ME 449 Robotic Manipulation
Fall 2014
Problem Set 1
Due Thursday October 9 at beginning of class

Show your work and explain your results. If you think a question is unclear, then state your assumptions to clarify it.

1. Give a formula, in terms of n, for the number of degrees of freedom of a rigid body in n-dimensional space. Indicate how many of those dof are translational and how many are rotational.

2. Refer to Figure 1.

(i) The four-legged walking robot in Figure 1(a) consists of a base and four URR legs (Universal-Revolute-Revolute, with the universal joint at the connection between the base and the leg and the two revolute joints distal of the universal joint). The feet are points. What is the dimension of the robot's C-space (the number of degrees of freedom) when no feet are on the ground? One? Two? Three? All four? (The feet are allowed to slip on the ground.)

(ii) Answer the same questions as in (i) but with each leg an SRR mechanism (Spherical-Revolute-Revolute).

(iii) Now turn the robot upside down and call it a hand with four fingers as in Figure 1(b). Each finger is a URR open chain. The palm of the hand is fixed in space, so it has zero degrees of freedom. Imagine the hand is grasping an object, with all fingertips in contact. If you assume the point fingertips don't slip on the object, how many degrees of freedom does the system have?

(iv) Now assume the fingertips of the hand are spheres, not points, as in Figure 1(c). The four fingertips can roll on the object, but cannot slip or break contact. How many degrees of freedom does the system have?

3. The C-space of a point in n-space is $\mathbb{R}^n$. The one-dimensional C-space of a revolute joint is S^1, which is topologically different than the one-dimensional line $\mathbb{R}^1$. The C-space of a point on a sphere is S^2, and the C-space of a point on an n-dimensional sphere is S^n. The C-space of a robot with n revolute joints is $S^1 \times S^1 \ldots \times S^1 = T^n$, the n-dimensional torus, which is topologically different than S^n. The set of two-dimensional rotations can be written $SO(2)$ and is topologically equivalent to S^1, but $SO(3)$ is different than S^3. The set of rigid-body configurations, $SE(3)$, is equivalent to $SO(3) \times \mathbb{R}^3$. A revolute joint with limited joint range can be expressed as a closed interval $[a, b]$.

What are the C-spaces of the following systems, using as examples the C-spaces above?

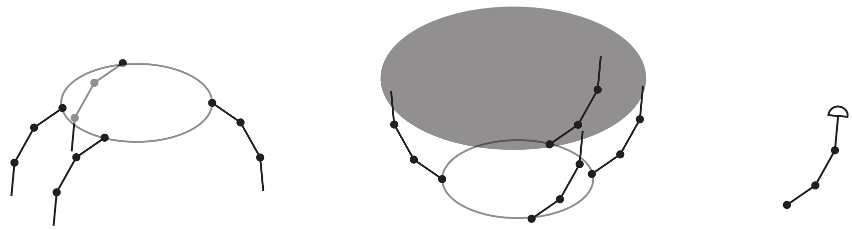

Figure 1: (a) A 4-legged robot. (b) A hand grasping a gray ellipsoid. (c) A rounded fingertip that can roll on the object without sliding.

(i) The chassis of a car-like mobile robot rolling on an infinite plane.

(ii) The car-like mobile robot, but including a representation of the wheel configurations.

(iii) The car-like mobile robot driving around on a spherical asteroid.

(iv) The car-like mobile robot on an infinite plane with an RRPR robot arm mounted on it. The prismatic joint has joint limits, but the revolute joints do not.

(v) The car-like mobile robot on an infinite plane with an RRPR robot arm mounted on it. All joints have joint limits.

(vi) A free-flying spacecraft with a 6R arm mounted on it, no joint limits.

4. A differential-drive mobile robot has two wheels which do not steer but whose speeds can be controlled independently. The robot goes forward and backward by spinning the wheels in the same direction at the same speed, and it turns by spinning the wheels at different speeds. The configuration of the robot is given by five variables: the (x, y) location of the point halfway between the wheels, the heading direction θ of the robot's chassis, and the rotation angles ϕ_1 and ϕ_2 of the two wheels relative to the axis through the centers of the wheels.

(i) Let $q = (x, y, \theta, \phi_1, \phi_2)$ be the configuration of the robot. If the two control inputs are the angular velocities of the wheels ω_1 and ω_2, write the vector differential equation $\dot{q} = g_1(q)\omega_1 + g_2(q)\omega_2$. The vector fields $g_1(q)$ and $g_2(q)$ are called *control vector fields*, expressing how the system moves when the respective control is applied.

(ii) Write the corresponding Pfaffian constraints $A(q)\dot{q} = 0$ for this system.

(iii) Are the constraints holonomic or nonholonomic?

5. While the C-space is a representation of all possible configurations of a system, the *task space* or *workspace* often refers to just the C-space of the end-effector of a robot, not the whole robot. Consider your arm, which we decided in class has 7 dof, from the shoulder to the palm. If we call your palm the end-effector, what is the task space? What is its dimension?

