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A Quantitative Stability Measure for Graspless Manipulation

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Abstract

In this paper, the stability of objects in graspless manipulation (or nonprehensile manipulation) is investigated. In contrast with the stability of grasps, it is crucial for the stability of graspless manipulation to evaluate frictional forces applied to objects in contact motion. We formulate the effect of Coulomb friction and propose a quantitative stability measure for graspless manipulation. The measure can be calculated approximately with linear programming. Finally, we show numerical examples. Our stability measure will be useful in planning of graspless manipulation.

1 Introduction

Graspless manipulation [1] (or nonprehensile manipulation [2]) is a method to manipulate objects without grasping. It includes tumbling, pushing, sliding, and so on (Figure 1). In graspless manipulation, robots do not have to support all the weight of the objects. That is a potential advantage of graspless manipulation over conventional pick-and-place. Thus graspless manipulation is important as a complement of pick-and-place for the dexterity of robots.

Graspless manipulation is obviously inferior to pick-and-place in terms of operation reliability. Therefore it is important to evaluate the stability of manipulated objects for planning and execution of graspless manipulation.

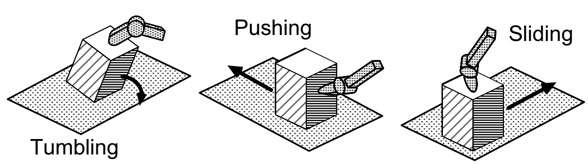

Figure 1: Graspless Manipulation

The stability of graspless manipulation is considerably affected by gravity and friction. Because frictional force is ruled by sliding of the manipulated objects, the stability depends on their motion. That means we should evaluate the stability of the objects in graspless manipulation considering the effect of their motion. In this paper, we call the stability of objects in manipulation "stability of manipulation," to distinguish from the stability of stationary objects.

Many researchers deal with the evaluation of the grasp stability [3, 4]. However, few of their results can be applied to graspless manipulation straightforwardly. Though Mason and Lynch coined "quasi-static closure" and "dynamic closure" for the stability of manipulation [5], they did not formulate them. Trinkle et al. formulated "first-order stability" in detail for whole-arm manipulation [6]. Although first-order stability will be also valid for graspless manipulation, they presented no quantitative measure. Kijimoto et al. proposed a performance index for quasi-static graspless manipulation [7]. The index evaluates the stability of graspless manipulation by the minimum margin of contact force at each contact point. However, the physical meaning of the index is ambiguous on the treatment of sliding contacts and substitution of surface contacts with multiple point contacts. Yu and Yoshikawa defined a stability measure, "constraint maintainability," for robotic contact tasks [8, 9]. The measure neglects the effect of gravity, therefore it is not suitable for graspless manipulation. Moreover, both [7] and [8, 9] did not deal with the contact forces of surface or line contacts in rotation.

The authors proposed a quantitative stability measure for objects in contact with the environment [10]. However, the measure can be applied only to graspless manipulation without sliding contacts. In this paper, we extend the measure to be applicable to quasi-static graspless manipulation with sliding.

2 Stability of Graspless Manipulation

In order to evaluate the "stability" of graspless manipulation, we should consider the following two properties:

(1) The ability of manipulated objects to resist disturbing force in any direction without changing their motion.

(2) The ability of manipulated objects to resume their original motion after a perturbation by disturbing force in any direction.

Graspless manipulation with loose constraints, such as pushing by position-controlled robots, requires the evaluation of the property (1). On the other hand, graspless manipulation with tight constraints, such as pivoting by force-controlled robot fingers, requires the evaluation of the property (2). We should evaluate the property (1) first for graspless manipulation because there exist contacts with the environment. The evaluation of the property (2) is very similar to that of conventional grasp stability (for example, [11]). Therefore, this paper deals with quantitative evaluation of the property (1) in graspless manipulation.

3 Modeling of Manipulation

3.1 Assumptions

In this paper, for graspless manipulation of an object by robot fingers, we make the following assumptions:

- The manipulated object and the environment are spatial rigid polyhedra.
- Position-controlled robot fingers are rigid enough to be equivalent to the environment.
- Each of force-controlled robot fingers is in one-point contact with the object.
- Contact surfaces between the object and the environment (or position-controlled robots) are polygons.
- Friction is under Coulomb's law. Friction coefficients on a contact surface are uniform.
- Static and kinetic friction coefficients are equal.
- Manipulation is quasi-static.

