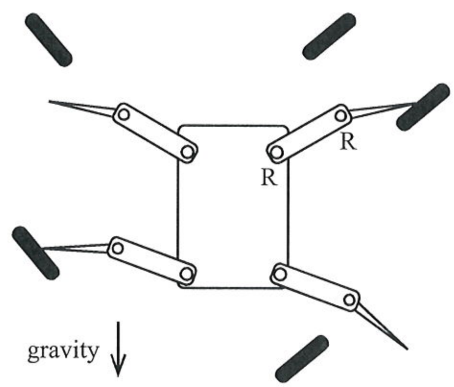
ME 449 Midterm
October 27, 2021

Name: Solutions

Always show your work or reasoning so your thought process is clear! If you need more space for your work, you can use the back side of a page. No electronics (phone, watch, calculator, computer, tablet, etc.) allowed.

1. **(8 pts)** The figure below shows a climbing robot that climbs in a vertical plane. Its motion is confined to that vertical plane. The climber consists of a body and four legs. Each leg has two revolute joints, and when a "foot" is in point contact with a foothold, it forms a revolute joint with the foothold in parts (b) and (c) (but not in (d); see below).



equivalently,
3 dof for body
2 dof for each leg
$3+4\times2 = 11$ dof

(a) When the robot has no feet in contact with a foothold, how many degrees of freedom does the robot have? Show your reasoning.

$m=3$ (planar) $N = \underbrace{1}_{\text{ground}} + 1 + 4\times2 = 10$ $J = 4\times2 = 8$ dof $=3(10-1-8)+8\times1$

dof $= 11$

(b) When the robot has one foot in contact with a foothold, forming a revolute joint at the contact, how many degrees of freedom does the robot have? Show your reasoning.

$J=9$ dof $= 3(10-1-9)+9\times1 = 9$

equivalently, started with 11 dof then placed 2 position constraints on a foot.

(c) When the robot has four feet in contact with footholds, forming revolute joints at the contacts, how many degrees of freedom does the robot have? Show your reasoning.

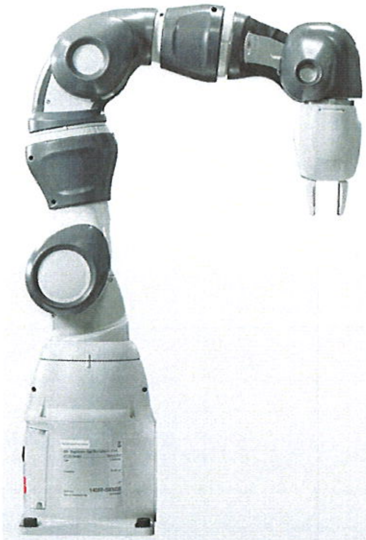
$J=12$ dof $= 3(10-1-12)+12\times1 = 3$

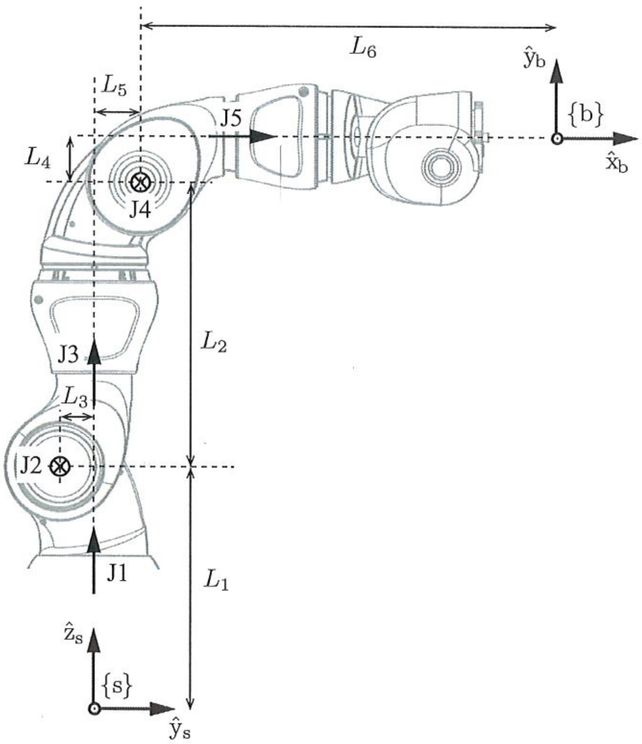
equivalently, $11-4\times2 = 3$

(d) Now assume the robot has two feet in contact with a foothold, but each foot can slide along the foothold in addition to rotate about the contact point. How many degrees of freedom does the robot have? Show your reasoning.

