

Ben Richardson
ME 449

Problem Set 3

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$$1) \quad f(x, y) = \begin{bmatrix} x^2 - 4 \\ y^2 - 9 \end{bmatrix} \quad (x_1, y_1) = (1, 1)$$

$$\nabla f = \begin{bmatrix} 2x & 0 \\ 0 & 2y \end{bmatrix} \quad 3$$

$$\text{we know } \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} - (\nabla f|_{x_1, y_1})^{-1} f(x_1, y_1)$$

$$\nabla f(x_1, y_1) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \Rightarrow (\nabla f(x_1, y_1))^{-1} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} -3 \\ -8 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 3/2 \\ 4 \end{bmatrix} = \begin{bmatrix} 5/2 \\ 5 \end{bmatrix}$$

1st iteration

$$\begin{bmatrix} x_3 \\ y_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} - (\nabla f(x_2, y_2))^{-1} f(x_2, y_2)$$

$$\nabla f(x_2, y_2) = \begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix} \Rightarrow (\nabla f(x_2, y_2))^{-1} = \begin{bmatrix} 1/5 & 0 \\ 0 & 1/10 \end{bmatrix} \quad 4$$

$$\Rightarrow \begin{bmatrix} x_3 \\ y_3 \end{bmatrix} = \begin{bmatrix} 5/2 \\ 5 \end{bmatrix} - \begin{bmatrix} 1/5 & 0 \\ 0 & 1/10 \end{bmatrix} \begin{bmatrix} 9/4 \\ 16 \end{bmatrix} = \begin{bmatrix} 5/2 \\ 5 \end{bmatrix} - \begin{bmatrix} 9/20 \\ 8/5 \end{bmatrix} = \begin{bmatrix} 41/20 \\ 17/5 \end{bmatrix} = \begin{bmatrix} x_3 \\ y_3 \end{bmatrix}$$

2nd iteration

The roots are $x = \pm 2, y = \pm 3$ 3

$$2) \quad S_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad S_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad S_3 = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \quad M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Using } e^{[\omega] \theta} = I + \sin \theta [\omega] + (1 - \cos \theta) [\omega]^2$$

$$\Rightarrow e^{[S_1] \theta_1} = \begin{bmatrix} C_1 & -S_1 & 0 \\ S_1 & C_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad e^{[S_2] \theta_2} = \begin{bmatrix} C_2 & 0 & S_2 \\ 0 & 1 & 0 \\ -S_2 & 0 & C_2 \end{bmatrix}; \quad e^{[S_3] \theta_3} = \begin{bmatrix} C_3 & -S_3 & 0 \\ S_3 & C_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$a) \quad \mathcal{V}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \mathcal{V}_2 = \text{Ad}_{e^{[S_1] \theta_1}}(S_2) = \begin{bmatrix} -S_1 \\ C_1 \\ 0 \end{bmatrix} \quad \mathcal{V}_3 = \begin{bmatrix} C_1 S_2 \\ S_1 S_2 \\ C_2 \end{bmatrix}$$

$$\Rightarrow J(\theta) = \begin{bmatrix} 0 & -S_1 & C_1 S_2 \\ 0 & C_1 & S_1 S_2 \\ 1 & 0 & C_2 \end{bmatrix}$$

Singularities for $\theta_2 = n\pi, n \in \mathbb{Z}$

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b) $T = e^{[S_1]\theta_1} e^{[S_2]\theta_2} e^{[S_3]\theta_3} M = \boxed{\begin{bmatrix} C_1 C_2 C_3 - S_1 S_3 & -C_3 S_1 - C_1 C_2 S_3 & C_1 S_2 \\ C_2 C_3 S_1 + C_1 S_3 & C_1 C_3 - C_2 S_1 S_3 & S_1 S_2 \\ -C_3 S_2 & S_2 S_3 & C_2 \end{bmatrix}}$ From Mathematica

