

Where we are:

Chap 2	Configuration Space
Chap 3	Rigid-Body Motions
Chap 4	Forward Kinematics
Chap 5	Velocity Kinematics and Statics
Chap 6	Inverse Kinematics
Chap 8	Dynamics of Open Chains
	8.1 Lagrangian Formulation
Chap 9	Trajectory Generation
Chap 11	Robot Control
Chap 13	Wheeled Mobile Robots

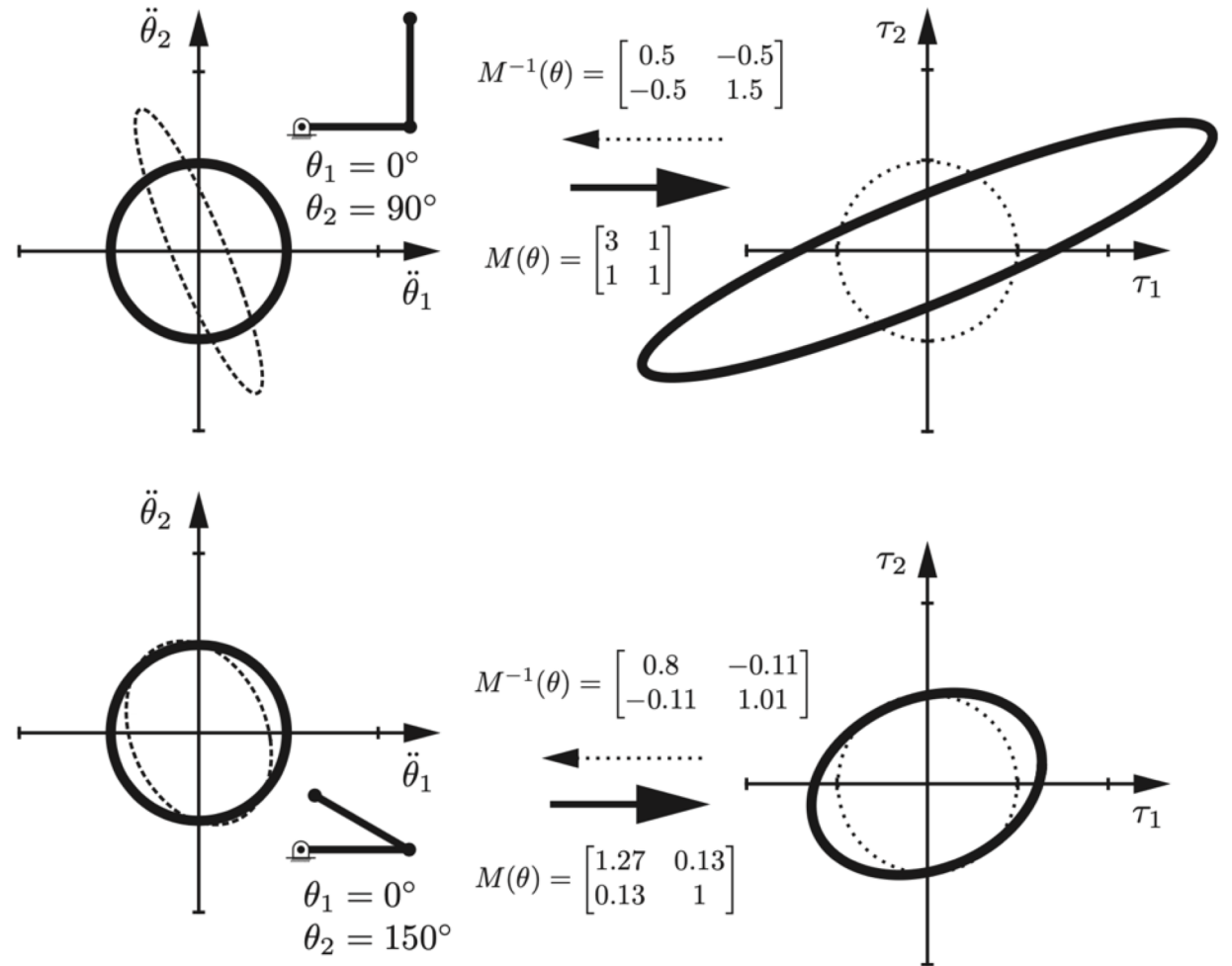
Important concepts, symbols, and equations

$$\tau = M(\theta)\ddot{\theta} + c(\theta, \dot{\theta}) + g(\theta)$$

kinetic energy of a robot:

$$\mathcal{K}(\theta, \dot{\theta}) = \frac{1}{2} \dot{\theta}^T M(\theta) \dot{\theta}$$

When $\dot{\theta} = 0$ and $g = 0$,
 $M(\theta)$ maps $\ddot{\theta}$ to τ and
 $M^{-1}(\theta)$ maps τ to $\ddot{\theta}$



Important concepts, symbols, and equations (cont.)

