Where we are:

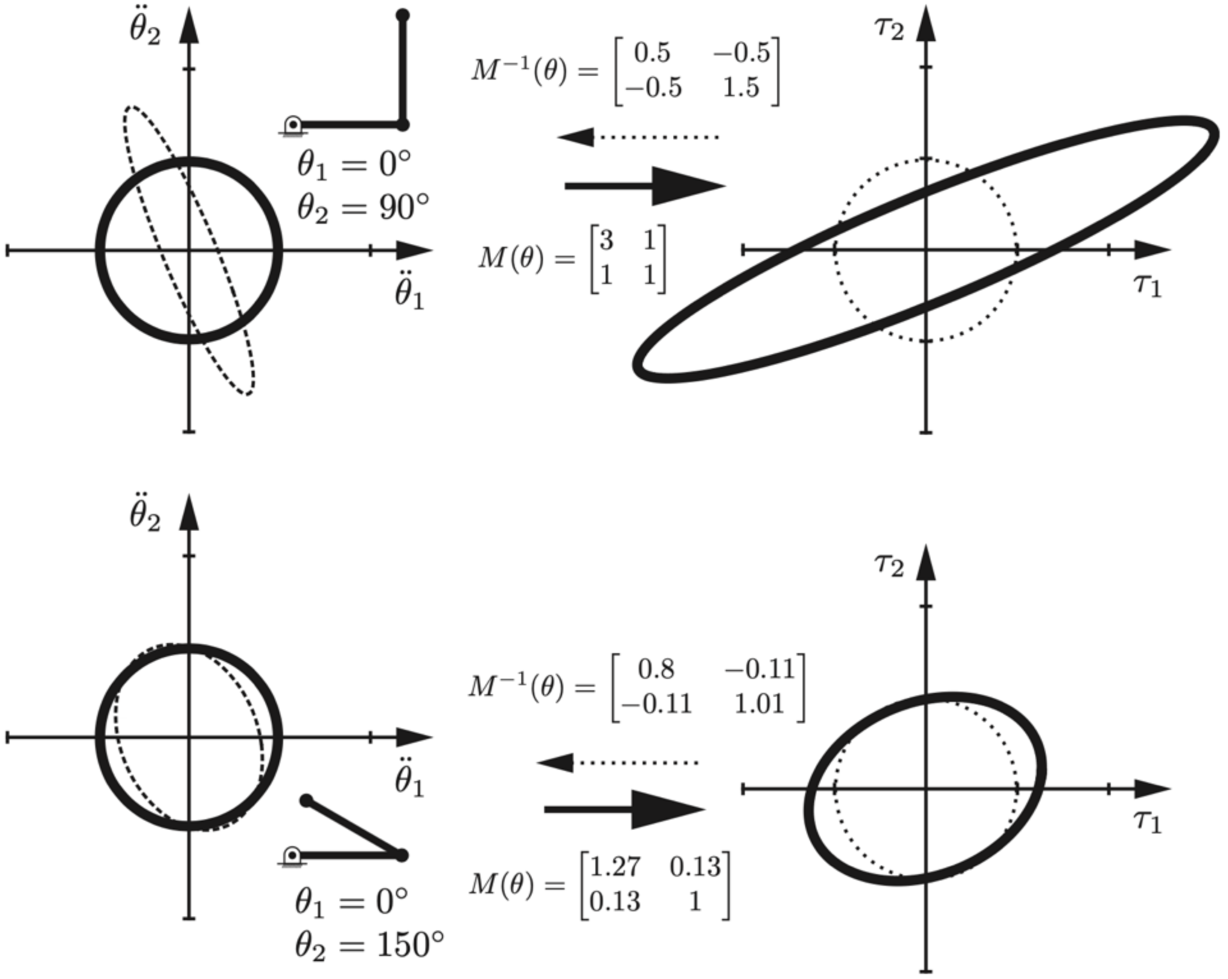
Important concepts, symbols, and equations

$$\tau = M(\theta)\ddot{\theta} + c(\theta,\dot{\theta}) + g(\theta)$$

kinetic energy of a robot:

$$\mathcal{K}(\theta,\dot{\theta}) = \frac{1}{2}\dot{\theta}^{\mathrm{T}} M(\theta)\,\dot{\theta}$$

When $\dot{\theta} = 0$ and $g = 0$,
$M(\theta)$ maps $\ddot{\theta}$ to τ and
$M^{-1}(\theta)$ maps τ to $\ddot{\theta}$



Important concepts, symbols, and equations (cont.)