Know $[S] = \log T^{-1}(\theta)X$

$\theta_i = (\theta_1=0, \theta_2=0, \theta_3=0)$

$$X = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = X^{-1} \cdot \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T^{-1}(\theta_i) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow T^{-1}X = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

From class/notes we know for $\log(T^{-1}X) = [\omega]\theta$

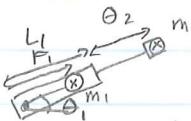
$$\theta = \cos^{-1}\left(\frac{\text{trace}(R)-1}{2}\right) = \pi/4 \quad [w] = \frac{1}{2\sin\theta}(R - R^T) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow S = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \text{Then } \Delta\theta = J^{-1}(\theta_i) S$$

However, because $J(\theta_i)$ is singular $\nexists J^{-1}(\theta_i)$

From inspection, we see that any combination of $\Delta\theta_1 + \Delta\theta_3 = \pi/4$ 3
for $\Delta\theta$, and therefore θ , works.

c) Clearly this is $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$. 3 Can also see $J(\theta_i)\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

3) 

$x_1 = r_1 \cos\theta_1$	$x_2 = (L_1 + \theta_2) \cos\theta_1$
$y_1 = r_1 \sin\theta_1$	$y_2 = (L_1 + \theta_2) \sin\theta_1$
$\dot{x}_1 = -r_1 \sin\theta_1 \dot{\theta}_1$	$\dot{x}_2 = \dot{\theta}_2 \cos\theta_1 - (L_1 + \theta_2) \sin\theta_1 \dot{\theta}_1$
$\dot{y}_1 = r_1 \cos\theta_1 \dot{\theta}_1$	$\dot{y}_2 = \dot{\theta}_2 \sin\theta_1 + (L_1 + \theta_2) \cos\theta_1 \dot{\theta}_1$

$$\Rightarrow K = \frac{1}{2} \left((m_1 r_1^2 + m_2 (L_1 + \theta_2)^2) \dot{\theta}_1^2 + m_2 \dot{\theta}_2^2 \right)$$

$$V = m_1 g r_1 \sin\theta_1 + m_2 g (L_1 + \theta_2) \sin\theta_1$$

$$L = K - V$$

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$$\frac{\partial L}{\partial \dot{\theta}_1} = [m_1 r_1^2 + m_2 (L + \theta_2)^2] \ddot{\theta}_1 \quad \frac{\partial L}{\partial \dot{\theta}_2} = m_2 \ddot{\theta}_2$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_1} = [m_1 r_1^2 + m_2 (L + \theta_2)^2] \ddot{\theta}_1 + 2m_2 (L_1 + \theta_2) \dot{\theta}_1 \dot{\theta}_2 \quad \frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_2} = m_2 \ddot{\theta}_2$$

$$\frac{\partial L}{\partial \theta_1} = -[m_1 g r_1 + m_2 g (L_1 + \theta_2)] \cos \theta_1 \quad \frac{\partial L}{\partial \theta_2} = m_2 (L_1 + \theta_2) \dot{\theta}_1^2 - m_2 g \sin \theta_1$$

$$C = \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}_1} \right) - \frac{\partial L}{\partial \theta_1} = \begin{bmatrix} m_1 r_1^2 + m_2 (L + \theta_2)^2 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 2m_2 (L_1 + \theta_2) \dot{\theta}_1 \dot{\theta}_2 \\ -m_2 (L_1 + \theta_2) \dot{\theta}_1^2 \end{bmatrix}$$

$$+ \begin{bmatrix} [m_1 g r_1 + m_2 g (L_1 + \theta_2)] \cos \theta_1 \\ m_2 g \sin \theta_1 \end{bmatrix}$$

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The Christoffel symbols:

$$\Gamma_{111} = \frac{1}{2} (0 + 0 + 0) \quad \Gamma_{112} = \frac{1}{2} (2m_2 (L + \theta_2) + 0 + 0)$$

$$\Gamma_{121} = \frac{1}{2} (0 + 2m_2 (L + \theta_2) + 0) \quad \Gamma_{122} = \frac{1}{2} (0 + 0 - 0)$$

$$\Gamma_{211} = \frac{1}{2} (0 + 0 - 2m_2 (L + \theta_2)) \quad \Gamma_{212} = 0 \quad \underline{\underline{3}}$$

$$\Gamma_{221} = 0 \quad \Gamma_{222} = 0$$

4)

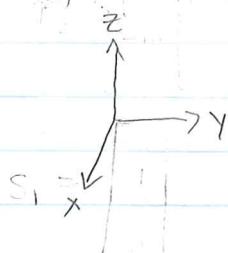


See attached Mathematica code.