8 revolute joints (1 freedom)
2 roll-slide foot contacts (2 freedoms)
dof $= 3(10-1-10)+8\times1+2\times2 = 9$
equivalently, start with 11 dof then add 1 constraint to each of 2 feet (these feet must be $d=0$ from footholds)

2. **(12 pts)** On the left is the 7R ABB Yumi collaborative robot arm, and on the right is a kinematic model that considers the first five joints, J1 to J5. (The last two joints near the end-effector are omitted.) Assume the Yumi is shown at its home configuration in the right image. Joint axes 1 and 3 are aligned and point up on the page, joint axes 2 and 4 point into the page, and joint axis 5 points to the right on the page. The various offsets are indicated by the lengths $L_1 \ldots L_6$. The {s} frame at the base of the robot and the {b} frame at the end-effector are shown.





(a) Write the matrix $M = T_{sb}(0)$ describing the configuration of {b} relative to {s} at this home configuration.

$$M = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & L_5 + L_6 \\ 0 & 1 & 0 & L_1 + L_2 + L_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) Write the 6×5 space Jacobian $J_s(0)$.

$$J_s(0) = \begin{bmatrix} 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -L_1 - L_2 - L_4 \\ 0 & -L_1 & 0 & -L_1 - L_2 & 0 \\ 0 & -L_3 & 0 & L_5 & 0 \end{bmatrix} \quad (\text{rank} = 4)$$

3. **(4 pts)** You are sitting in an aircraft simulator cockpit, and the frame of your seat is initially aligned with a frame {a} fixed to the room. Your seat first follows a twist $\mathcal{V}_{a1}$ (represented in the {a} frame) for t_1 seconds, achieving a new frame location {b}. Your seat then follows the screw axis $\mathcal{S}_b$ (represented in the {b} frame) a rotational distance α, achieving a new frame location {c}. Finally, your seat follows a twist $\mathcal{V}_{a2}$ (represented in the {a} frame) for a time t_2, achieving a new frame location {d}.

   Your final configuration relative to the {a} frame can be represented by exponential coordinates $\mathcal{S}_a\theta$, where $\mathcal{S}_a$ is a screw axis, represented in the {a} frame, that you would have had to follow a distance θ to move to the frame {d}. Give the $se(3)$ representation $[\mathcal{S}_a\theta]$ of these exponential coordinates, expressed in terms of the functions log and exp and the information given above.

$$T_{ab} = \exp([\mathcal{V}_{a1} t_1])$$
$$T_{ac} = \exp([\mathcal{V}_{a1} t_1])\exp([\mathcal{S}_b \alpha])$$
$$T_{ad} = \exp([\mathcal{V}_{a2} t_2])\, T_{ac} = \exp([\mathcal{V}_{a2} t_2])\exp([\mathcal{V}_{a1} t_1])\exp([\mathcal{S}_b \alpha])$$
$$[\mathcal{S}_a \theta] = \log\left(\exp([\mathcal{V}_{a2} t_2])\exp([\mathcal{V}_{a1} t_1])\exp([\mathcal{S}_b \alpha])\right)$$

4. **(4 pts)** A human-collaborative robot arm is mounted to the floor, and a frame {s} is located at the mounting location. A frame {b} is at the robot's end-effector. The robot's joint vector θ, end-effector configuration $T_{sb}(\theta)$, and body Jacobian $J_b(\theta)$ are known. A human grabs the end-effector and applies a wrench to it, expressed in the {s} frame as $\mathcal{F}_s$. Give the expression for the joint torques τ the robot must apply to resist motion in terms of the quantities defined above.

$$F_b = [Ad_{T_{sb}(\theta)}]^T F_s$$
$$\tau = J_b^T(\theta) F_b$$
$$\tau = J_b^T(\theta) [Ad_{T_{sb}(\theta)}]^T F_s \quad \text{or} \quad [Ad_{T_{sb}(\theta)}] J_b(\theta) F_s$$

To resist the wrench, apply the *negative* of these joint torques

5. **(2 pts)** The exponential coordinates $\mathcal{S}_c\theta$ representing a frame {d} relative to a frame {c} are $(0, 1, 1, 10, 5, 0)$. Give the corresponding screw axis $\mathcal{S}_c$ and the angle of travel θ along it.

$$\mathcal{S}_c = \frac{1}{\|\omega\|}\begin{bmatrix}0\\1\\1\\10\\5\\0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\1\\10\\5\\0\end{bmatrix} \qquad \theta = \sqrt{2}$$