**Where we are:**

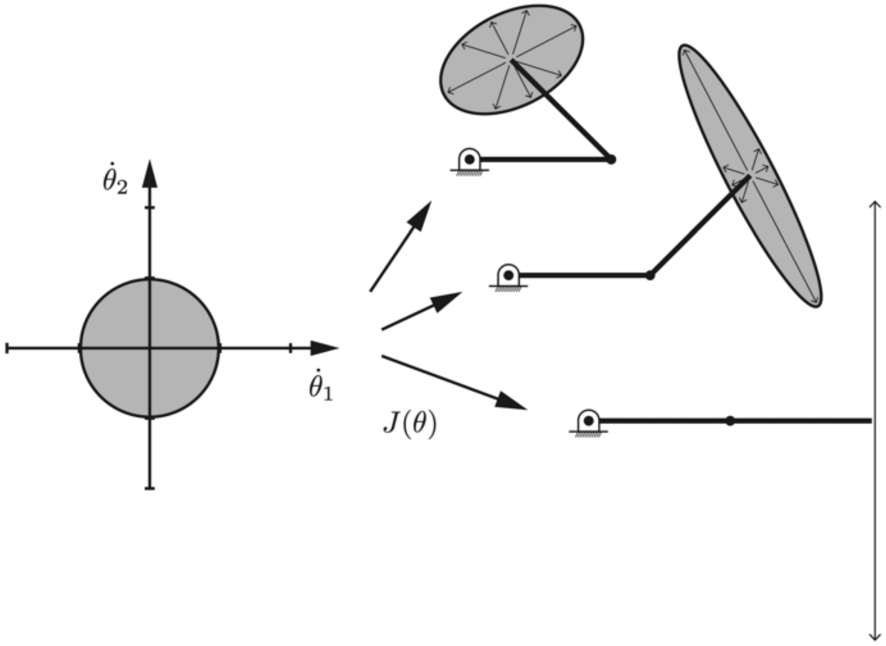
# Important concepts, symbols, and equations

- A configuration $\theta \in \mathbb{R}^n$ is **singular** if $\text{rank}(J(\theta))$ is less than its maximum rank over all $\theta$.

- $\text{rank}(J_s(\theta)) = \text{rank}(J_b(\theta))$ ; singularities are independent of frame.

- Some common sources of singularities for six-dof spatial open chains:

  i. two collinear revolute joint axes
  ii. three coplanar and parallel revolute joint axes
  iii. four revolute joint axes intersecting at a common point
  iv. four coplanar revolute joints
  v. six revolute joint axes intersecting a common line



## Important concepts, symbols, and equations (cont.)

- A **fat** or **wide** Jacobian (or matrix generally) has more columns than rows. A **skinny** or **tall** Jacobian has more rows than columns.

- A robot is **redundant** for a task if there exists a family of $\dot{\theta}$ satisfying $v = J(\theta)\,\dot{\theta}$ at most $\theta$, where $v$ could be a twist $\mathcal{V}$ or a coordinate velocity. (wide Jacobians)

- A robot is **kinematically deficient** for a task if there are required velocities $v$ for which there is no $\dot{\theta}$ satisfying $v = J(\theta)\,\dot{\theta}$ at most $\theta$. (tall Jacobians)

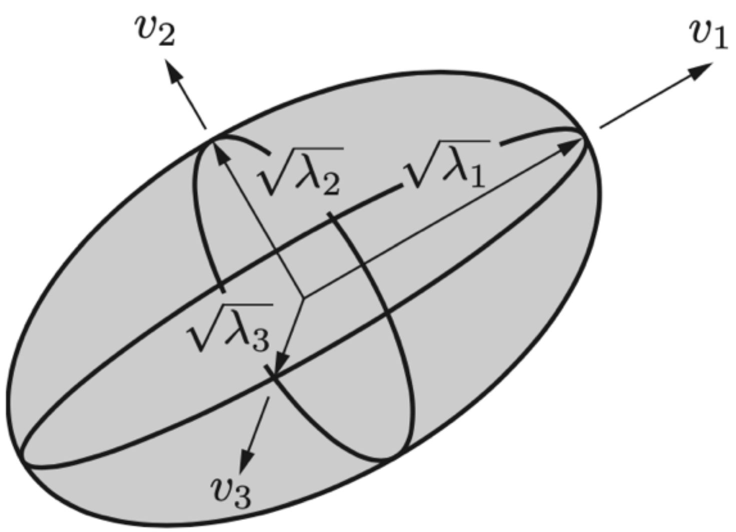
- Redundancy and deficiency depend on the robot and the task (e.g., dimensions of the Jacobian); singularities depend on $\theta$.