The final 2 values (τ_2, τ_1) match with

problem 3.

5)



$$\bar{x}_{xx} = \int_{-1.5}^{1.5} \int_{-1}^1 \int_{-0.5}^{0.5} 10(y^2 + z^2) dz dy dx = [2.5 \times 10^{-4}]$$

$$\bar{x}_{yy} = \int_{-1.5}^{1.5} \int_{-1}^1 \int_{-0.5}^{0.5} 10(x^2 + z^2) dz dy dx = [5 \times 10^{-4}]$$

$$\bar{x}_{zz} = \int_{-1.5}^{1.5} \int_{-1}^1 \int_{-0.5}^{0.5} 10(x^2 + y^2) dz dy dx = [6.5 \times 10^{-4}]$$

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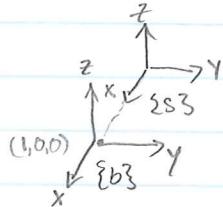
⇒ A.

All the diagonal terms are 0.

$$\bar{x} = \begin{bmatrix} 2.5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 6.5 \end{bmatrix} \times 10^{-4}$$

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$$6) \quad \dot{\mathcal{X}}_b = \begin{bmatrix} 2.5 \times 10^{-4} & 0 & 0 \\ 0 & 5 \times 10^{-4} & 0 \\ 0 & 0 & 6.5 \times 10^{-4} \end{bmatrix} \quad \dot{\mathcal{V}}_S = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$



$$a) \quad \dot{\mathcal{V}}_b = \text{Ad}_{T_{bs}}(\dot{\mathcal{V}}_S)$$

$$T_{sb} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow T_{bs} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{Ad}_{T_{bs}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \dot{\mathcal{V}}_b = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \quad \underline{4}$$

$$b) \text{ Find } F_b = G_b \dot{\mathcal{V}}_b - \text{Ad}_{\dot{\mathcal{V}}_b}^T (G_b \dot{\mathcal{V}}_b)$$

$\dot{\mathcal{V}}_b = 0$ because constant body velocity.

$$G_b = \begin{bmatrix} \dot{\mathcal{X}}_b & 0 \\ 0 & m_b I \end{bmatrix} \quad m_b = 10(1)(.2)(.3) = .06$$

$$\Rightarrow F_b = -\text{ad}_{\dot{\mathcal{V}}_b}^T (G_b \dot{\mathcal{V}}_b) \quad \text{ad}_{\dot{\mathcal{V}}_b}^T = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}^T$$

$$\Rightarrow F_b = - \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2.5 \times 10^{-4} & 0 & 0 \\ 0 & 5 \times 10^{-4} & 0 \\ 0 & 0 & 6.5 \times 10^{-4} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.06 \\ 0.06 \\ 0 \end{bmatrix} = F_b$$

$\begin{bmatrix} 0 \\ 0 \\ 6.5 \times 10^{-4} \\ 0.06 \\ 0.06 \\ 0 \end{bmatrix}$

