = \underbrace{\frac{1}{r_i}}_{\text{wheel radius}} \underbrace{\begin{bmatrix} 1 & \tan\gamma_i \end{bmatrix}}_{\text{component in driving direction}} \underbrace{\begin{bmatrix} \cos\beta_i & \sin\beta_i \\ -\sin\beta_i & \cos\beta_i \end{bmatrix} \begin{bmatrix} -y_i & 1 & 0 \\ x_i & 0 & 1 \end{bmatrix} \mathcal{V}_b}_{\text{linear velocity at wheel, in wheel frame}}$$

linear velocity at wheel, in {b}: $\begin{bmatrix} -y_i & 1 & 0 \\ x_i & 0 & 1 \end{bmatrix} \mathcal{V}_b$

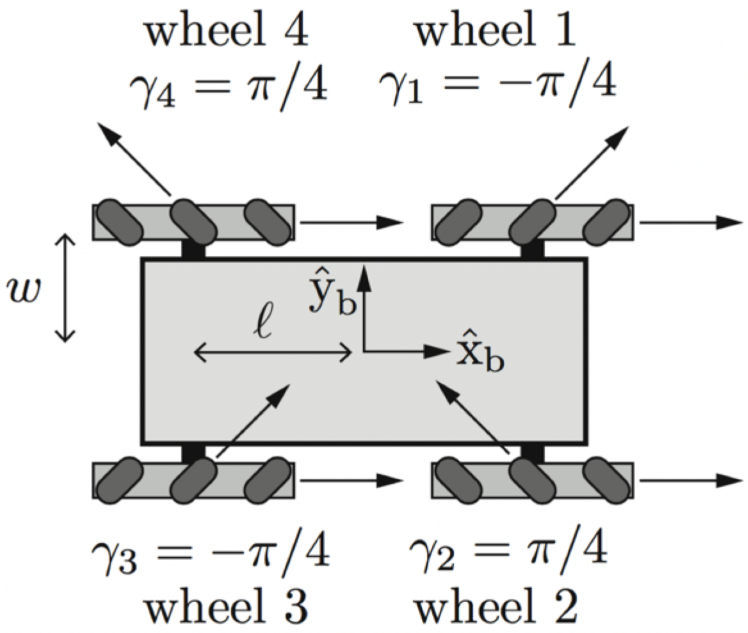
Important concepts, symbols, and equations (cont.)



Why are at least three wheels required for omnidirectional motion?

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = H(0)\mathcal{V}_b = \frac{1}{r}\left[\begin{array}{ccc} \text{spin in place} & \text{right} & \text{up} \\ -d & 1 & 0 \\ -d & -1/2 & -\sin(\pi/3) \\ -d & -1/2 & \sin(\pi/3) \end{array}\right]\begin{bmatrix} \omega_{bz} \\ v_{bx} \\ v_{by} \end{bmatrix}$$

Important concepts, symbols, and equations (cont.)



Wheel velocities are 4d.
Chassis twist is 3d.
Implications?

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = H(0)\mathcal{V}_b = \frac{1}{r} \begin{array}{ccc} \text{spin in place} & \text{forward} & \text{sideways} \\ -\ell - w & 1 & -1 \\ \ell + w & 1 & 1 \\ \ell + w & 1 & -1 \\ -\ell - w & 1 & 1 \end{array} \begin{bmatrix} \omega_{bz} \\ v_{bx} \\ v_{by} \end{bmatrix}$$

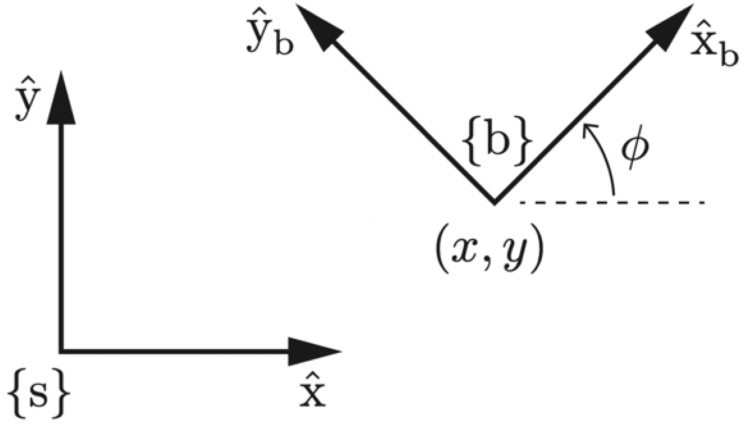
Important concepts, symbols, and equations (cont.)