6. Elements of the Special Orthogonal Group $SO(2)$ (planar rotations) commute, while those of $SO(3)$ do not. (Recall that a matrix in $SO(2)$ consists of the 2×2 upper-left submatrix of a matrix in $SO(3)$.) Prove these statements.

7. In terms of the $\hat{x}$-$\hat{y}$-$\hat{z}$ coordinates of a fixed space frame {s}, the frame {a} has an $\hat{x}_a$-axis pointing in the direction $(0, 0, 1)$ and a $\hat{y}_a$-axis pointing in the direction $(-1, 0, 0)$, and the frame {b} has an $\hat{x}_b$-axis pointing in the direction $(1, 0, 0)$ and a $\hat{y}_b$-axis pointing in the direction $(0, 0, -1)$.

(i) Give your best hand drawing of the three frames. Draw them at different locations so they are easy to see.

(ii) Write the rotation matrices R_{sa} and R_{sb}.

(iii) Given R_{sb}, how do you calculate R_{sb}^{-1} without using a matrix inverse? Write R_{sb}^{-1} and verify its correctness with your drawing.

(iv) Given R_{sa} and R_{sb}, how do you calculate R_{ab} (again no matrix inverses)? Compute the answer and verify its correctness with your drawing.

(v) R_{sb} is obtained by rotating the frame {s} about its $\hat{x}_s$-axis by $-90°$. Let $R = R_{sb}$. Calculate $R_1 = R_{sa}R$, and think of R_{sa} as a representation of an orientation, R as a rotation of R_{sa}, and R_1 as the new orientation after performing the rotation. Does the new orientation R_1 correspond to rotating R_{sa} by $-90°$ about the world-fixed $\hat{x}_s$-axis or the body-fixed $\hat{x}_a$-axis? Now calculate $R_2 = RR_{sa}$. Does the new orientation R_2 correspond to rotating R_{sa} by $-90°$ about the world-fixed $\hat{x}_s$-axis or the body-fixed $\hat{x}_a$-axis?

(vi) Use R_{sb} to change the representation of the point $p_b = (1, 2, 3)$ (in {b} coordinates) to {s} coordinates.

(vii) Choose a point p represented by $p_s = (1, 2, 3)$ in {s} coordinates. Calculate $p' = R_{sb}p_s$ and $p'' = R_{sb}^T p_s$. For each operation, did we change coordinates (from the {s} frame to {b}) or transform of the location of the point without changing the reference frame of the representation?

8. (This exercise is similar to the previous one, but now using transformation matrices instead of rotation matrices.) In terms of the $\hat{x}$-$\hat{y}$-$\hat{z}$ coordinates of a fixed space frame {s}, the frame {a} has an $\hat{x}_a$-axis pointing in the direction $(0, 0, 1)$ and a $\hat{y}_a$-axis pointing in the direction $(-1, 0, 0)$, and the frame {b} has an $\hat{x}_b$-axis pointing in the direction $(1, 0, 0)$ and a $\hat{y}_b$-axis pointing in the direction $(0, 0, -1)$. The origin of {a} is at (3,0,0) in {s} and the origin of {b} is at (0,2,0).

This exercise deals with some of the major uses of a rotation matrix: to represent the orientation of a frame relative to another, to change coordinate frames of the representation of a frame or point, and to rotate a frame or point.

(i) Give your best hand drawing of the three frames.

(ii) Write the rotation matrices R_{sa} and R_{sb}.

(iii) Given T_{sb}, how do you calculate T_{sb}^{-1} without using a matrix inverse? Write T_{sb}^{-1} and verify its correctness with your drawing.

(iv) Given T_{sa} and T_{sb}, how do you calculate T_{ab} (again no matrix inverses)? Compute the answer and verify its correctness with your drawing.

(v) Let $T = T_{sb}$. Calculate $T_1 = T_{sa}T$. Does T_1 correspond to a body-fixed or world-fixed transformation of T_{sa}, or neither? Now calculate $T_2 = TT_{sa}$. Does T_2 correspond to a body-fixed or world-fixed transformation of T_{sb}, or neither?

(vi) Use T_{sb} to change the representation of the point $p_b = (1, 2, 3)$ (in {b} coordinates) to {s} coordinates.

(vii) Choose a point p represented by $p_s = (1, 2, 3)$ in {s} coordinates. Calculate $p' = T_{sb}p_s$ and $p'' = T_{sb}^{-1}p_s$. For each operation, did we change coordinates (from the {s} frame to {b}) or transform of the location of the point without changing the reference frame of the representation?

9. Find the exponential coordinates $\omega\theta \in \mathbb{R}^3$ corresponding to the $SO(3)$ matrix

$$\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}.$$

10. Assume that the body frame angular velocity is $\omega_b = (1, 2, 3)$ for a moving body at

$$R = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

in the world frame {s}. Calculate the angular velocity ω_s in {s}.