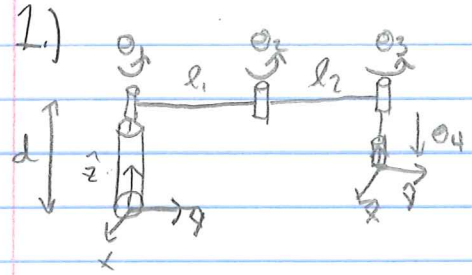
Zack Woodruff

74/73

ME 449

1.)



$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & L_1+L_2 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ 2

Find forward Kinematics using space frame and body frame screws.

1a) Space frame:

$$S_{1s} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad S_{2s} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad S_{3s} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ L_1+L_2 \\ 0 \\ 0 \end{bmatrix} \quad S_{4s} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

1 1 1 1

$$S_{1b} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -(l_1+l_2) \\ 0 \\ 0 \end{bmatrix} \quad S_{2b} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -l_2 \\ 0 \\ 0 \end{bmatrix} \quad S_{3b} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad S_{4b} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

1 1 1 1

$$T_{04s} = e^{[S]\theta_1} \cdots e^{[S_n]\theta_n} M \qquad T_{04b} = M e^{[B]\theta_1} \cdots e^{[b_n]\theta_n}$$

1 1

The attached mathematica code shows that the forward kinematics are identical and =

$$\boxed{T_{01} = \begin{bmatrix} C_{123} & -S_{123} & 0 & -L_1S_1 - L_2S_{12} \\ S_{123} & C_{123} & 0 & L_1C_1 + L_2C_{12} \\ 0 & 0 & 1 & d - \theta_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$ 2

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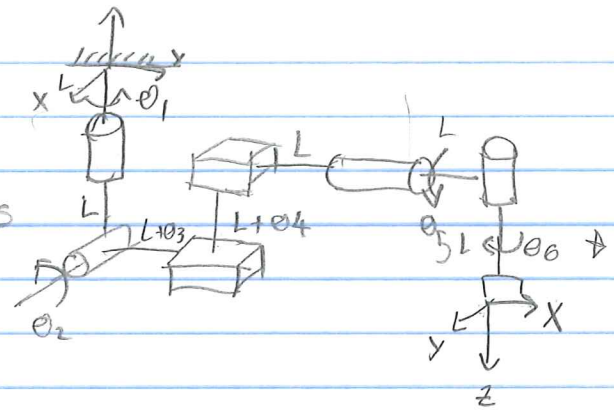
2 Chapter 4 Problem 10
RRPPRR



a.) Space frame forward kinematics

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 3L \\ 0 & 0 & -1 & -2L \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ 1

$$S_{1s} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad S_{2s} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ -2L \\ 0 \end{bmatrix} \quad S_{3s} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad S_{4s} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad S_{5s} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ L \\ 0 \\ 0 \end{bmatrix} \quad S_{6s} = \begin{bmatrix} 0 \\ 0 \\ -1 \\ -3L \\ 0 \\ 0 \end{bmatrix}$$

1 1 1 1 1 1

$$T_{06s} = e^{[S_1]\theta_1} \cdots e^{[S_6]\theta_6} M$$

b.) $$T_{06b} = M e^{[B_1]\theta_1} \cdots e^{[B_6]\theta_6}$$

$$S_{1b} = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ -3L \\ 0 \end{bmatrix} \quad S_{2b} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ -3L \end{bmatrix} \quad S_{3b} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad S_{4b} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} \quad S_{5b} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ -L \\ 0 \end{bmatrix} \quad S_{6b} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

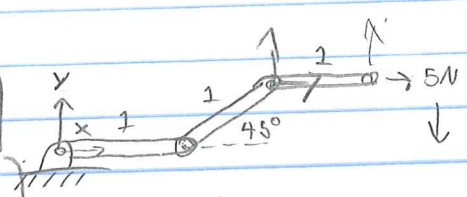
1 1 1 1 1 1

T_{06s} and T_{06b} were shown to be equal in mathematica. 1

14

3. Prob 16 chapter 5

$$F = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} \quad M = \begin{bmatrix} 0 \\ 0 \\ -5/\sqrt{2} \end{bmatrix}$$



zero position

$$S_{1s} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad S_{2s} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

$$F_{sx} = \begin{bmatrix} 0 \\ 0 \\ -5/\sqrt{2} \\ 5 \\ 0 \\ 0 \end{bmatrix} \quad F_{sy} = \begin{bmatrix} 0 \\ 0 \\ 5(2+1/\sqrt{2}) \\ 0 \\ 5 \\ 0 \end{bmatrix}$$