In this paper, we regard line contacts as special surface contacts, and may refer to lines as "surfaces."

3.2 Model of Contact Forces

Let us denote the set of positions of contacts between the object and the environment (or equivalently, position-controlled robots) by $\mathcal{C}_{\text{env}}$. Similarly, let $\mathcal{C}_{\text{rob}}$ be the set of positions of contacts between the object and force-controlled robots. We write the sets of positions of sliding and non-sliding contacts as $\mathcal{C}_{\text{slide}}$ and $\mathcal{C}_{\text{stat}}$, respectively. The set of positions of all the contacts, $\mathcal{C}$, is:

$$\mathcal{C} = \mathcal{C}_{\text{slide}} \cup \mathcal{C}_{\text{stat}} = \mathcal{C}_{\text{env}} \cup \mathcal{C}_{\text{rob}}. \tag{1}$$

We set a reference frame whose origin coincides with the center of mass of the object, and denote each position of contacts in the frame by $\boldsymbol{p} (\in \mathcal{C})$. $\boldsymbol{\mathcal{A}}(\boldsymbol{p}) \subset \Re^6$ is the set of generalized forces (forces and moments about the center of mass of the object) that can be applied through the contact point. We approximate each friction cone at the contact points by a polyhedral convex cone with unit edge vectors, $\boldsymbol{c}_1(\boldsymbol{p}), \ldots, \boldsymbol{c}_s(\boldsymbol{p}) \in \Re^3$. For $\boldsymbol{p} \in \mathcal{C}_{\text{env}} \cap \mathcal{C}_{\text{stat}}$, we have:

$$\boldsymbol{\mathcal{A}}(\boldsymbol{p}) = \left\{ \begin{pmatrix} \boldsymbol{f} \\ \boldsymbol{p} \times \boldsymbol{f} \end{pmatrix} \middle| \boldsymbol{f} \in \text{span}\{\boldsymbol{c}_1(\boldsymbol{p}) \ldots \boldsymbol{c}_s(\boldsymbol{p})\} \right\}, \tag{2}$$

where span$\{\ldots\}$ is a polyhedral convex cone spanned by its element vectors [12]. $\boldsymbol{\mathcal{A}}(\boldsymbol{p})$ also forms a polyhedral convex cone.

For $\boldsymbol{p} \in \mathcal{C}_{\text{env}} \cap \mathcal{C}_{\text{slide}}$,

$$\boldsymbol{\mathcal{A}}(\boldsymbol{p}) = \left\{ \begin{pmatrix} \boldsymbol{f} \\ \boldsymbol{p} \times \boldsymbol{f} \end{pmatrix} \middle| \boldsymbol{f} \in \text{span}\{\boldsymbol{c}'(\boldsymbol{p})\} \right\}, \tag{3}$$

where $\boldsymbol{c}'(\boldsymbol{p}) \in \Re^3$ is a unit edge vector of the friction cone at the contact point opposite to the sliding direction.

For $\boldsymbol{p} \in \mathcal{C}_{\text{rob}} \cap \mathcal{C}_{\text{stat}}$, we have:

$$\boldsymbol{\mathcal{A}}(\boldsymbol{p}) = \left\{ \begin{pmatrix} \boldsymbol{f} \\ \boldsymbol{p} \times \boldsymbol{f} \end{pmatrix} \middle| \boldsymbol{f} \in \text{span}\{\boldsymbol{c}_1(\boldsymbol{p}) \ldots \boldsymbol{c}_s(\boldsymbol{p})\}, \boldsymbol{\tau}(\boldsymbol{p}) = \boldsymbol{J}(\boldsymbol{p})^T \boldsymbol{f} \right\}, \tag{4}$$

where $\boldsymbol{\tau}(\boldsymbol{p})$ is a joint torque vector, and $\boldsymbol{J}(\boldsymbol{p})$ is a Jacobian matrix of the finger corresponding to the contact point $\boldsymbol{p}$. We assume $\boldsymbol{\tau}(\boldsymbol{p})$ to be constant, and thus $\boldsymbol{\mathcal{A}}(\boldsymbol{p})$ forms a convex hyperpolyhedron.

