

ME449 – Problem Set 4 – Solution Set

Quit[]

Question 2

(*Function has been added to the library*)

```

(*JointTorquesNum[MiPreToi_,Si_,Gi_,g_,Ftip_,θ_,dθ_,ddθ_] :=
Module[{n,i,Mi,Ai,TiPreToTi,Fi,Vi,dVi,τ,τi},
n=Length[Si];
(*Initialization*)
Mi[0]=IdentityMatrix[4];
Vi[0]= {0,0,0,0,0,0};
dVi[0] = {0,0,0,g[[1]],g[[2]],g[[3]]};
Fi[n+1]= Ftip;

(*Forward Iteration*)
For[i=1,i≤ n,i++,
(*Calculate Mi*)
Mi[i]=Mi[i-1].MiPreToi[[i]];
(*Calculate Ai*)
Ai[i]= AdjTrans[TransfInv[Mi[i]]].Si[[i]];
(*Calculate TiPreToTi*)
TiPreToTi[i]= MiPreToi[[i]].ExpToTrans[Ai[i],θ[[i]]];
(*Calculate Vi*)
Vi[i]=AdjTrans[TransfInv[TiPreToTi[i]]].Vi[i-1]+Ai[i]*dθ[[i]];
(*Calculate dVi*)
dVi[i]=AdjTrans[TransfInv[TiPreToTi[i]]].dVi[i-1]+
LieBrack[Vi[i],Ai[i]]*dθ[[i]]+Ai[i]*ddθ[[i]];
];

(*Backward Iteration*)
(*Create array to store τ*)
τ=Array[τi,n];
TiPreToTi[n+1]=IdentityMatrix[4];
(*Assumes that tip coordinate frame is coincident with frame of last link*)

For[i=n,i≥ 1,i--,
(*Calculate Fi*)
Fi[i] = AdjTrans[TransfInv[TiPreToTi[i+1]]]^T.Fi[i+1] +
Gi[[i]].dVi[i] -adV[Vi[i]]^T.(Gi[[i]].Vi[i]);

τi[i]= Fi[i].{Ai[i]}^T;
];
Return[τ];
];*)

```

Question 3

Robot Model

```

L1 = 3;
L2 = 3;
m1 = 2;
m2 = 1;
gval = -10;

M01 = {{1, 0, 0, L1}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}};
M12 = {{1, 0, 0, L2}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}};
MiPreToi = {M01, M12};

S1 = {{0, 0, 1, 0, 0, 0}};
S2 = {{0, 0, 1, 0, -L1, 0}};
Si = {S1[[1]], S2[[1]]};

G1 = {{0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0},
      {0, 0, 0, m1, 0, 0}, {0, 0, 0, 0, m1, 0}, {0, 0, 0, 0, 0, m1}};
G2 = {{0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0},
      {0, 0, 0, m2, 0, 0}, {0, 0, 0, 0, m2, 0}, {0, 0, 0, 0, 0, m2}};
Gi = {G1, G2};

g = {0, gval, 0};

```

Question 3 a)

```
In[79]:= Ftip = {0, 0, 0, 0, 0, 0};
```

```

 $\theta$  = {0, 0};
d $\theta$  = {0, 0};
dd $\theta$  = {0, 0};

```

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 $\tau_a$  = JointTorquesNum[MiPreToi, Si, Gi, -g, Ftip,  $\theta$ , d $\theta$ , dd $\theta$ ];
Print["The joint torques for question 3a) are: \n  $\tau_a$ =",  $\tau_a$  // MatrixForm, "\n"];

```

The joint torques for question 3a) are:

$$\tau_a = \begin{pmatrix} 120 \\ 30 \end{pmatrix}$$

Question 3 b)

```
In[85]:= Ftip = {0, 0, 0, 0, 5, 0};
```

```

 $\tau_b$  = JointTorquesNum[MiPreToi, Si, Gi, -g, Ftip,  $\theta$ , d $\theta$ , dd $\theta$ ];
Print["The joint torques for question 3b) are: \n  $\tau_b$ =",  $\tau_b$  // MatrixForm, "\n"];

```

The joint torques for question 3b) are:

$$\tau_b = \begin{pmatrix} 150 \\ 45 \end{pmatrix}$$

Question 3 c)

```
In[88]:= dθ = {1, 2};  
ddθ = {3, 4};
```

```
τc = JointTorquesNum[MiPreToi, Si, Gi, -g, Ftip, θ, dθ, ddθ];  
Print["The joint torques for question 3c) are: \n τc=", τc // MatrixForm, "\n"];
```

The joint torques for question 3c) are:

$$\tau_c = \begin{pmatrix} 384 \\ 135 \end{pmatrix}$$