Where we are:

Chap 2 Configuration Space
Chap 3 Rigid-Body Motions
Chap 4 Forward Kinematics
Chap 5 Velocity Kinematics and Statics
Chap 6 Inverse Kinematics
Chap 8 Dynamics of Open Chains
Chap 9 Trajectory Generation
- 9.1 Definitions
- 9.2 Point-to-Point Trajectories
- 9.3 Polynomial Via Point Trajectories
- 9.4 Time-Optimal Time Scaling

Chap 11 Robot Control
Chap 13 Wheeled Mobile Robots

Important concepts, symbols, and equations

Time-optimal time scaling: Find the time scaling $s(t)$ that moves the robot along a path from $\theta(0)$ to $\theta(1)$ in minimum time, given the robot's dynamics and actuator limits.

$$\boxed{\begin{aligned} \dot\theta &= \frac{d\theta}{ds}\dot s, \\ \ddot\theta &= \frac{d\theta}{ds}\ddot s + \frac{d^2\theta}{ds^2}\dot s^2 \end{aligned}}$$

Dynamics:

$$M(\theta)\ddot\theta + \dot\theta^{\mathrm{T}}\Gamma(\theta)\dot\theta + g(\theta) = \tau$$

Dynamics along the path:

$$\underbrace{\left(M(\theta(s))\frac{d\theta}{ds}\right)}_{m(s)\in\mathbb{R}^n}\ddot s + \underbrace{\left(M(\theta(s))\frac{d^2\theta}{ds^2} + \left(\frac{d\theta}{ds}\right)^{\mathrm{T}}\Gamma(\theta(s))\frac{d\theta}{ds}\right)}_{c(s)\in\mathbb{R}^n}\dot s^2 + \underbrace{g(\theta(s))}_{g(s)\in\mathbb{R}^n} = \tau$$

$$m(s)\ddot s + c(s)\dot s^2 + g(s) = \tau$$

Important concepts, symbols, and equations (cont.)

Force/torque limits, joint i:

$$\tau_i^{\min}(s, \dot{s}) \le \tau_i \le \tau_i^{\max}(s, \dot{s})$$

Limits on $\ddot{s}$ as a function of $(s, \dot{s})$, joint i:

$$\tau_i^{\min}(s, \dot{s}) \le m_i(s)\ddot{s} + c_i(s)\dot{s}^2 + g_i(s) \le \tau_i^{\max}(s, \dot{s})$$

Important concepts, symbols, and equations (cont.)

Upper and lower bounds on $\ddot{s}$ due to joint i:

$$\text{if } m_i(s) > 0, \quad L_i(s,\dot{s}) = \frac{\tau_i^{\min}(s,\dot{s}) - c(s)\dot{s}^2 - g(s)}{m_i(s)},$$

$$U_i(s,\dot{s}) = \frac{\tau_i^{\max}(s,\dot{s}) - c(s)\dot{s}^2 - g(s)}{m_i(s)}$$

$$\text{if } m_i(s) < 0, \quad L_i(s,\dot{s}) = \frac{\tau_i^{\max}(s,\dot{s}) - c(s)\dot{s}^2 - g(s)}{m_i(s)},$$

$$U_i(s,\dot{s}) = \frac{\tau_i^{\min}(s,\dot{s}) - c(s)\dot{s}^2 - g(s)}{m_i(s)}$$

Important concepts, symbols, and equations (cont.)

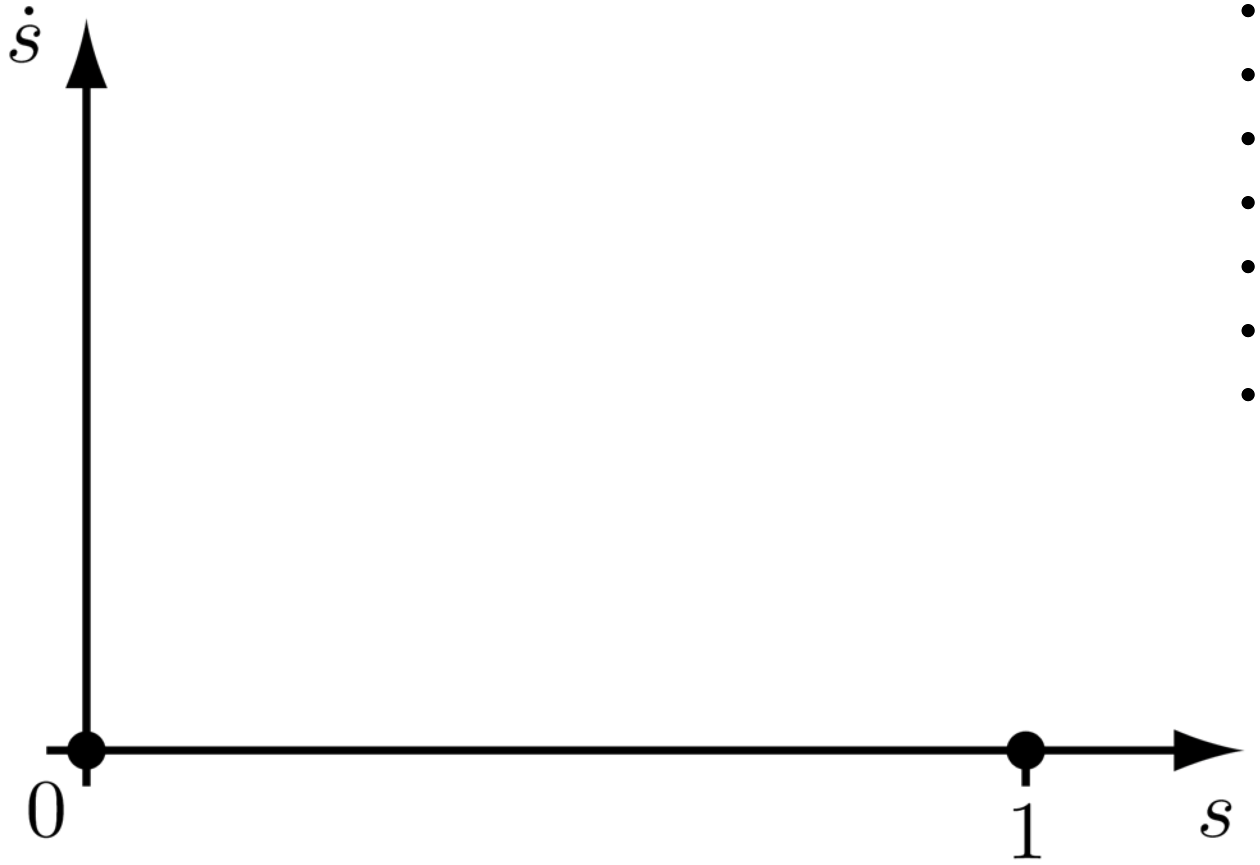
Upper and lower bounds on $\ddot{s}$ at $(s, \dot{s})$:

$$L(s,\dot{s}) = \max_i L_i(s,\dot{s}) \qquad \text{and} \qquad U(s,\dot{s}) = \min_i U_i(s,\dot{s})$$

$$\boxed{L(s,\dot{s}) \leq \ddot{s} \leq U(s,\dot{s})}$$

Important concepts, symbols, and equations (cont.)

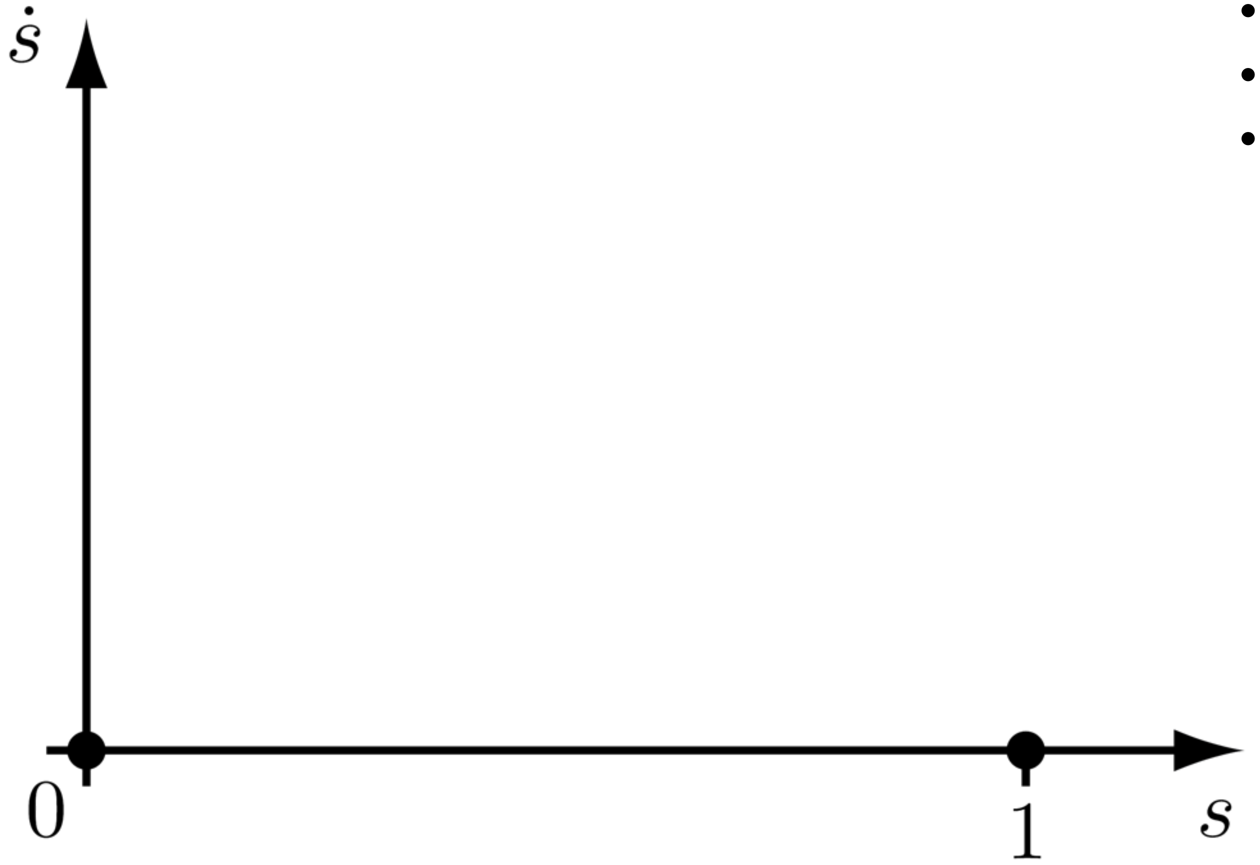
The $(s, \dot{s})$ plane



- Why we only consider quadrant I.
- A slow motion along the path.
- An example rate of change $(\dot{s}, \ddot{s})$ plotted at $(s, \dot{s})$.
- A **motion cone** defined by $L(s, \dot{s})$ and $U(s, \dot{s})$.
- Cones on the $\dot{s} = 0$ line.
- What does a constant accel curve look like?
- What does a constant decel curve look like?