If $V = J(\theta)\,\dot{\theta}$ is the e-e velocity and J is invertible (there exists a unique joint velocity for each e-e velocity):

$$\frac{1}{2} V^{\mathrm{T}} \Lambda(\theta) V = \frac{1}{2} \dot{\theta}^{\mathrm{T}} M(\theta)\, \dot{\theta}$$

$$\Downarrow$$

$$\dot{\theta}^{\mathrm{T}} J^{\mathrm{T}}(\theta)\, \Lambda(\theta)\, J(\theta)\, \dot{\theta} = \dot{\theta}^{\mathrm{T}} M(\theta)\, \dot{\theta}$$

$$\Downarrow$$

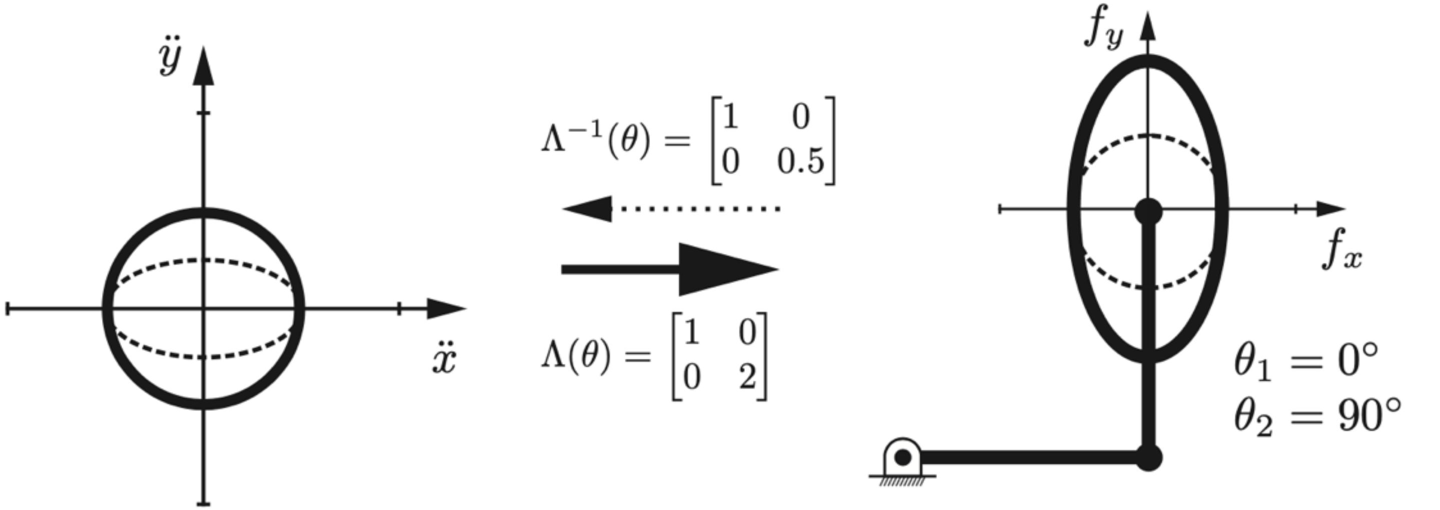
$$\Lambda(\theta) = J^{-\mathrm{T}}(\theta)\, M(\theta)\, J^{-1}(\theta)$$

end-effector mass matrix

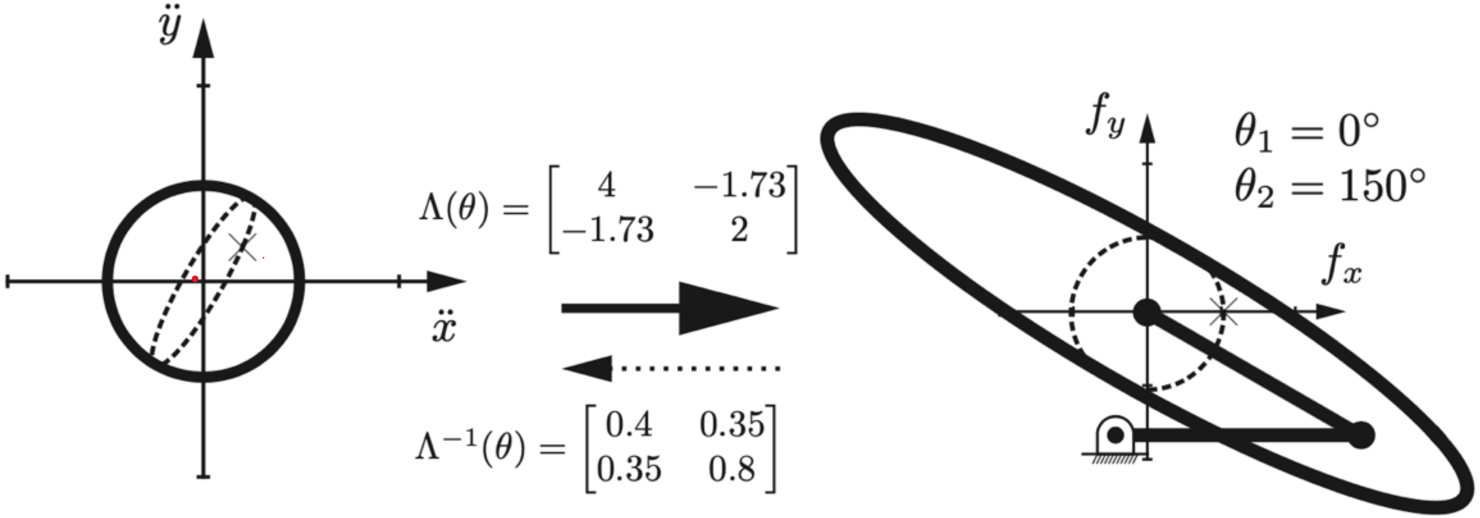
What if J is tall? wide?