Similarly, for $\boldsymbol{p} \in \mathcal{C}_{\text{rob}} \cap \mathcal{C}_{\text{slide}}$,

$$\boldsymbol{\mathcal{A}}(\boldsymbol{p}) = \left\{ \begin{pmatrix} \boldsymbol{f} \\ \boldsymbol{p} \times \boldsymbol{f} \end{pmatrix} \middle| \boldsymbol{f} \in \text{span}\{\boldsymbol{c}'(\boldsymbol{p})\}, \boldsymbol{\tau}(\boldsymbol{p}) = \boldsymbol{J}(\boldsymbol{p})^T \boldsymbol{f} \right\}. \tag{5}$$

For simplicity, we assume that there exist no defective contacts [13, 14], i.e., rank $\boldsymbol{J}(\boldsymbol{p}) = 3$.

When we replace all the contacts by finite contact points, $\boldsymbol{p}_1, \ldots, \boldsymbol{p}_m \in \mathcal{C}$, the set of generalized forces applicable to the object through the contacts, $\boldsymbol{\mathcal{A}} \subset \Re^6$, can be approximated as follows [8, 9]:

$$\boldsymbol{\mathcal{A}} = \bigcup_i \{ \bigoplus_{\boldsymbol{p}_j \in \mathcal{C}_{\text{rob}} \cup \mathcal{C}_{\text{env}i}} \boldsymbol{\mathcal{A}}(\boldsymbol{p}_j) \}, \qquad (6)$$

where

$$\mathcal{C}_{\text{env}i} \in \left\{ \{\boldsymbol{p}_j\} \middle| \boldsymbol{p}_j \in \mathcal{C}_{\text{env}}, \begin{pmatrix} \boldsymbol{n}_j \\ \boldsymbol{p}_j \times \boldsymbol{n}_j \end{pmatrix} \text{ are linearly independent} \right\}.$$

$\boldsymbol{n}_j$ is the unit normal vector at $\boldsymbol{p}_j$ and $\oplus$ denotes the Minkowski sum. Note that we cannot necessarily apply arbitrary forces in $\boldsymbol{\mathcal{A}}$ actively. This is because $\boldsymbol{\mathcal{A}}$ contains forces that can be generated only as reaction.

4 Formulation of Contact Forces

4.1 Manipulation without Surface Contacts in Rotation

In the case of manipulation without surface contacts in rotation, we can replace a surface contact with its equivalent point contacts, vertices of the convex hull of the surface. Eq. (6) can be rewritten as follows:

$$\boldsymbol{\mathcal{A}} = \bigcup_i \{ \boldsymbol{Q} \mid \boldsymbol{Q} = \boldsymbol{W}_i \boldsymbol{C}_i \boldsymbol{k}, \boldsymbol{k} \geq \boldsymbol{0}, \boldsymbol{\tau}_i = \boldsymbol{J}_i^T \boldsymbol{C}_i \boldsymbol{k} \}, \quad (7)$$

where

$$\boldsymbol{W}_i = \begin{bmatrix} \cdots & \boldsymbol{I}_3 & \cdots \\ \cdots & \boldsymbol{p}_j \times \boldsymbol{I}_3 & \cdots \end{bmatrix} (\boldsymbol{p}_j \in \mathcal{C}_{\text{rob}} \cup \mathcal{C}_{\text{env}i})$$

$$\boldsymbol{C}_i = \text{diag}(\ldots, \boldsymbol{C}_{ij}, \ldots)$$

$$\boldsymbol{C}_{ij} = \begin{cases} [\boldsymbol{c}_1(\boldsymbol{p}_j) \ldots \boldsymbol{c}_s(\boldsymbol{p}_j)] \in \Re^{3\times s} \\ \qquad (\boldsymbol{p}_j \in \mathcal{C}_{\text{stat}} \cap (\mathcal{C}_{\text{rob}} \cup \mathcal{C}_{\text{env}i})) \\ [\boldsymbol{c}'(\boldsymbol{p}_i)] \in \Re^{3\times 1} \\ \qquad (\boldsymbol{p}_i \in \mathcal{C}_{\text{slide}} \cap (\mathcal{C}_{\text{rob}} \cup \mathcal{C}_{\text{env}i})) \end{cases}$$

$$\boldsymbol{J}_i = \text{diag}(\ldots, \boldsymbol{J}_{ij}, \ldots)$$

$$\boldsymbol{J}_{ij} = \begin{cases} \boldsymbol{J}(\boldsymbol{p}_j) & (\boldsymbol{p}_j \in \mathcal{C}_{\text{rob}}) \\ \boldsymbol{O} & (\boldsymbol{p}_j \in \mathcal{C}_{\text{env}i}) \end{cases}$$

$$\boldsymbol{\tau}_i = [\ldots, \boldsymbol{\tau}_{ij}^T, \ldots]^T$$

$$\boldsymbol{\tau}_{ij} = \begin{cases} \boldsymbol{\tau}(\boldsymbol{p}_j) & (\boldsymbol{p}_j \in \mathcal{C}_{\text{rob}}) \\ \boldsymbol{0} & (\boldsymbol{p}_j \in \mathcal{C}_{\text{env}i}). \end{cases}$$

