ME 449 Robotic Manipulation
Homework 1
Due Tuesday April 17, 2012

Reading: Mason pp. 1-19, 41-47, 58-60; Choset chapter 3.5; Murray Li Sastry pp. 19-61 (through Section 4; you can skip Section 2.3 on "Other representations")

This homework covers the representation of rigid body configurations and velocities. Some things you should be able to do: represent the configuration of one frame relative to another; represent the velocity of one frame relative to another; perform a coordinate transformation to represent a point, frame, or velocity expressed in a frame A in a new frame B; displace (move) a point or frame; and express planar velocities using rotation centers.

In the Mason text, the frame B relative to the frame A is written as ${}^A_BT$; in the Choset text it is T_{AB}; and in the Murray Li Sastry text it is g_{ab} (or $\bar{g}_{ab}$). Given a point r, represented as either ${}^Ar$ (in the A frame) or ${}^Br$ (in the B frame), then by the superscript-subscript canceling rule, ${}^A_BT\ {}^Br$ simply gives ${}^Ar$ (change of coordinates to the A frame), and ${}^A_BT\ {}^Ar$, which does not satisfy the superscript-subscript canceling rule, moves the point r by first rotating the point by ${}^A_BR$ and then adding ${}^A_Bp$ (where these two make up ${}^A_BT$). The new point is r', and the result is the representation in the A frame ${}^Ar'$.

1. Mason 2.1.
2. Mason 2.2.
3. Mason 2.3. You don't have to construct the fixed and moving centrodes; just indicate the rotation center for each of the motions A to B, B to C, and C to A.
4. Mason 2.4. Again, you only have to plot the rotation centers for your sequence, not the centrodes.
5. Mason 3.2.
6. Mason 3.3.
7. Choset 3.2.
8. Choset 3.6.
9. Choset 3.16.
10. Choset 3.17.
11. We can represent the same rigid body velocity in either frame A or frame B. The frame A (respectively B) is stationary (inertial), and the rigid body velocity consists of the linear velocity of the point currently at the origin of A (resp. B) as well as the angular velocity relative to the axes of A (resp. B). We say that the two representations are related by ${}^AV = \mathrm{Ad}_{{}^A_BT}\ {}^BV$, where Ad is called the *adjoint matrix*. Using the equation for linear velocity often written $v = \omega \times r$, derive the form of this adjoint matrix, and show it is equivalent to the form we used in class.
12. Murray Li Sastry 3.18 (a)-(d).
13. The configuration of a body frame in the fixed world frame is $T \in SE(3)$. If it is changing in time, we call it $T(t)$. One representation of its velocity is $\dot{T}$, a 4×4 matrix with all zeros in the bottom row, $\dot{R} = \omega \times R$ in the top left 3×3 submatrix, and $\dot{p}$ in the top right 3×1 submatrix. That ω is the angular velocity of the body expressed in the fixed world frame and $\dot{p}$ is the linear velocity of the origin of the body frame expressed in the world frame. Prove that $\dot{T}T^{-1}$ is the spatial velocity and $T^{-1}\dot{T}$ is the body velocity. (You can use the expression for T^{-1} you derived earlier.)

14. Let V^s be the spatial velocity, a six-vector $[v_x, v_y, v_z, \omega_x, \omega_y, \omega_z]^T$. We restrict to planar motion, so $v_z = \omega_x = \omega_y = 0$. Write the corresponding instantaneous velocity center $[x_c, y_c]^T$ in the space frame in terms of (v_x, v_y, ω_z).

15. The transform T_1 is given by R_1 and p_1, and the transform T_2 is given by R_2 and p_2. Express the 4×4 matrices T_1T_2 and T_2T_1 in terms of the R_i and p_i. If you consider T_1 as the initial position and orientation of a frame, and T_2 as a transformation of that frame, explain in words what R_2 and p_2 do to T_1 when T_2 is multiplied on the left and when it is multiplied on the right. Be sure to indicate the axes in which these transformations are happening.

16. You are looking at an LCD monitor that is 16 units wide and 9 units tall. It is sitting on a desk so the screen is facing you. You attach three coordinate frames to the monitor, A at the bottom left, B at the bottom right, and C at the top right. The x-axis of the A frame points to the right and the y-axis points up. The x-axis of the B frame points up and the y-axis points to the left. The x-axis of the C frame points away from the screen (toward you) and the y-axis points down. Draw these frames and give ${}^A_BT$, ${}^A_CT$, and ${}^C_BT$.