2 2

$$V_{s_i}(\theta) = Ad_{e^{[S_1]\theta_1} \cdots e^{[S_{i-1}]\theta_{i-1}}}(S_i)$$

$$\omega_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad q_3 = \begin{bmatrix} 1 + \frac{1}{\sqrt{2}} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$$

$$Ad_T = \begin{bmatrix} R & 0 \\ [P]R & R \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 + 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} \\ -(1 + 1/\sqrt{2}) \\ 0 \end{bmatrix} \quad S_{3s} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1/\sqrt{2} \\ -(1 + 1/\sqrt{2}) \\ 0 \end{bmatrix}$$

$$\begin{array}{cc} 3\times3 & 3\times1 \\ \end{array} \begin{bmatrix} R & P \\ 0 & 1 \end{bmatrix} \quad [P]$$

$$V_{s1} = S_{1s}$$

$$V_{s2} = \left[Ad_{(e^{[S_1]\theta_1})}\right](S_{2s}) = \begin{bmatrix} 0 \\ 0 \\ 1 \\ \sin(\theta_1) \\ -\cos(\theta_1) \\ 0 \end{bmatrix}$$

$$V_{s3} = Ad_{e^{[S_1]\theta_1} e^{[S_2]\theta_2}}(S_{3s}) = \begin{bmatrix} 0 \\ 0 \\ 1 \\ \frac{C_{12}}{\sqrt{2}} + \frac{S_{12}}{\sqrt{2}} + S_1 \\ \frac{-C_{12} + S_{12}}{\sqrt{2}} - C_1 \\ 0 \end{bmatrix}$$

4

$$\boxed{J_s(\theta) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & S_1 & \frac{C_{12} + S_{12}}{\sqrt{2}} + S_1 \\ 0 & -C_1 & \frac{-C_{12} + S_{12}}{\sqrt{2}} - C_1 \\ 0 & 0 & 0 \end{bmatrix}}$$

$$\tau = J_s(\theta)^T F_s$$

x dir

$$\boxed{\tau = \begin{bmatrix} -5/\sqrt{2} \\ -5/\sqrt{2} \\ 0 \end{bmatrix}}$$

2

y dir

$$\boxed{\tau = \begin{bmatrix} 5(2 + 1/\sqrt{2}) \\ 5(1 + 1/\sqrt{2}) \\ 5 \end{bmatrix}}$$

2

12

P3 Continued

$$B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1/\sqrt{2} \\ 2+1/\sqrt{2} \\ 0 \end{bmatrix} \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1/\sqrt{2} \\ 1+1/\sqrt{2} \\ 0 \end{bmatrix} \quad b_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad F_{bx} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 5 \\ 0 \\ 0 \end{bmatrix}$$

$$V_{bi}(\theta) = Ad_{e^{-[B_n]\theta_n} \cdots e^{-[B_{i-1}]\theta_{i-1}}}(B_i) \quad F_{by} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 5 \\ 0 \end{bmatrix}$$

$$V_{b1} = Ad_{\left(e^{-[B_3]\theta_3} e^{-[B_2]\theta_2}\right)} B_1$$

$$V_{b2} = Ad_{e^{-[B_3]\theta_3}} B_2 \qquad V_{b3} = B_3$$

$$J_b(\theta) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ \frac{-C_3+S_3}{\sqrt{2}} + S_{23} & \frac{-C_3+S_3}{\sqrt{2}} & 0 \\ \frac{C_3+S_3}{\sqrt{2}} + C_{23} + 1 & \frac{C_3+S_3}{\sqrt{2}} + 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

(from mathematica)

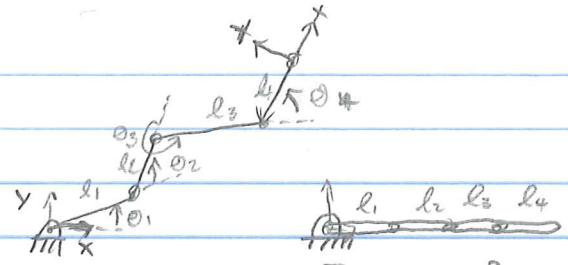
✓

τ calculation matches those for Js

$$\boxed{\begin{array}{l} \text{Case 1} \quad \tau = J_b(\theta)^T F_{bx} \\ \\ \text{Case 2} \quad \tau = J_b(\theta)^T F_{by} \end{array}}$$

1

4.) Ch 5. P17



Zero config

a.) $T(\theta) = e^{[S_1]\theta_1} e^{[S_2]\theta_2} e^{[S_3]\theta_3} e^{[S_4]\theta_4} M$

$$S_1 = \begin{bmatrix} \omega_s \\ \dot{x}_s \\ \dot{y}_s \end{bmatrix} \quad S_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad S_2 = \begin{bmatrix} 1 \\ 0 \\ -l_1 \end{bmatrix} \quad S_3 = \begin{bmatrix} 1 \\ 0 \\ -(l_1+l_2) \end{bmatrix} \quad S_4 = \begin{bmatrix} 1 \\ 0 \\ -(l_1+l_2+l_3) \end{bmatrix} \quad M = \begin{bmatrix} 1 & 0 & l_1+l_2+l_3+l_4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