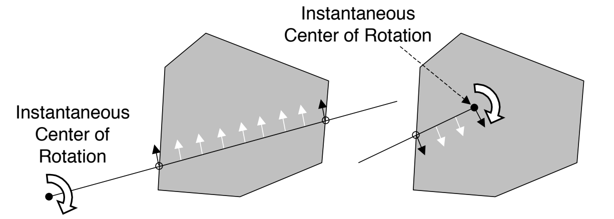

Figure 2: Frictional Force of Half-Line Contact in Rotation

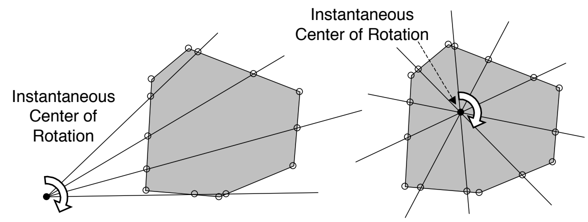

Figure 3: Representative Points for Surface Contact in Rotation

$\boldsymbol{I}_3$ is the 3×3 identity matrix, and $\boldsymbol{p}_j \times \boldsymbol{I}_3 \in \Re^{3\times 3}$ is a linear transformation equivalent to cross product with $\boldsymbol{p}_j$.

Eq. (7) forms a union of polyhedral convex cones in $\Re^6$.

4.2 Manipulation with Surface Contacts in Rotation

In the case of manipulation with surface contacts in rotation, the set of contact forces by the surface cannot be replaced equivalently by the effect of vertices of the convex hull of the surface (See numerical examples in Section 6). This fact was not referred to in [7, 8, 9]. Here we formulate the set of applicable (generalized) forces by a surface contact in rotation, $\boldsymbol{\mathcal{A}}_{\text{rot}} \subset \Re^6$.

To begin with, when the instantaneous center of rotation (COR) of a contact surface is *outside* the surface, $\boldsymbol{\mathcal{A}}_{\text{rot}}$ can be replaced equivalently by the effect of all the contact points on the *boundary* of the surface. This is because all the contact forces on a half-line that passes the instantaneous COR have the same direction vector; therefore the resultant generalized force of all the contact points on the intersection between the half-line and the surface can be represented by the effect of point contacts at the intersections between the half-line and the *boundary* of the surface (see Figure 2, left). Similarly, when the instantaneous COR is *on* the surface, $\boldsymbol{\mathcal{A}}_{\text{rot}}$ can be replaced equivalently by the effect of all the contact points on the *boundary* of the surface and the COR (see Figure 2, right).

However, we still have to consider infinite contact points to obtain $\boldsymbol{\mathcal{A}}_{\text{rot}}$. Thus we approximate $\boldsymbol{\mathcal{A}}_{\text{rot}}$

with the effect of contact forces at finite representative points as follows:

1. If the COR is *outside* the contact surface (Figure 3, left), we draw several half-lines that pass the COR. We choose intersections of these half-lines and the boundary of the surface as representative points. Besides, vertices of the surface are also chosen as representative points.
2. If the COR is *on* the contact surface (Figure 3, right), we also draw several half-lines that pass the COR. We choose intersections of these half-lines and the boundary of the surface, vertices of the surface, and the COR as representative points. The COR is regarded as a non-sliding contact point.

We can express the set of applicable contact forces by a union of polyhedral convex cones as Eq. (7) approximately using the above-mentioned representative points, even if the object has surface contacts in rotation. When we draw more half-lines and choose a lot of representative points, the approximation can be to arbitrary precision.

Even when we calculate performance indices in [7] and [8, 9], the above treatment is still necessary for manipulation with surface contacts in rotation.