If $V = J(\theta) \dot{\theta}$ is the e-e velocity and J is invertible (there exists a unique joint velocity for each e-e velocity):

$$\frac{1}{2} V^T \Lambda(\theta) V = \frac{1}{2} \dot{\theta}^T M(\theta) \dot{\theta}$$

$$\dot{\theta}^T J^T(\theta) \Lambda(\theta) J(\theta) \dot{\theta} = \dot{\theta}^T M(\theta) \dot{\theta}$$

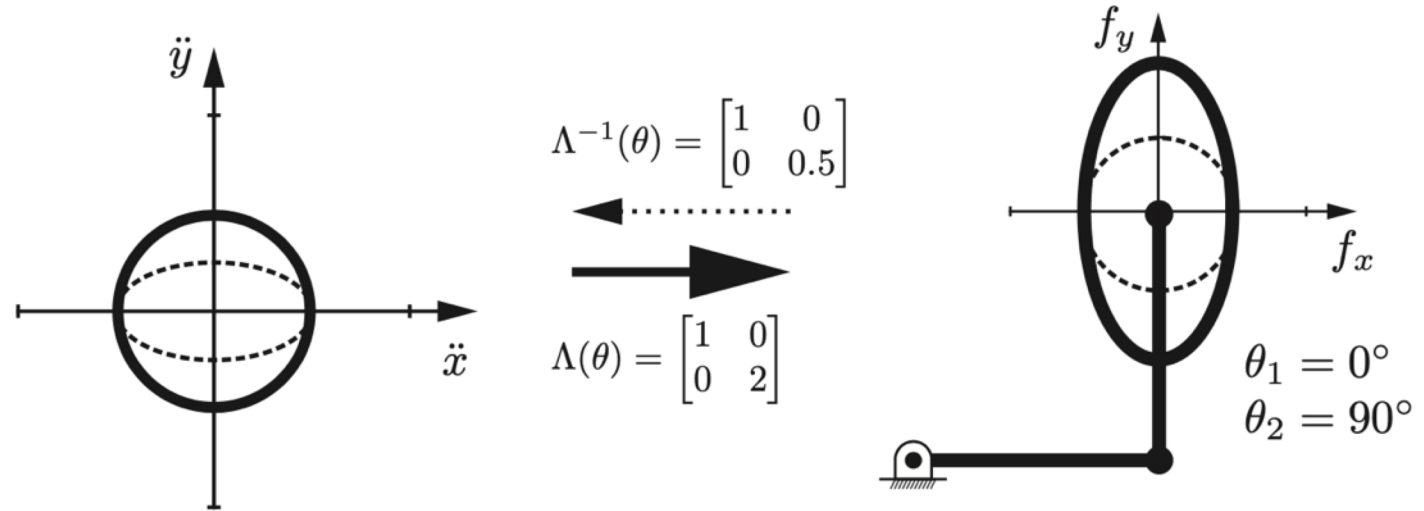
$$\Lambda(\theta) = J^{-T}(\theta) M(\theta) J^{-1}(\theta)$$

end-effector mass matrix

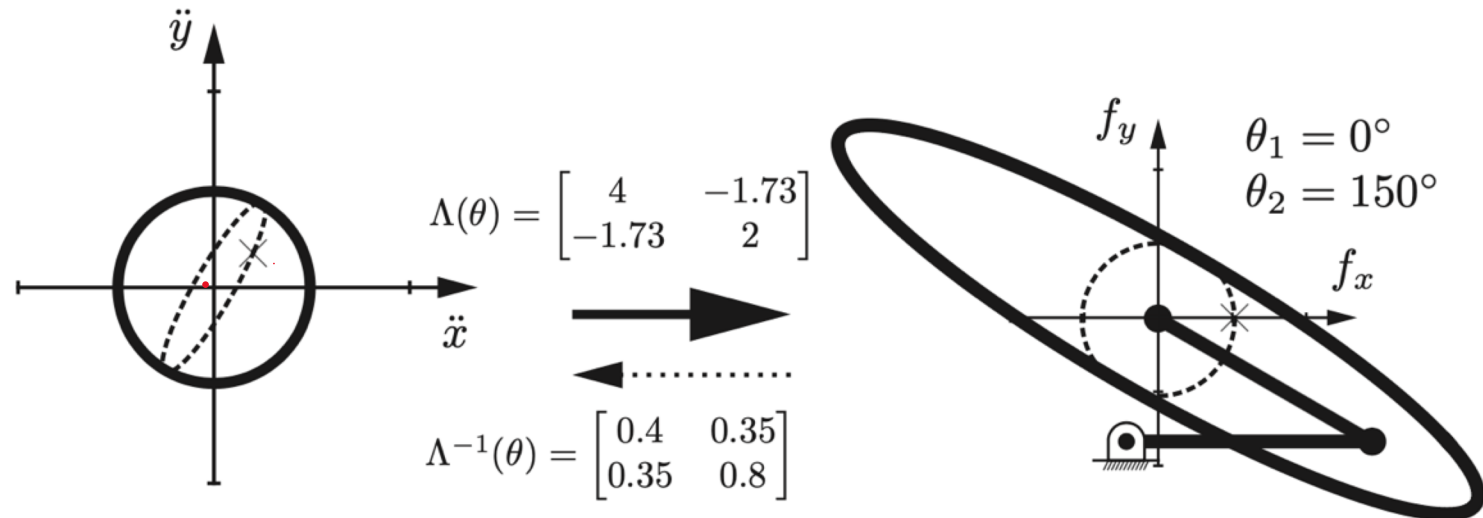
What if J is tall? wide?

