Homework 2 Solutions for Chapter 3

Exercise 3.16.

(a) The three frames are shown in the figure.

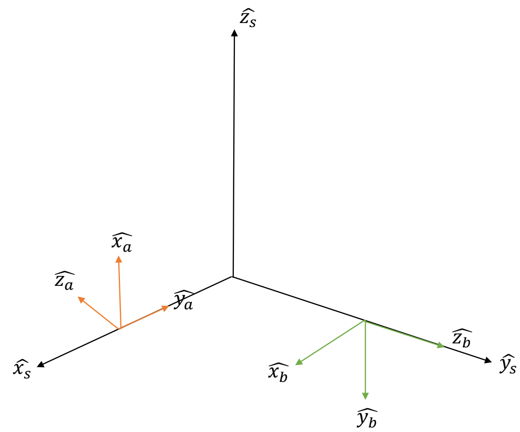

Figure 1

(b)

$$R_{sa} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} R_{sb} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$T_{sa} = \begin{bmatrix} 0 & -1 & 0 & 3 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} T_{sb} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(c)

$$T_{sb}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(d)

$$T_{ab} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 3 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(e)

$$T_1 = \begin{bmatrix} 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This corresponds to a transformation in a body frame.

$$T_2 = \begin{bmatrix} 0 & -1 & 0 & 3 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This corresponds to a transformation in s world frame

(f)

$$p_s = \begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix}$$

(g)

$$p' = \begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix}$$

$$p'' = \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}$$

Therefore p' is a change in location and p'' is change in reference frame.

(h)

$$\mathbb{V}_a = \begin{bmatrix} 1 \\ -3 \\ -2 \\ -9 \\ 1 \\ -1 \end{bmatrix}$$

(i)

$$\begin{aligned} s &= [0.5774, \ -0.5774, \ 0.5774, \ 1.0548, \ -1.0548, \ -0.6772]^{\mathrm{T}} \qquad \Longrightarrow \dot{\theta} = 1 \\ h &= 0.827 \\ q &= [-1 \ 1 \ 0]^{\mathrm{T}} \end{aligned}$$

(j)

$$e^{[s]\theta} = \begin{bmatrix} -0.6173 & -0.7037 & 0.3518 & 1.0555 \\ 0.7037 & -0.2938 & 0.6469 & 1.9407 \\ -0.3518 & 0.6469 & 0.6765 & -0.9704 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

Exercise 3.17.

(a)

$$T_{ad} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \ T_{cd} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

(b)

$$T_{ab} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & -3 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Exercise 3.27.

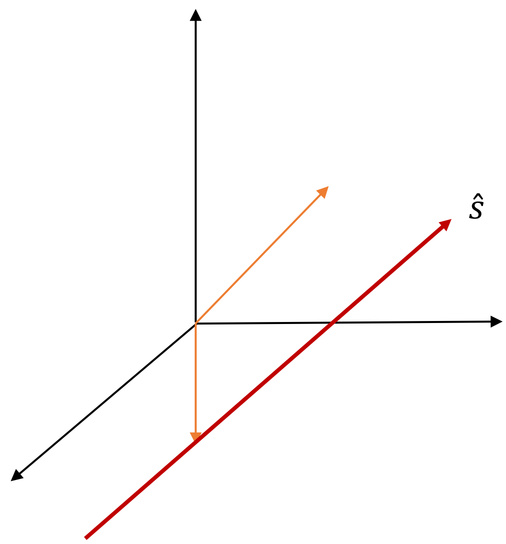

Figure 2

$$\hat{s} = [0\ 1/\sqrt{2}\ 1/\sqrt{2}]$$
$$v = [\sqrt{2}\ 0\ 0]^{\mathrm{T}}$$
$$h = 0$$
$$q = [0\ 1\ \ -1]^{\mathrm{T}}$$

Drawn as Figure 2.

Exercise 3.30.

(a) Case 1:

$$\omega = (0, 0, 2n\pi)^{\intercal}, v = (2n\pi v_1, 0, 1)^{\intercal}.$$

Case 2:

$$\omega = (0, 0, (2n+1)\pi)^T, v = ((2n+1)\pi v_1', 0, 1)^T.$$

(b) Restrict the norm of w as $\|w\| \leq \pi$. From the answers of (a),
Case 1:

$$\omega = (0, 0, 0)^T, v = (0, 0, 1)^T.$$

Case 2:

$$\omega = (0, 0, \pm\pi)^T, v = (\pi, 0, 1)^T.$$

Exercise 3.31.

Since the force is applied at the center of mass there is no torque, therefore the wrench vector in the

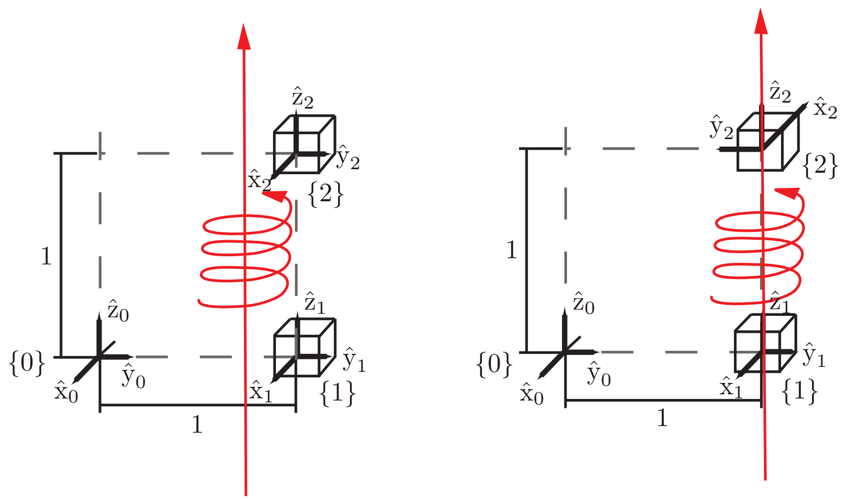

Figure 3

effector frame can be written as:

$$\mathcal{F}_e = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 10 \end{bmatrix}$$

$$\mathcal{F}_c = \begin{bmatrix} 0 \\ 919.2 \\ 1838.5 \\ 10 \\ 0 \\ 0 \end{bmatrix}$$

Exercise 3.49.
Programming assignment, no solution provided.