# Important concepts, symbols, and equations (cont.)

- The **manipulability** of a robot at a configuration $\theta$ measures how close it is to being singular.

- The **manipulability ellipsoid** defines the ease of motion in different directions.

- The principal axes of the ellipsoid are given by the eigenvectors and square roots of the eigenvalues of the square matrix $JJ^{\mathrm{T}}$.




- Manipulability measures include
  1. the ratio of the largest to smallest principal axis half-lengths
  2. the ratio of the largest to smallest eigenvalues
  3. the volume of the ellipsoid (proportional to $\sqrt{\lambda_1 \lambda_2 \ldots}$).

## Important concepts, symbols, and equations (cont.)

- Split the body Jacobian into angular and linear components to to get angular velocity and linear velocity ellipsoids.

$$J_b(\theta) = \begin{bmatrix} J_{b\omega}(\theta) \\ J_{bv}(\theta) \end{bmatrix} \in \mathbb{R}^{6\times n}$$

$$J_{b\omega}(\theta) \in \mathbb{R}^{3\times n} \rightarrow \text{angular velocity/moment ellipsoids}$$

$$J_{bv}(\theta) \in \mathbb{R}^{3\times n} \rightarrow \text{linear velocity/force ellipsoids}$$

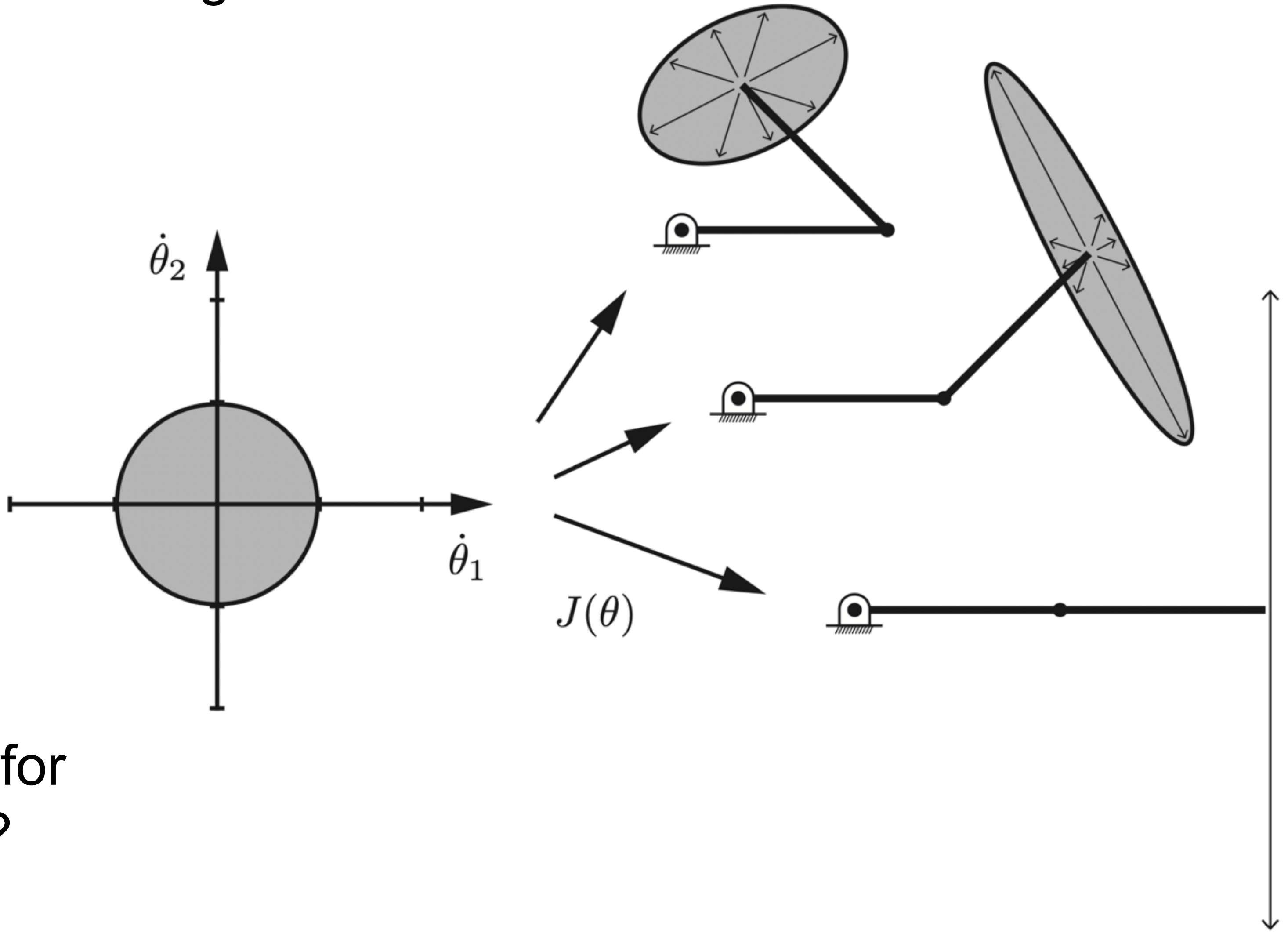
## Important concepts, symbols, and equations (cont.)

