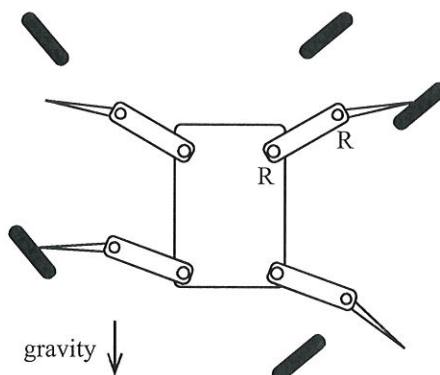


Always show your work or reasoning so your thought process is clear! If you need more space for your work, you can use the back side of a page. No electronics (phone, watch, calculator, computer, tablet, etc.) allowed.

1. (8 pts) The figure below shows a climbing robot that climbs in a vertical plane. Its motion is confined to that vertical plane. The climber consists of a body and four legs. Each leg has two revolute joints, and when a "foot" is in point contact with a foothold, it forms a revolute joint with the foothold in parts (b) and (c) (but not in (d); see below).



equivalently,  
3 dof for body  
2 dof for each leg  
 $3 + 4 \times 2 = 11$  dof

- (a) When the robot has no feet in contact with a foothold, how many degrees of freedom does the robot have? Show your reasoning.

$m = 3$  (planar)  $N = 1 + 1 + 4 \times 2 = 10$   $J = 4 \times 2 = 8$   $dof = 3(10 - 1 - 8) + 8 \times 1$   
 $dof = 11$

- (b) When the robot has one foot in contact with a foothold, forming a revolute joint at the contact, how many degrees of freedom does the robot have? Show your reasoning.

$J = 9$   $dof = 3(10 - 1 - 9) + 9 \times 1 = 9$   
 equivalently, started with 11 dof then placed 2 position constraints on a foot.

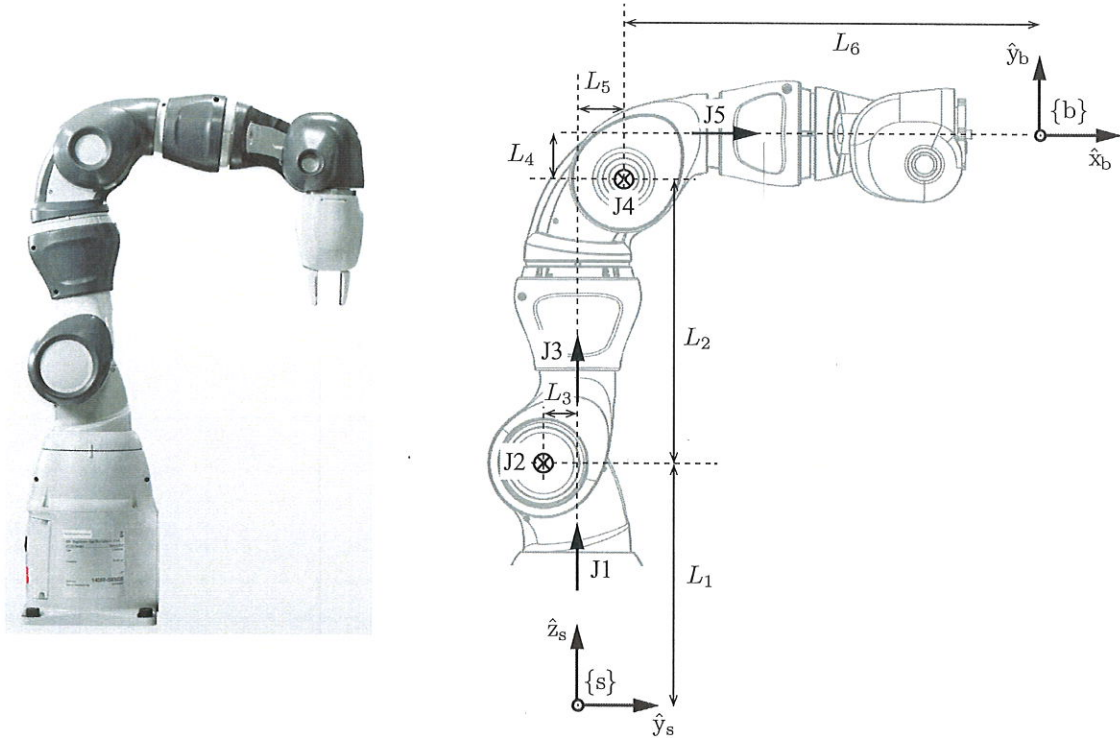
- (c) When the robot has four feet in contact with footholds, forming revolute joints at the contacts, how many degrees of freedom does the robot have? Show your reasoning.

$J = 12$   $dof = 3(10 - 1 - 12) + 12 \times 1 = 3$   
 equivalently,  $11 - 4 \times 2 = 3$

- (d) Now assume the robot has two feet in contact with a foothold, but each foot can slide along the foothold in addition to rotate about the contact point. How many degrees of freedom does the robot have? Show your reasoning.