Wheel speeds in terms of $\dot{q}$:

$$u = H(0)\mathcal{V}_b \ , \quad H(0) \in \mathbb{R}^{m\times 3}$$

$$u = \underbrace{H(0)\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}}_{H(\phi)} \underbrace{\begin{bmatrix} \dot{\phi} \\ \dot{x} \\ \dot{y} \end{bmatrix}}_{\dot{q}}$$

$$u = H(\phi)\dot{q}$$



Important concepts, symbols, and equations (cont.)

Feedforward + PI feedback stabilization of a planned trajectory:

$$\dot{q}(t) = \dot{q}_d(t) + K_p(q_d(t) - q(t)) + K_i \int_0^t (q_d(\mathrm{t}) - q(\mathrm{t}))\, d\mathrm{t}$$

$$u = H(\phi)\dot{q}$$

Stability and steady-state error for different control laws and desired trajectories?

Important concepts, symbols, and equations (cont.)

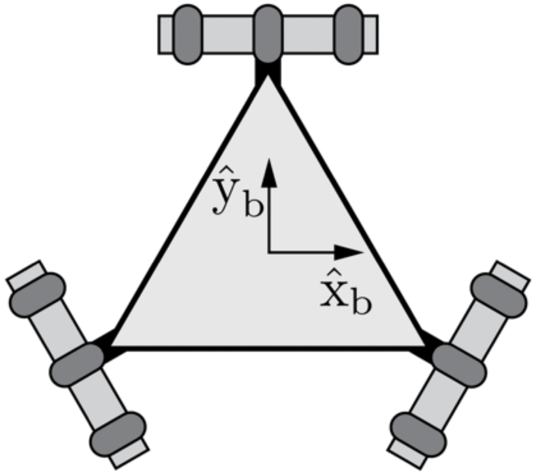
Given wheel velocity limits, the chassis' feasible twists lie inside a $2m$-sided convex polyhedron:

$$\begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} = H(0)\mathcal{V}_b = \begin{bmatrix} h_1(0) \\ \vdots \\ h_m(0) \end{bmatrix} \mathcal{V}_b$$

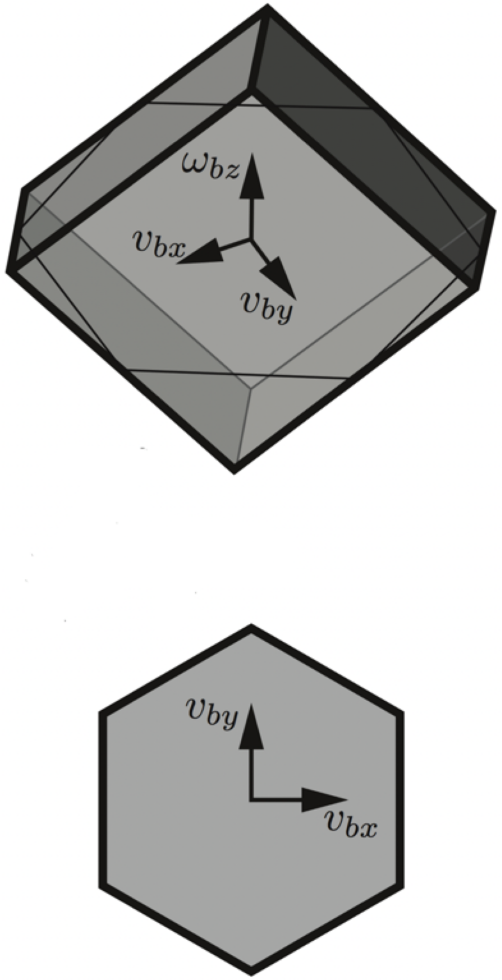
$$-u_{i,\max} \leq u_i = h_i(0)\mathcal{V}_b \leq u_{i,\max}$$

$$\left.\begin{aligned} -u_{i,\max} &= h_i(0)\mathcal{V}_b \\ u_{i,\max} &= h_i(0)\mathcal{V}_b \end{aligned}\right\} \text{define two parallel bounding planes in twist space}$$

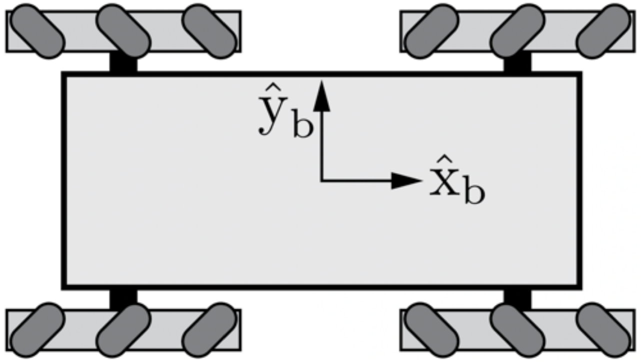
Important concepts, symbols, and equations (cont.)



