Where we are:

Chap 2  Configuration Space
Chap 3  Rigid-Body Motions
         3.2.1 Rotation Matrices
Chap 4  Forward Kinematics
Chap 5  Velocity Kinematics and Statics
Chap 6  Inverse Kinematics
Chap 8  Dynamics of Open Chains
Chap 9  Trajectory Generation
Chap 11 Robot Control
Chap 13 Wheeled Mobile Robots
Important concepts, symbols, and equations

• We often define a fixed space frame \{s\} and a body frame \{b\} attached to some body of interest. All frames are instantaneously stationary.

• Right-handed frames, and right-hand rule for positive rotation.

• Special orthogonal group $SO(3)$: matrices $R$ in $\mathbb{R}^{3\times3}$ where $R^TR = I$, $\det R = 1$. $R$ is a rotation matrix. Implicit representation with 9 numbers for 3 dof.
Important concepts, symbols, and equations (cont.)

• A group is a set of elements $G = \{a, b, c \ldots\}$ and a binary operation $\cdot$ satisfying

  closure  
  $a \cdot b \in G$ for all $a, b \in G$

  associativity  
  $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

  identity element exists  
  there is an $I \in G$ such that $a \cdot I = I \cdot a = a$ for each $a \in G$

  inverse exists  
  for each $a \in G$, there exists $a^{-1} \in G$ such that $a \cdot a^{-1} = a^{-1} \cdot a = I$

Integers under addition? Nonnegative integers under addition? Square real matrices under multiplication? What is a Lie group?
Important concepts, symbols, and equations (cont.)

- $SO(3)$ is a matrix (Lie) group (the group operation is matrix multiplication).

  closure: $R_1R_2 \in SO(3)$
  associative: $(R_1R_2)R_3 = R_1(R_2R_3)$ (not commutative! $R_1R_2 \neq R_2R_1$ generally)
  identity: identity matrix $I$
  inverse: matrix inverse

  $R^T R = I$, so $R^{-1} = R^T$.

  For $x \in \mathbb{R}^3$, $\|x\| = \|Rx\|$. 

Modern Robotics, Lynch and Park, Cambridge University Press
Important concepts, symbols, and equations (cont.)

• Uses of a rotation matrix:

  1. Represent an orientation.  $R_{ab}$ represents orientation of \{b\} in \{a\}.
  2. Change the reference frame of a vector or frame.

    subscript cancellation:
    \[
    R_{ab} R_{bc} = R_{ab} R_{bc} = R_{ac} \\
    R_{ab} p_b = R_{ab} p_b = p_a
    \]

  3. Rotate a vector or frame.  $R = R_{cd} = \text{Rot}(\hat{w}, \theta)$, axis $\hat{w}$ expressed in \{c\}.

    \[
    p'_c = R_{cd} p_c \quad \text{(no subscript cancellation)} \\
    R_{ab}' = RR_{ab} \quad \text{(after rotating about axis in \{a\})} \\
    R_{ab}'' = R_{ab}R \quad \text{(after rotating about axis in \{b\})}
    \]
\[ R_{ab} = \quad \quad p_b = \]
Given $R_1 = R_{ab}$, $R_2 = R_{bc}$, and $R_3 = R_{ad}$, write $R_{dc}$ in terms of $R_1$, $R_2$, and $R_3$ (no inverses!).

Given $p_b$, what is $p_d$ in terms of $R_1$, $R_2$, and $R_3$ (no inverses)?
\[ R = R_{ba} = \text{Rot}(\hat{w}, \theta): \theta = \pi/2, \text{ axis } \hat{w} = \]

\[ R_{bc} = R R_{bc} = \quad R_{bc}'' = R_{bc} R = \]
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