5 Quantitative Evaluation of Stability of Graspless Manipulation

5.1 Stability Measure

In this section we define a measure to evaluate the stability of graspless manipulation in terms of the magnitude of disturbing (generalized) force that the object can resist without changing its motion. This is an extended version of a stability measure defined in [10], which was applicable only to non-sliding cases. In the literature of power grasp, similar stability measures for grasped objects were proposed [15, 16]. However, there is a difference between the important point for the stability of tightly-constrained objects in power grasp and that of loosely-constrained objects in graspless manipulation. Our stability measure is distinct from those in [15] and [16] in the following ways:

1. Our measure can deal with the stability of manipulated objects in motion. The measures in [15] and [16] deal with the stability of stationary objects in power grasp, and cannot be applied straightforward to graspless manipulation, where the stability of the objects is considerably affected by whether they are in motion or not.
2. Our measure takes gravity into account. In [16], gravity is neglected. The measure in [15] deals with only gravity that affects joint torques, therefore it is not suitable for graspless manipulation in which objects may be loosely constrained.

A sort of scaling is necessary for quantitative evaluation of the magnitude of generalized forces, because forces and moments have different physical dimensions. In this paper, we use a norm for generalized forces, $\boldsymbol{Q} \in \Re^6$, as follows:

$$\|\boldsymbol{Q}\|_{\boldsymbol{M}} = \sqrt{\boldsymbol{Q}^T \boldsymbol{M}^{-1} \boldsymbol{Q}}, \tag{8}$$

where $\boldsymbol{M} \in \Re^{6\times 6}$ is the inertia matrix of the manipulated object. The norm is coordinate-invariant. Physically, Eq. (8) evaluates the magnitude of a generalized force in terms of kinetic energy given to the object when the generalized force is applied to the object in a certain infinitesimal time.

We denote the known external force applied to the object, such as gravity, by $\boldsymbol{Q}_{\text{known}} \in \Re^6$. The unknown disturbing force is denoted by $\boldsymbol{Q}_{\text{dist}} \in \Re^6$. The equilibrium equation of the quasi-statically manipulated object is:

$$\begin{cases} \boldsymbol{Q}_{\text{known}} + \boldsymbol{Q} = -\boldsymbol{Q}_{\text{dist}} \\ \boldsymbol{Q} \in \mathcal{A} \end{cases} \tag{9}$$

Then we define a stability measure z for graspless manipulation as the solution of the following minimax problem:

$$z = \min_{\|\hat{\boldsymbol{Q}}_{\text{dist}}\|_{\boldsymbol{M}}=1} \max_{\substack{\boldsymbol{Q}_{\text{known}}+\boldsymbol{Q}=-t\hat{\boldsymbol{Q}}_{\text{dist}}, \\ t>0, \boldsymbol{Q}\in\mathcal{A}}} \|\boldsymbol{Q}_{\text{known}} + \boldsymbol{Q}\|_{\boldsymbol{M}}. \tag{10}$$

If $\|\boldsymbol{Q}_{\text{dist}}\|_{\boldsymbol{M}} < z$, the object motion is not perturbed. When the above minimax problem is infeasible, the quasi-static manipulation is impossible even with no disturbance.

Actually, stability evaluation is not enough to plan and execute graspless manipulation, because it is also necessary to avoid excessive internal force applied to the object. We can judge the possibility of excessive internal force with linear programming [10], and exclude improper manipulation.

5.2 Calculation of Stability Measure

Here we show a method to solve the minimax problem (Eq. (10)) approximately with linear programming. The value of z is the radius of the inscribed hypersphere whose center is $-\boldsymbol{Q}_{\text{known}}$ in $\mathcal{A}$. Approximating the hypersphere with a circumscribed convex hyperpolyhedron (see Figure 4 for a three-dimensional

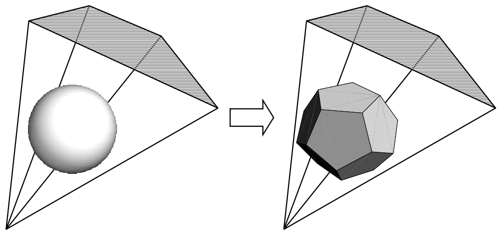

Figure 4: Approximate Calculation of Stability Measure

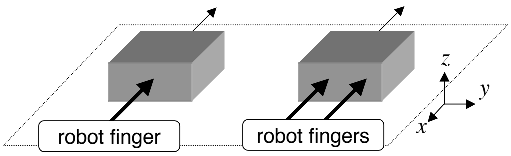

Figure 5: Example: Pushing Cuboids

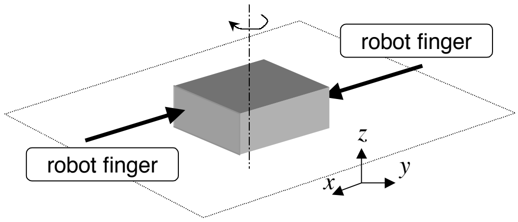

Figure 6: Example: Rotating a Cuboid

schematic sketch), we can calculate the approximate value of z as the following linear programming problems:

$$z = \min_{i=1,\dots,N} z_i \tag{11}$$

$$z_i = \max_j z_{ij} \tag{12}$$

$$\text{subject to} \begin{cases} z_{ij}\boldsymbol{l}_i = \boldsymbol{Q}_{\text{known}} + \boldsymbol{W}_j \boldsymbol{C}_j \boldsymbol{k}_{ij} \\ \boldsymbol{\tau}_j = \boldsymbol{J}_j^T \boldsymbol{C}_j \boldsymbol{k}_{ij} \\ \boldsymbol{k}_{ij} \geq \boldsymbol{0}, \end{cases}$$

where $\boldsymbol{l}_1, \dots, \boldsymbol{l}_N \in \Re^6$ are position vectors of vertices of a hyperpolyhedron circumscribed to a six-dimensional unit hypersphere whose center is the origin. Stability evaluation by the above linear programming is conservative when the set of generalized forces represented by Eq. (7) is convex. We can approximate the value of z to arbitrary precision by increasing N.

6 Numerical Examples

Let us consider graspless manipulation of a cuboid on a plane for numerical examples. The size of the object is $2 \times 2 \times 1$ and the center of mass is located at $(0,0,0)^T$. The mass of the object

Table 1: Approximate Values of Stability Measure

Representative Points	Calculated Value
the instantaneous COR, four vertices, and four other points on the boundary	$z = 1.2$
the instantaneous COR and four vertices	$z = 1.1$
four vertices of the bottom surface	$z = 0.7$

is 1 and the mass distribution is uniform, then we have $\boldsymbol{Q}_{\text{known}} = (0,0,-9.8,0,0,0)^T$ and $\boldsymbol{M} = \text{diag}(1,1,1,5/12,5/12,2/3)$. Friction coefficient is 0.2 and each friction cone is represented as a polyhedral convex cone with 12 unit edge vectors. For simplicity, all the robot fingers are position-controlled ($\mathcal{C}_{\text{rob}} = \emptyset$). Here we approximate the 6-dimensional unit hypersphere as a circumscribed hyperpolyhedron with 76 vertices, and calculate the stability measure in several cases.

In the case of one-point pushing of the object at $(1,0,0)^T$ toward $(-1,0,0)^T$-direction (Figure 5, left), the stability measure $z = 0$. That is, this manipulation is not stable because an infinitesimal disturbance can perturb the motion of the object. If the object is stationary, of course, the object is stable and $z = 1.3$. This affords an example that shows the importance of evaluating the stability of the object in manipulation.

On the other hand, in the case of two-point pushing at $(1, \pm 1/2, 0)^T$ (Figure 5, right), $z = 0.23$. That is, the object can resist disturbing force whose magnitude is at least 0.23 (in the sense of Eq. (8)) without changing its motion. That corresponds to a *stable push* [17] by a position-controlled pusher with line contact.

As an example of manipulation with a surface contact in rotation, let us consider rotating the cuboid around a vertical axis that passes the center of mass (Figure 6). The cuboid is in two-point contact at $(\pm 1, \mp 1/2, 0)^T$ with robot fingers. In this case, the approximate value of z depends largely on the choice of representative points for the bottom surface. As shown in Table 1, the stability is underestimated when we choose only the four vertices of the surface as representative points. Therefore it is not appropriate to model a surface contact in rotation using point contacts only at vertices of the surface.

It takes about 0.1 through 1.3 CPU seconds to calculate a value of the stability measure for the above examples on a Linux PC with Pentium4–1.5GHz.

7 Conclusion

We proposed a quantitative measure to evaluate the stability of graspless manipulation. The measure can be calculated with linear programming. We also presented a method to approximate the set of contact forces of surface contacts in rotation by multiple point contacts.

Now we are trying to incorporate this measure into our planning algorithm of graspless manipulation [18] to obtain robuster manipulation.

Acknowledgments

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