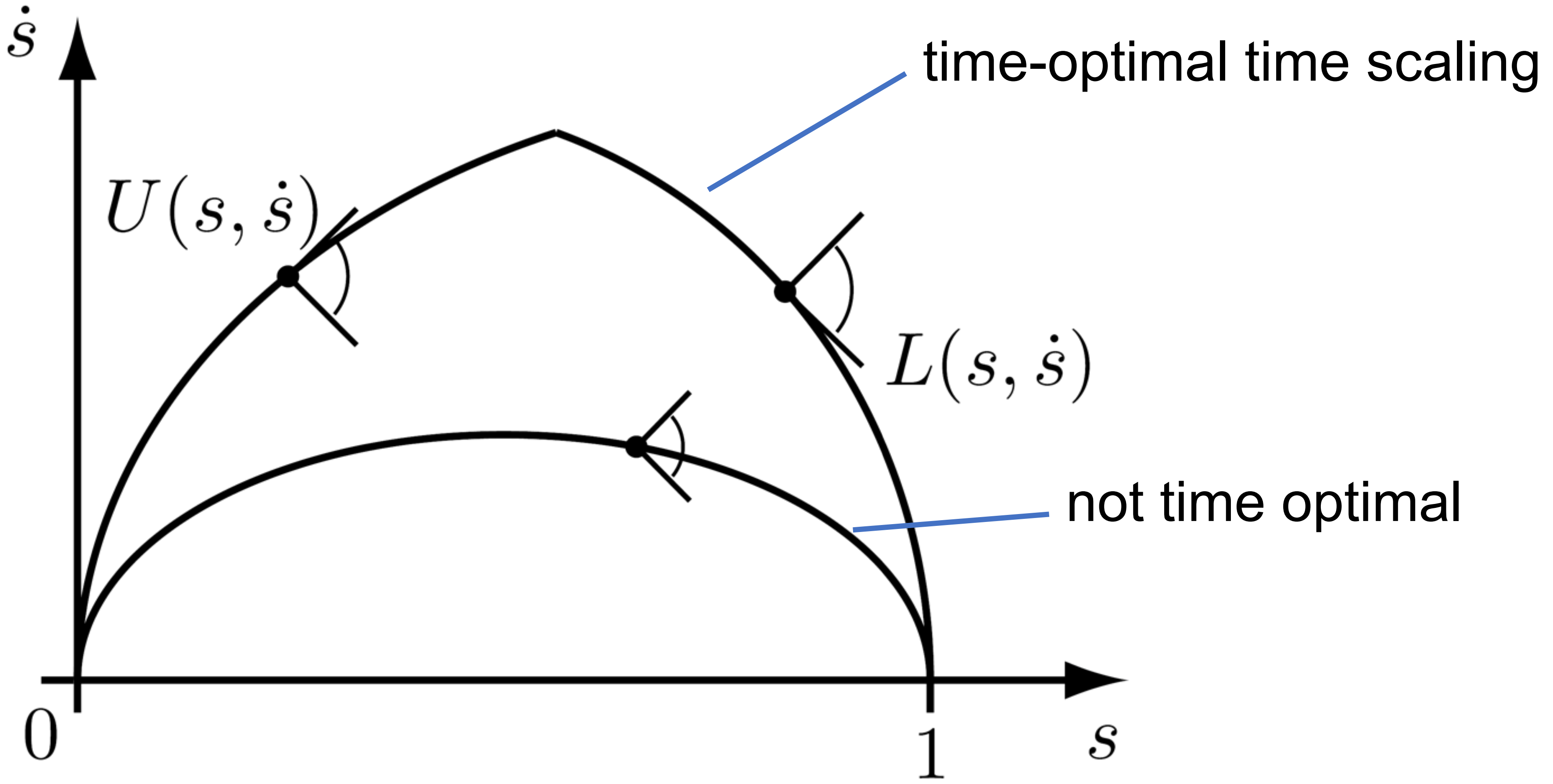
Important concepts, symbols, and equations (cont.)