Important concepts, symbols, and equations (cont.)

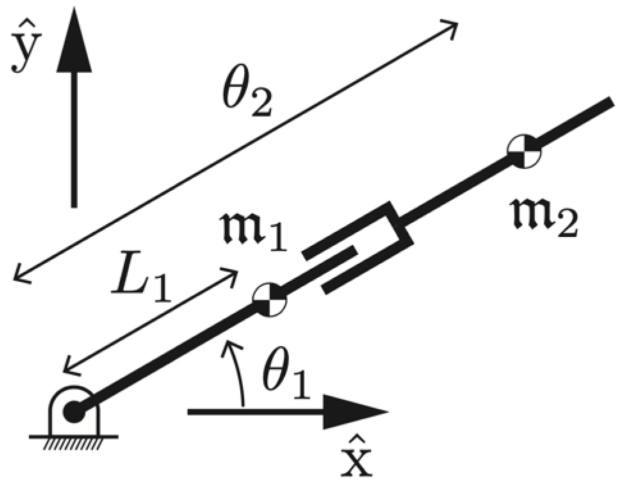
When $\dot{\theta} = 0$ and $g = 0$, $\Lambda(\theta)$ maps $\dot{V}$ to F and $\Lambda^{-1}(\theta)$ maps F to $\dot{V}$



Force and acceleration are only parallel along principal axes.



RP robot



$\dot{\theta} = 0$ and $g = 0$

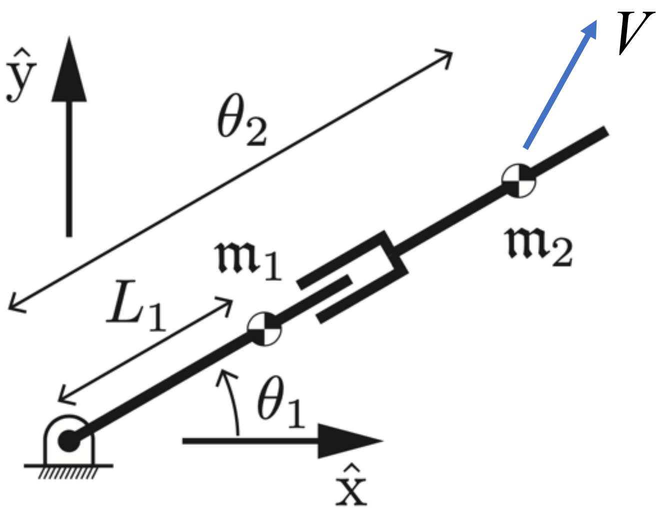
$$\tau = M(\theta)\ddot{\theta} + c(\theta, \dot{\theta}) + g(\theta)$$

$$M(\theta) = \begin{bmatrix} \mathcal{I}_1 + \mathcal{I}_2 + \mathfrak{m}_1 L_1^2 + \mathfrak{m}_2 \theta_2^2 & 0 \\ 0 & \mathfrak{m}_2 \end{bmatrix}$$

What are the e-vals and e-vecs of M?

Draw the ellipse of τ corresponding to a unit circle of $\ddot{\theta}$ as θ_2 increases from zero and $I_1 = I_2 = \mathfrak{m}_1 = \mathfrak{m}_2 = L_1 = 1$.

RP robot



$\dot{\theta} = 0$ and $g = 0$

At $\theta_1 = 0$, the e-e mass matrix is

$$\Lambda(\theta) = \begin{bmatrix} \mathfrak{m}_2 & 0 \\ 0 & (\mathcal{I}_1 + \mathcal{I}_2 + \mathfrak{m}_1 L_1^2 + \mathfrak{m}_2 \theta_2^2)/\theta_2^2 \end{bmatrix}$$

Draw the ellipse of F corresponding to a unit circle of $\dot{V}$ as θ_2 increases from zero and $I_1 = I_2 = \mathfrak{m}_1 = \mathfrak{m}_2 = L_1 = 1$. How does it change as θ_1 changes?