ME 449 Robotic Manipulation
Fall 2014
Problem Set 3
Due Thursday November 6 at beginning of class

Turn in your homework on canvas, as one pdf file with your answers to the questions and one text file with your code.

1. Derive the coordinate-based forward kinematics $z = f(\theta)$ of the 2R robot shown in Figure 1, where the end-effector configuration is $z = (x, y) \in \mathbb{R}^2$ and the two joint angles are (θ_1, θ_2). (We've done it plenty of times in class!) Then derive the Jacobian $J(\theta) = \partial f / \partial \theta$.

2. Write functions to calculate the body and space Jacobian of a manipulator, given the joint screw axes (expressed in the proper frame) and the joint angles. Test your code by printing the numerical 6×2 body and space Jacobians returned by your code for the 2R robot with the space and body frames shown in the figure when (a) $(\theta_1, \theta_2) = (0, \pi/2)$ and (b) $(\theta_1, \theta_2) = (\pi/4, \pi/4)$. (Note the figure does not show the robot at its home configuration; the home configuration is where $\theta_1 = \theta_2 = 0$.)

3. Using the coordinate-based Jacobian, write code to plot the polygon of feasible endpoint velocities $(\dot{x}, \dot{y})$ for the 2R robot at the joint angles $(\pi/8, \pi/4)$, $(-\pi/4, \pi/2)$, and $(-\pi/3, 2\pi/3)$ when the joint velocities are limited to the set $|\dot{\theta}_i| \leq 1$.

4. Write code to perform numerical inverse kinematics using the body frame Jacobian. Test your code on the 2R robot. The goal configuration is

$$T_{sd} = \begin{bmatrix} 0 & -1 & 0 & 2 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The initial joint angle guess is $(-10^\circ, 80^\circ)$. Run the numerical inverse kinematics until the total error in the (x, y) position is less than 0.01. After each iteration of the numerical method, print the joint angle guess and the (x, y) position of the end-effector.

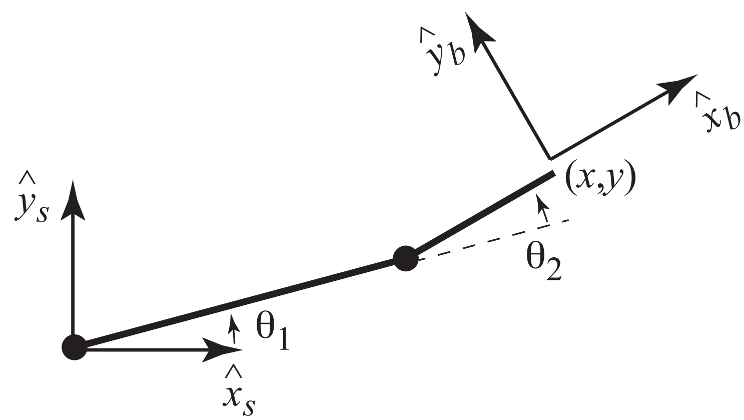

Figure 1: 2R robot. The length of the first link is 2 and the length of the second link is 1.