The $(s, \dot{s})$ plane

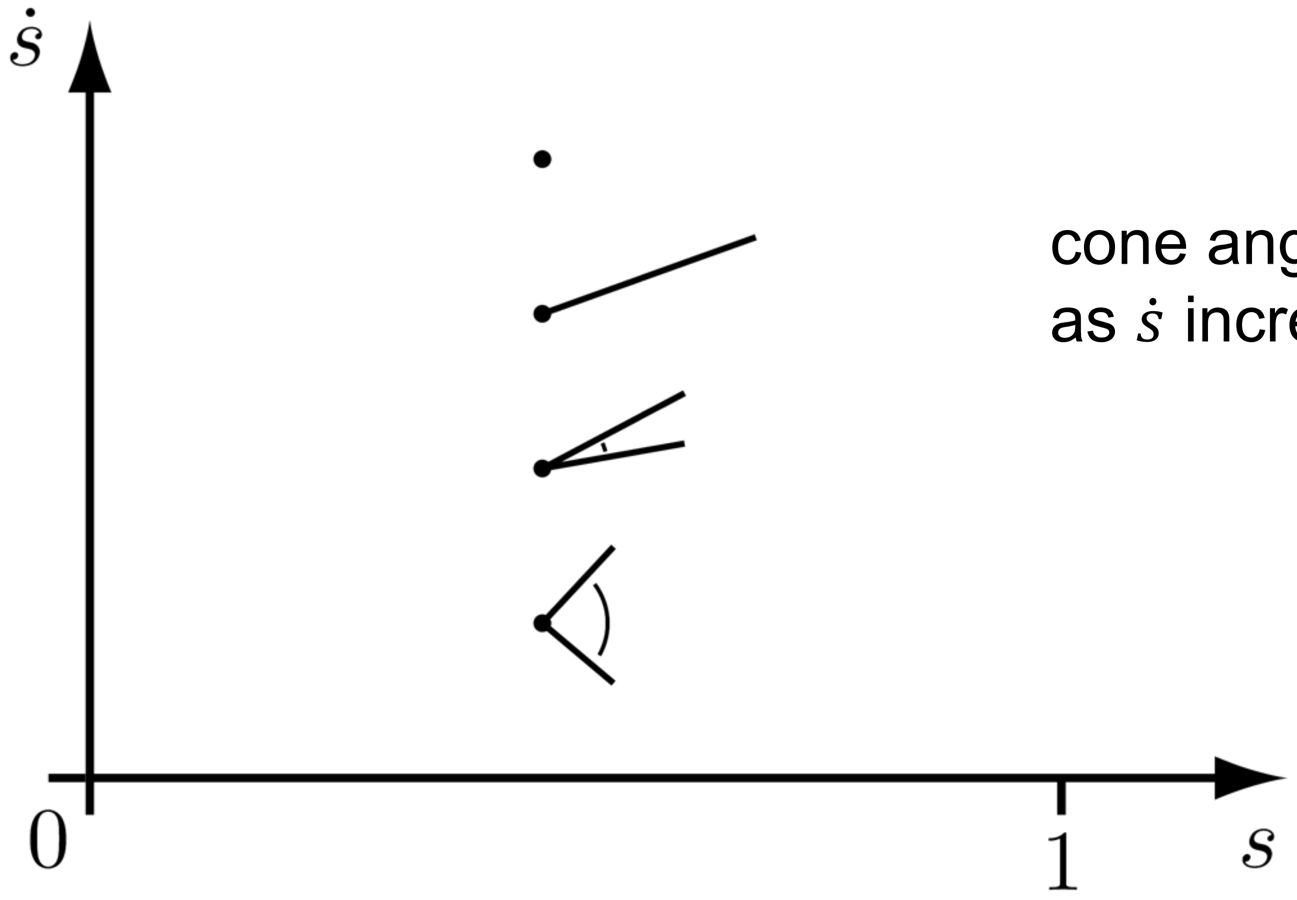


- Plot a complete time scaling.
- Check the feasibility of a time scaling.
- What happens at $(s, \dot{s})$ where $L(s, \dot{s})$ > $U(s, \dot{s})$?

Important concepts, symbols, and equations (cont.)