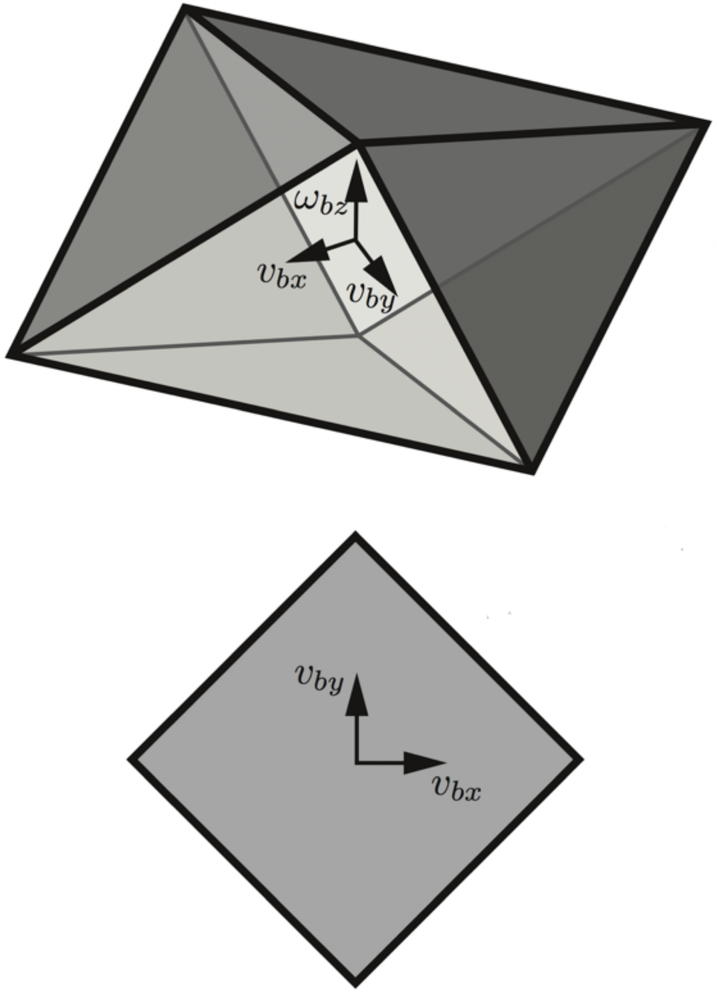
3 omniwheels



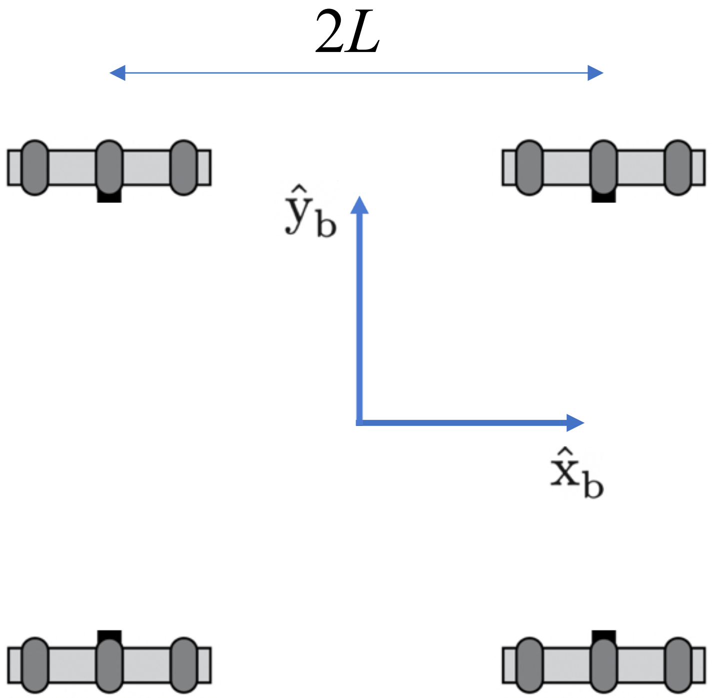
Important concepts, symbols, and equations (cont.)



4 mecanum wheels



What does it mean if the convex polyhedron is unbounded?



The centers of the omniwheels of a mobile robot are at the corners of a square a distance $2L$ from each other. The radius of the wheels is r, and the forward driving direction for each wheel is to the right. What is $H(0)$?