Important concepts, symbols, and equations (cont.)

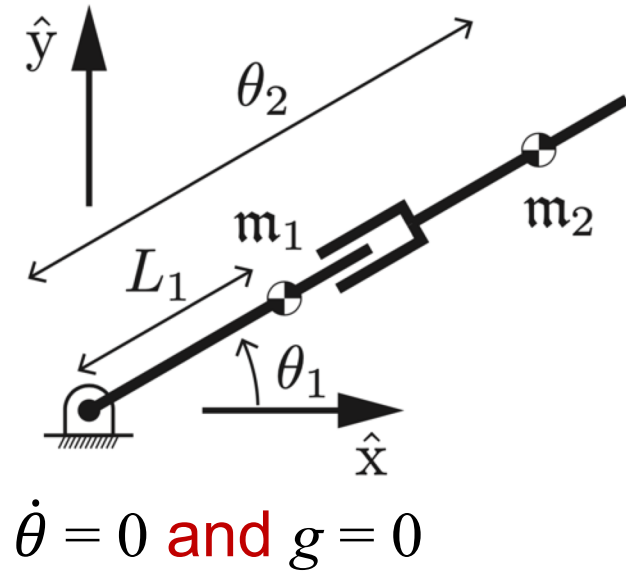
When $\dot{\theta} = 0$ and $g = 0$,
 $\Lambda(\theta)$ maps \dot{V} to F and
 $\Lambda^{-1}(\theta)$ maps F to \dot{V}



Force and acceleration
are only parallel along
principal axes.



RP robot



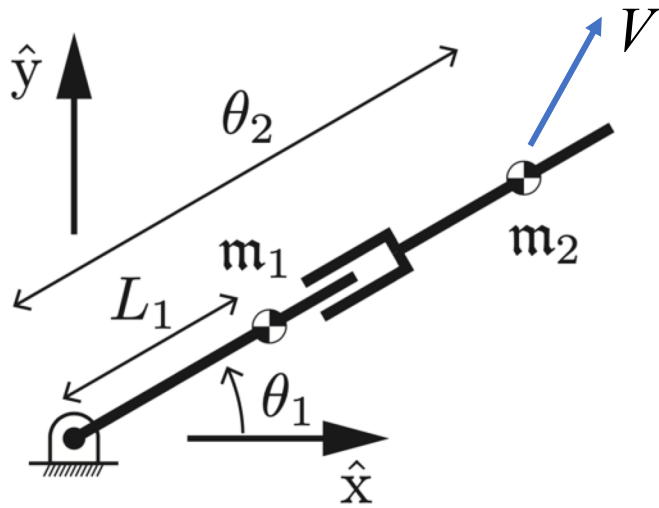
$$\tau = M(\theta)\ddot{\theta} + c(\theta, \dot{\theta}) + g(\theta)$$

$$M(\theta) = \begin{bmatrix} \mathcal{I}_1 + \mathcal{I}_2 + m_1 L_1^2 + m_2 \theta_2^2 & 0 \\ 0 & m_2 \end{bmatrix}$$

What are the e-vals and e-vecs of M ?

Draw the ellipse of τ corresponding to a unit circle of $\ddot{\theta}$ as θ_2 increases from zero and $I_1 = I_2 = m_1 = m_2 = L_1 = 1$.

RP robot



$\dot{\theta} = 0$ and $g = 0$

At $\theta_1 = 0$, the e-e mass matrix is

$$\Lambda(\theta) = \begin{bmatrix} m_2 & 0 \\ 0 & (\mathcal{I}_1 + \mathcal{I}_2 + m_1 L_1^2 + m_2 \theta_2^2) / \theta_2^2 \end{bmatrix}$$

Draw the ellipse of F corresponding to a unit circle of \dot{V} as θ_2 increases from zero and $I_1 = I_2 = m_1 = m_2 = L_1 = 1$. How does it change as θ_1 changes?