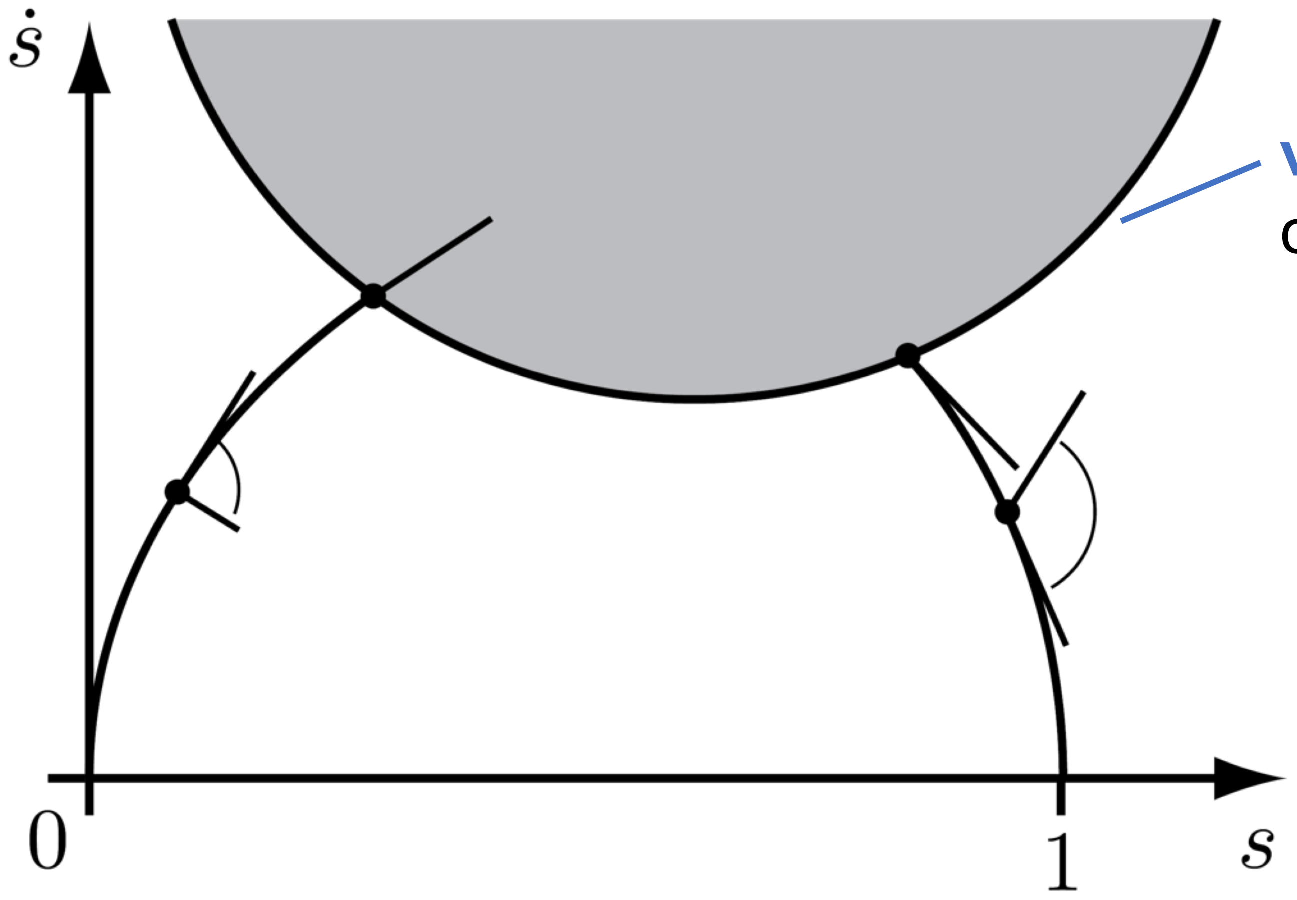


Important concepts, symbols, and equations (cont.)



cone angle tends to shrink
as $\dot{s}$ increases

Important concepts, symbols, and equations (cont.)



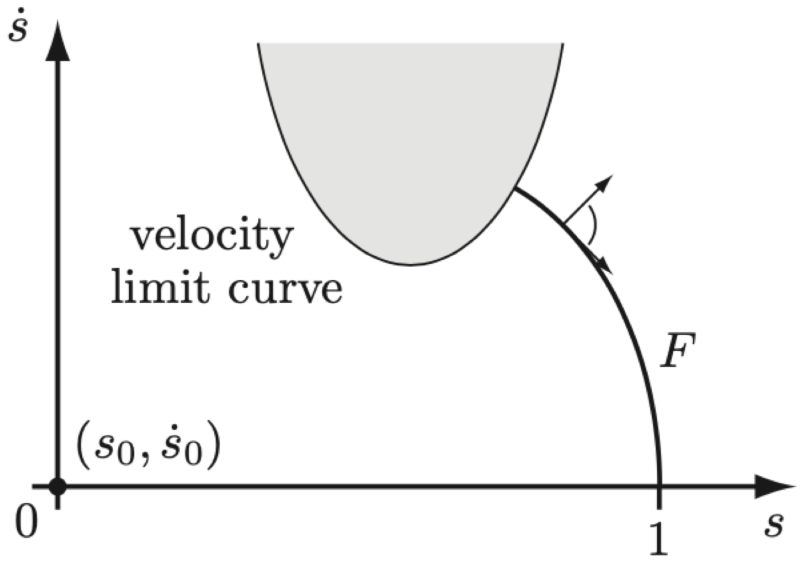
velocity limit curve where the cone is only a single tangent vector

Important concepts, symbols, and equations (cont.)

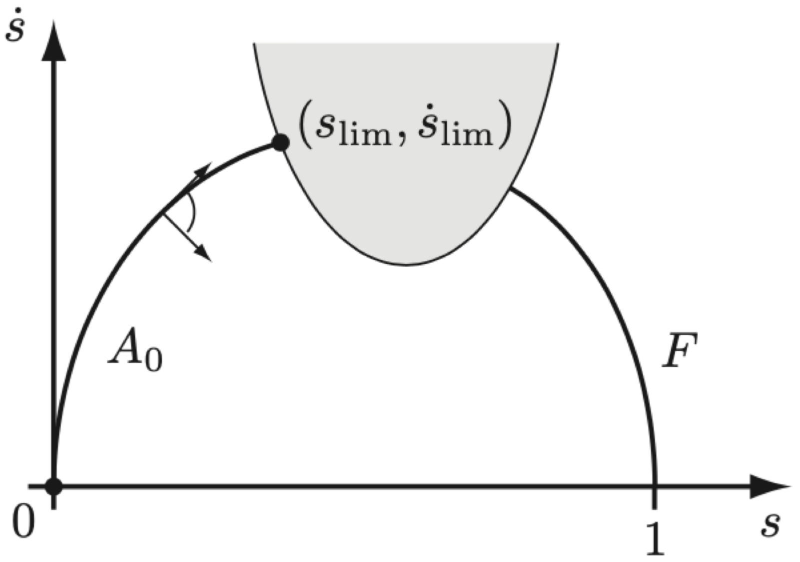
Time-optimal time-scaling algorithm

1. Initialize with $i = 0$ and $(s_0, \dot{s}_0) = (0, 0)$.
2. Integrate L backward from $(1, 0)$ to get the curve F.
3. Integrate U forward from $(s_i, \dot{s}_i)$ to get A_i. If it intersects F, finished.
4. Binary search to find how to start decelerating from A_i.
5. Increment i and integrate backwards to get A_i.
6. Update i and $(s_i, \dot{s}_i)$ and go back to step 3.

Important concepts, symbols, and equations (cont.)