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Problem 4

First Step

```
In[1579]:= (M1 = {{1, 0, 0, r1}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}) // MatrixForm
(M2 = {{1, 0, 0, L1}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}) // MatrixForm
M12 = Inverse[M1].M2;
(S1 = {{0}, {0}, {1}, {0}, {0}, {0}}) // MatrixForm
(S2 = {{0}, {0}, {0}, {1}, {0}, {0}}) // MatrixForm
AdM1inv = {{1, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0},
{0, 0, 0, 1, 0, 0}, {0, 0, r1, 0, 1, 0}, {0, -r1, 0, 0, 0, 1}};
AdM2inv = {{1, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0},
{0, 0, 0, 1, 0, 0}, {0, 0, L1, 0, 1, 0}, {0, -L1, 0, 0, 0, 1}};
Out[1579]//MatrixForm=

$$M_1 = \begin{pmatrix} 1 & 0 & 0 & r1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Out[1560]//MatrixForm=

$$M_2 = \begin{pmatrix} 1 & 0 & 0 & L1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Out[1562]//MatrixForm=

$$S_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Out[1563]//MatrixForm=

$$S_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

```

Second Step

```
In[1528]:= (G1 = {{0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0},
{0, 0, 0, m1, 0, 0}, {0, 0, 0, 0, m1, 0}, {0, 0, 0, 0, 0, m1}}) // MatrixForm;
(G2 = {{0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}, {0, 0, 0, m2, 0, 0},
{0, 0, 0, 0, m2, 0}, {0, 0, 0, 0, 0, m2}}) // MatrixForm;
```

Find A_i 's

```
In[1530]:= (A1 = AdM1inv.S1) // MatrixForm;
(A2 = AdM2inv.S2) // MatrixForm;
A1sk =
{{0, -A1[[3, 1]], A1[[2, 1]], A1[[4, 1]]}, {A1[[3, 1]], 0, -A1[[1, 1]], A1[[5, 1]]},
{-A1[[2, 1]], A1[[1, 1]], 0, A1[[6, 1]]}, {0, 0, 0, 0}};
A2sk =
{{0, -A2[[3, 1]], A2[[2, 1]], A2[[4, 1]]}, {A2[[3, 1]], 0, -A2[[1, 1]],
A2[[5, 1]]}, {-A2[[2, 1]], A2[[1, 1]], 0, A2[[6, 1]]}, {0, 0, 0, 0}};
```

Define Initial Conditions

```
In[1534]:= (V0 = {{0}, {0}, {0}, {0}, {0}, {0}}) // MatrixForm;
(V0dot = {{0}, {0}, {0}, {0}, {g}, {0}}) // MatrixForm;
```

Find Ti's

```
In[1536]:= (T01 = M1.MatrixExp[A1sk * θ1]) MatrixForm;
(T12 = M12.MatrixExp[A2sk * θ2]) // MatrixForm;
```

Find Velocities

```
In[1586]:= (V1 = A1 * θ1') // MatrixForm
V1sk =
{{0, -V1[[3, 1]], V1[[2, 1]], V1[[4, 1]]}, {V1[[3, 1]], 0, -V1[[1, 1]], V1[[5, 1]]},
{-V1[[2, 1]], V1[[1, 1]], 0, V1[[6, 1]]}, {0, 0, 0, 0}};
V2sk = Inverse[T12].V1sk.T12 + A2sk * θ2';
(V2 = FullSimplify[{{V2sk[[3, 2]]}, {V2sk[[1, 3]]}, {V2sk[[2, 1]]},
{V2sk[[1, 4]]}, {V2sk[[2, 4]]}, {V2sk[[3, 4]]}}]) // MatrixForm
```

Out[1586]/MatrixForm=

$$\alpha_V_1 = \begin{pmatrix} 0 \\ 0 \\ \theta_1' \\ 0 \\ r_1 \theta_1' \\ 0 \end{pmatrix}$$

Out[1589]/MatrixForm=

$$\alpha_V_2 = \begin{pmatrix} 0 \\ 0 \\ \theta_1' \\ \theta_2' \\ (L_1 + \theta_2) \theta_1' \\ 0 \end{pmatrix}$$