- The principal axes of the force (wrench) ellipsoid are given by the eigenvectors and square roots of the eigenvalues of $(JJ^{\mathrm{T}})^{-1}$.

- The force ellipsoid has the same principal axes as the manipulability ellipsoid, and the principal axis half-lengths are the reciprocal of those of the manipulability ellipsoid.

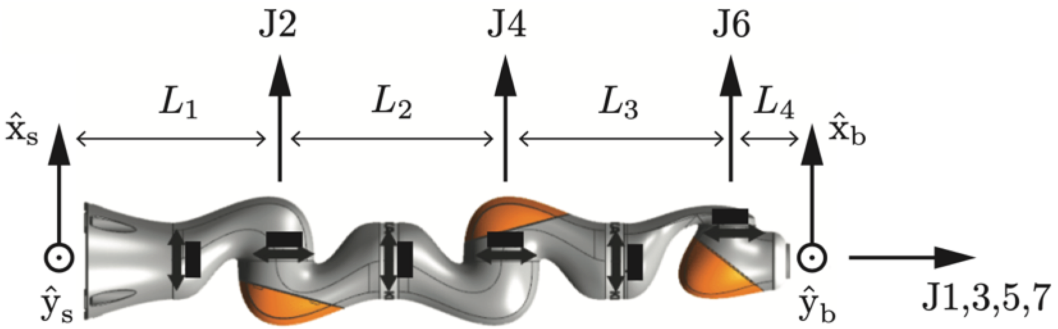
- The product of the volumes of the force and manipulability ellipsoids is constant over $\theta$.

Approximately draw the corresponding force ellipsoids at the three configurations.




Why use $J_b$ instead of $J_s$ for manipulability ellipsoids?


Adept SCARA RRRP robot




KUKA LBR iiwa 7R robot

UR5 6R robot

If using a twist to represent the e-e velocity, what are the dimensions of each Jacobian?

For the task of manipulating a rigid body, which of these is redundant or deficient?

What is the rank of the Jacobian of the iiwa at its home configuration (shown)?