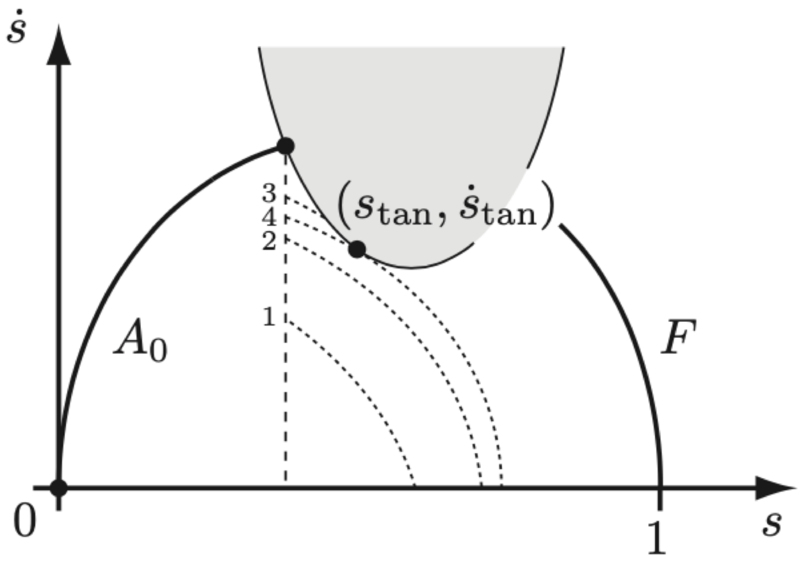


Step 2: $i = 0, \mathcal{S} = \{\}$

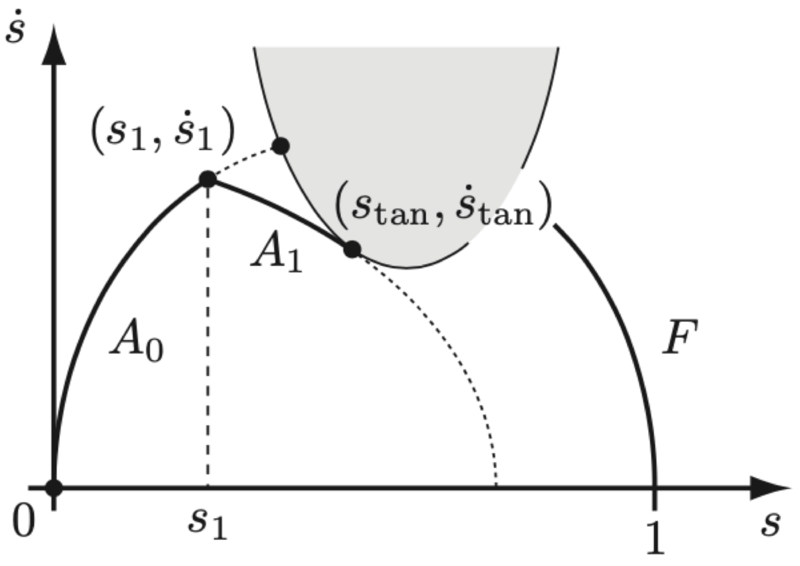


Step 3: $i = 0, \mathcal{S} = \{\}$

Important concepts, symbols, and equations (cont.)