Find Accelerations

```
In[1590]:= (T10 = FullSimplify[Inverse[T01]]) // MatrixForm;
(AdT10 = {{Cos[\theta1], Sin[\theta1], 0, 0, 0, 0}, {-Sin[\theta1], Cos[\theta1], 0, 0, 0, 0},
{0, 0, 1, 0, 0, 0}, {0, 0, -r1 Sin[\theta1], Cos[\theta1], Sin[\theta1], 0},
{0, 0, -r1 (-2 + Cos[\theta1]), -Sin[\theta1], Cos[\theta1], 0},
{-r1 (-2 + Cos[\theta1]) Sin[\theta1] - r1 Cos[\theta1] Sin[\theta1],
r1 (-2 + Cos[\theta1]) Cos[\theta1] - r1 Sin[\theta1]^2, 0, 0, 0, 1}}) // MatrixForm;
(AdV1 = {{0, -V1[[3, 1]], V1[[2, 1]], 0, 0, 0}, {V1[[3, 1]], 0, -V1[[1, 1]], 0, 0, 0},
{-V1[[2, 1]], V1[[1, 1]], 0, 0, 0, 0}, {0, -V1[[6, 1]], V1[[5, 1]], 0, -V1[[3, 1]],
V1[[2, 1]]}, {V1[[6, 1]], 0, -V1[[4, 1]], V1[[3, 1]], 0, -V1[[1, 1]]},
{-V1[[5, 1]], V1[[4, 1]], 0, V1[[2, 1]], V1[[1, 1]], 0}}) // MatrixForm;
(V1dot = FullSimplify[AdT10.V0dot + AdV1.A1 * \theta1' + A1 * \theta1'']) // MatrixForm;
(T21 = Inverse[T12]) // MatrixForm
(AdT21 = {{1, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0}, {0, 0, 1, 0, 0, 0}, {0, 0, 0, 1, 0, 0},
{0, 0, -(-L1 + r1 - \theta2), 0, 1, 0}, {0, -L1 + r1 - \theta2, 0, 0, 0, 1}}) // MatrixForm;
(AdV2 = {{0, -V2[[3, 1]], V2[[2, 1]], 0, 0, 0}, {V2[[3, 1]], 0, -V2[[1, 1]], 0, 0, 0},
{-V2[[2, 1]], V2[[1, 1]], 0, 0, 0, 0}, {0, -V2[[6, 1]], V2[[5, 1]], 0, -V2[[3, 1]],
V2[[2, 1]]}, {V2[[6, 1]], 0, -V2[[4, 1]], V2[[3, 1]], 0, -V2[[1, 1]]},
{-V2[[5, 1]], V2[[4, 1]], 0, V2[[2, 1]], V2[[1, 1]], 0}}) // MatrixForm;
(V2dot = FullSimplify[AdT21.V1dot + AdV2.A2 * \theta2' + A2 * \theta2'']) // MatrixForm
```

Out[1594]:= MatrixForm=

$$\ddot{V}_1 = \begin{pmatrix} 1 & 0 & 0 & -L1 + r1 - \theta2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Out[1597]:= MatrixForm=

$$\ddot{V}_2 = \begin{pmatrix} 0 \\ 0 \\ \theta1'' \\ g \sin[\theta1] + \theta2'' \\ g \cos[\theta1] + \theta1' \theta2' + (L1 + \theta2) \theta1'' \\ 0 \end{pmatrix}$$

Now Forces

```
In[1562]:= (F2 = FullSimplify[G2.V2dot - AdV2^T.G2.V2]) // MatrixForm;
Flatten[F2^T.A2] // FullSimplify
(F1 = FullSimplify[AdT21^T.F2 + G1.V1dot - AdV1^T.G1.V1]) // MatrixForm;
Flatten[F1^T.A1] // FullSimplify
```

$$\text{Out[1563]} = \{m2 (g \sin[\theta1] - (L1 + \theta2) (\theta1')^2 + \theta2'')\} = \underline{T}_2$$

$$\text{Out[1565]} = \{g (m1 r1 + m2 (L1 + \theta2)) \cos[\theta1] + 2 m2 (L1 + \theta2) \theta1' \theta2' + (m1 r1^2 + m2 (L1 + \theta2)^2) \theta1''\} = \underline{T}_1$$

Same as #3 10

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