8 revolute joints (1 freedom)  
 2 roll-slide foot contacts (2 freedoms)  
 $dof = 3(10 - 1 - 10) + 8 \times 1 + 2 \times 2 = 9$   
 equivalently, start with 11 dof then add 1 constraint to each of 2 feet (these feet must be  $d = 0$  from footholds)

2. (12 pts) On the left is the 7R ABB Yumi collaborative robot arm, and on the right is a kinematic model that considers the first five joints, J1 to J5. (The last two joints near the end-effector are omitted.) Assume the Yumi is shown at its home configuration in the right image. Joint axes 1 and 3 are aligned and point up on the page, joint axes 2 and 4 point into the page, and joint axis 5 points to the right on the page. The various offsets are indicated by the lengths  $L_1 \dots L_6$ . The  $\{s\}$  frame at the base of the robot and the  $\{b\}$  frame at the end-effector are shown.



- (a) Write the matrix  $M = T_{sb}(0)$  describing the configuration of  $\{b\}$  relative to  $\{s\}$  at this home configuration.

$$M = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & L_5 + L_6 \\ 0 & 1 & 0 & L_1 + L_2 + L_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- (b) Write the  $6 \times 5$  space Jacobian  $J_s(0)$ .

$$J_s(0) = \begin{bmatrix} 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -L_1 - L_2 - L_4 \\ 0 & -L_1 & 0 & -L_1 - L_2 & 0 \\ 0 & -L_3 & 0 & L_5 & 0 \end{bmatrix} \quad (\text{rank} = 4)$$

3. (4 pts) You are sitting in an aircraft simulator cockpit, and the frame of your seat is initially aligned with a frame {a} fixed to the room. Your seat first follows a twist  $\mathcal{V}_{a1}$  (represented in the {a} frame) for  $t_1$  seconds, achieving a new frame location {b}. Your seat then follows the screw axis  $\mathcal{S}_b$  (represented in the {b} frame) a rotational distance  $\alpha$ , achieving a new frame location {c}. Finally, your seat follows a twist  $\mathcal{V}_{a2}$  (represented in the {a} frame) for a time  $t_2$ , achieving a new frame location {d}.

Your final configuration relative to the {a} frame can be represented by exponential coordinates  $\mathcal{S}_a\theta$ , where  $\mathcal{S}_a$  is a screw axis, represented in the {a} frame, that you would have had to follow a distance  $\theta$  to move to the frame {d}. Give the  $se(3)$  representation  $[\mathcal{S}_a\theta]$  of these exponential coordinates, expressed in terms of the functions log and exp and the information given above.

$$\begin{aligned}
 T_{ab} &= \exp([\mathcal{V}_{a1} t_1]) \\
 T_{ac} &= \exp([\mathcal{V}_{a1} t_1]) \exp([\mathcal{S}_b \alpha]) \\
 T_{ad} &= \exp([\mathcal{V}_{a2} t_2]) T_{ac} = \exp([\mathcal{V}_{a2} t_2]) \exp([\mathcal{V}_{a1} t_1]) \exp([\mathcal{S}_b \alpha]) \\
 [\mathcal{S}_a \theta] &= \log(\exp([\mathcal{V}_{a2} t_2]) \exp([\mathcal{V}_{a1} t_1]) \exp([\mathcal{S}_b \alpha]))
 \end{aligned}$$

4. (4 pts) A human-collaborative robot arm is mounted to the floor, and a frame {s} is located at the mounting location. A frame {b} is at the robot's end-effector. The robot's joint vector  $\theta$ , end-effector configuration  $T_{sb}(\theta)$ , and body Jacobian  $J_b(\theta)$  are known. A human grabs the end-effector and applies a wrench to it, expressed in the {s} frame as  $F_s$ . Give the expression for the joint torques  $\tau$  the robot must apply to resist motion in terms of the quantities defined above.

$$\begin{aligned}
 F_b &= [Ad_{T_{sb}(\theta)}]^T F_s \\
 \tau &= J_b^T(\theta) F_b \\
 \tau &= J_b^T(\theta) [Ad_{T_{sb}(\theta)}]^T F_s \quad \text{or} \quad [Ad_{T_{sb}(\theta)}] J_b(\theta) F_s
 \end{aligned}$$

To resist the wrench, apply the \*negative\* of these joint torques

5. (2 pts) The exponential coordinates  $\mathcal{S}_c\theta$  representing a frame {d} relative to a frame {c} are  $(0, 1, 1, 10, 5, 0)$ . Give the corresponding screw axis  $\mathcal{S}_c$  and the angle of travel  $\theta$  along it.

$$\mathcal{S}_c = \frac{1}{\|\omega\|} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 10 \\ 5 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 10 \\ 5 \\ 0 \end{bmatrix} \quad \theta = \sqrt{2}$$