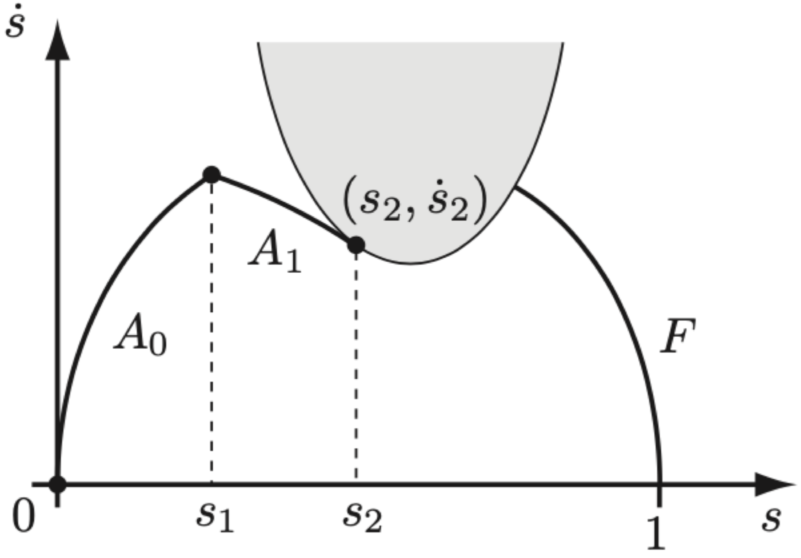


Step 4: $i = 0, \mathcal{S} = \{\}$

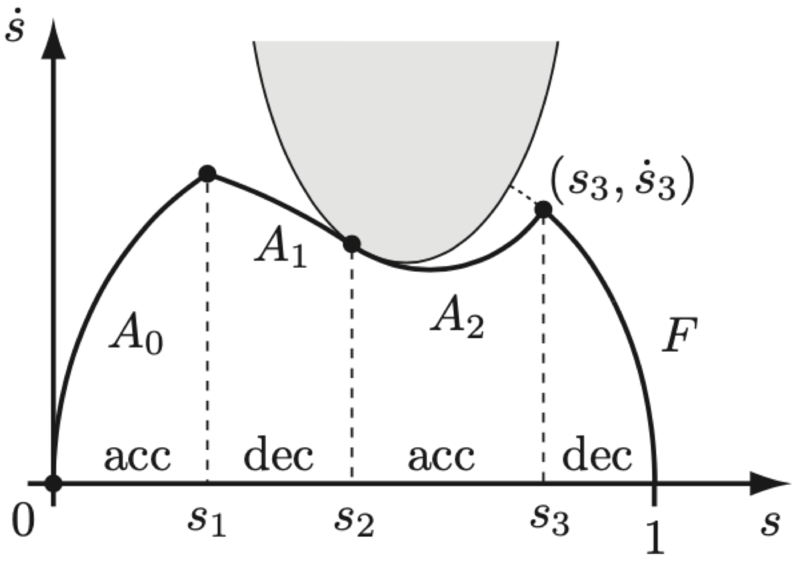


Step 5: $i = 1, \mathcal{S} = \{s_1\}$

<u>Important concepts, symbols, and equations</u> (cont.)

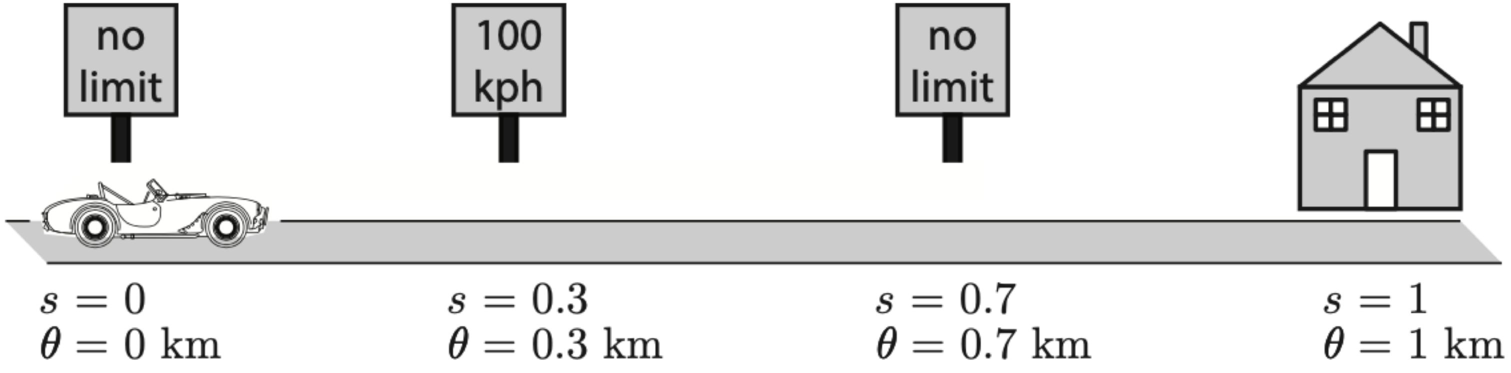


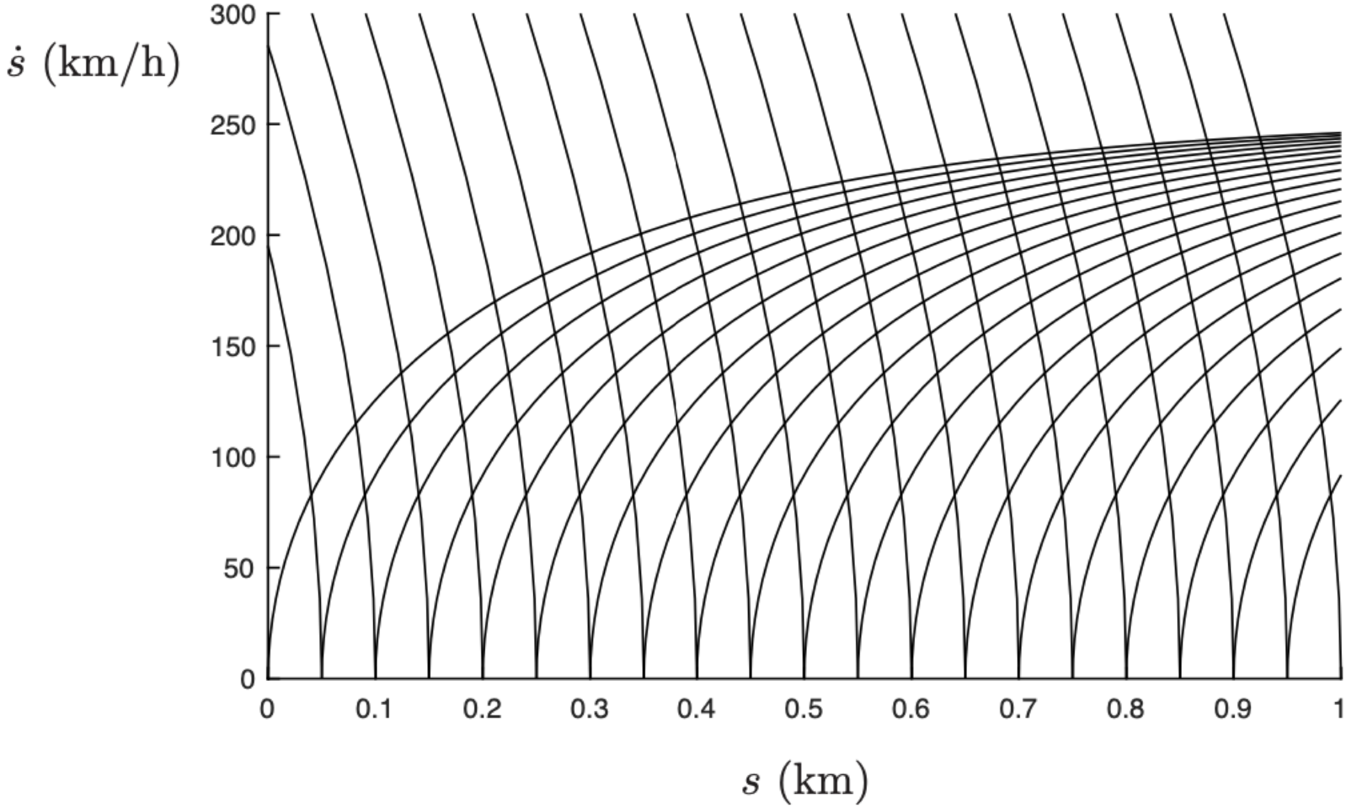
Step 6: $i = 2, \mathcal{S} = \{s_1, s_2\}$

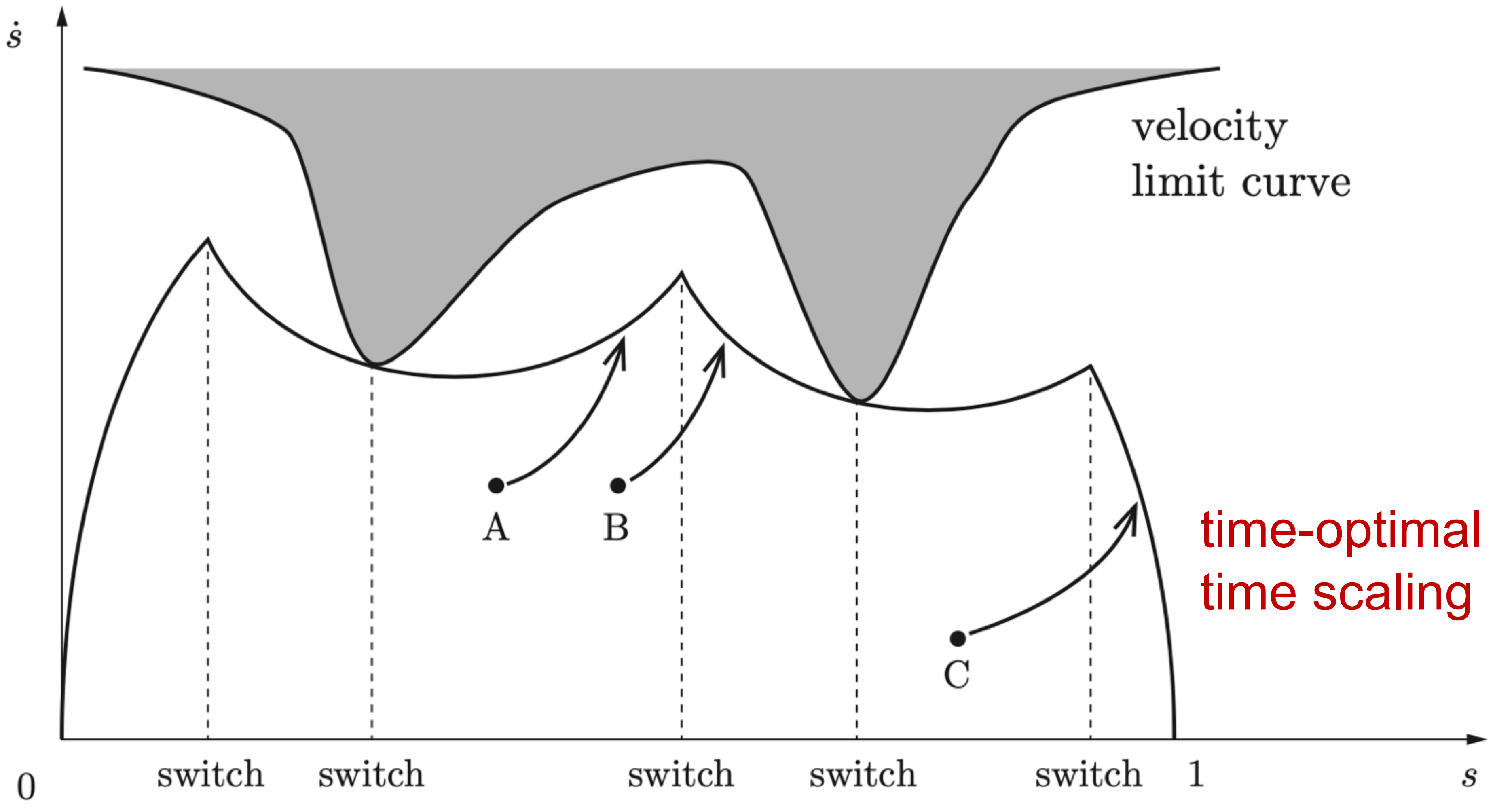


Step 3: $i = 3, \mathcal{S} = \{s_1, s_2, s_3\}$

Draw the **legal** time-optimal time scaling for a driver rushing home with the max braking and max acceleration **integral curves** shown.







1. Where do we know how to draw the motion cones, just from this plot?
2. Where is the robot able to stay on the path, but is doomed to leave it?
3. Can we get back to the optimal time scaling by the trajectories shown from A, B, C?