1 1 1 1 1

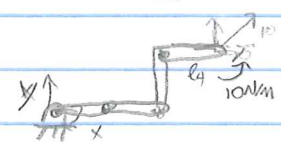
$$\boxed{T(\theta) = \begin{bmatrix} C_{1234} & -S_{1234} & L_1C_1 + l_2C_{12} + l_3C_{123} + l_4C_{1234} \\ S_{1234} & C_{1234} & L_1S_1 + L_2S_{12} + l_3S_{123} + l_4S_{1234} \\ 0 & 0 & 1 \end{bmatrix}}$$ 1

b.) $V_{b,i}(\theta) = Ad_{e^{-[B_n]\theta_n} \cdots e^{-[B_{i-1}]\theta_{i+1}}}(B_i)$

(From Mathematica):

$$\boxed{J_b(\theta) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ L_3S_4 + L_2S_{34} + L_1S_{234} & L_3S_4 + L_2S_{34} & L_3S_4 & 0 \\ L_3C_4 + L_2C_{34} + L_1C_{234} + l_4 & L_4 + L_3C_4 + L_2C_{34} & L_4 + L_3C_4 & L_4 \end{bmatrix}}$$ 5

c.) $\theta_1 = \theta_2 = 0 \quad \theta_3 = \pi/2 \quad \theta_4 = -\pi/2$



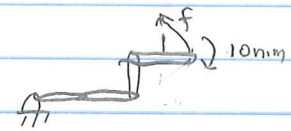
Fixed frame = Body frame

$f = \begin{bmatrix} 10 \\ 10 \end{bmatrix} \quad M = [10] + [10 \cdot l_4] = 10[1 + l_4]$

$$F_b = \begin{bmatrix} 10 \\ 10 \\ 10 \end{bmatrix} = \begin{bmatrix} m_b \\ f_x \\ f_y \end{bmatrix}$$

$$\boxed{\tau = J_b^T F_b = 10 \begin{bmatrix} 1 + L_1 + L_2 - L_3 + L_4 \\ 1 + L_2 - L_3 + L_4 \\ 1 - L_3 + L_4 \\ 1 + L_4 \end{bmatrix}}$$ 3

d.) $f = \begin{bmatrix} -10 \\ 10 \end{bmatrix}$ $m = [-10]$



$F_b = \begin{bmatrix} -10 \\ -10 \\ 10 \end{bmatrix}$ $\tau = J_b(\theta)^T F_b$

$$\tau = 10\begin{bmatrix} -1 + L_1 + L_2 + L_3 + L_4 \\ -1 + L_2 + L_3 + L_4 \\ -1 + L_3 + L_4 \\ -1 + L_4 \end{bmatrix}$$

3

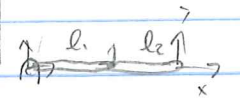
e.) Kinematic singularities occur when all links are in line. This happens when the links are in line, so when θ_2, θ_3 and θ_4 are all 0 or 180°

3

MORE GENERAL THAN THAT BUT WORKS HERE

6

P5) $S_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $S_2 = \begin{bmatrix} 1 \\ 0 \\ -l_1 \end{bmatrix}$ $M = \begin{bmatrix} 1 & 0 & l_1+l_2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$



$$T_{02} = \begin{bmatrix} C_{12} & -S_{12} & 0 & L_1C_1 + L_2C_{12} \\ C_{12} & S_{12} & 0 & L_1S_1 + L_2S_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$J(\theta) = \frac{\partial}{\partial \theta} \begin{bmatrix} L_1C_1 + L_2C_{12} \\ L_1S_1 + L_2S_{12} \end{bmatrix} = \begin{bmatrix} -L_1S_1 - L_2S_{12} & -L_2S_{12} \\ L_1C_1 + L_2C_{12} & L_2C_{12} \end{bmatrix}$$

$$\dot{X} = J(\theta)\dot{\theta}$$

Pluged in $(\dot{\theta}_1, \dot{\theta}_2)$ pairs from unit circle to find $(\dot{x}_1, \dot{y}_1)$ pairs.

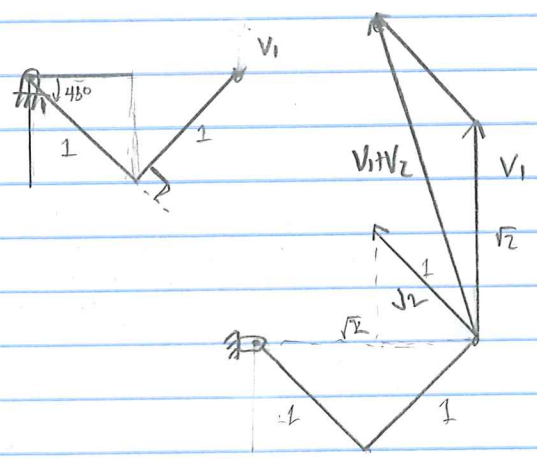
a.) Yes it depends on θ_2 but not θ_1.

3

$\dot{\theta}_1$ can only make the velocity change perpendicular to the line from (0,0) to the manipulator. This defines the direction of the major axis but not the eccentricity.

b.) It appears the most isotropic at 135°, 90°
(Plot attached)

5

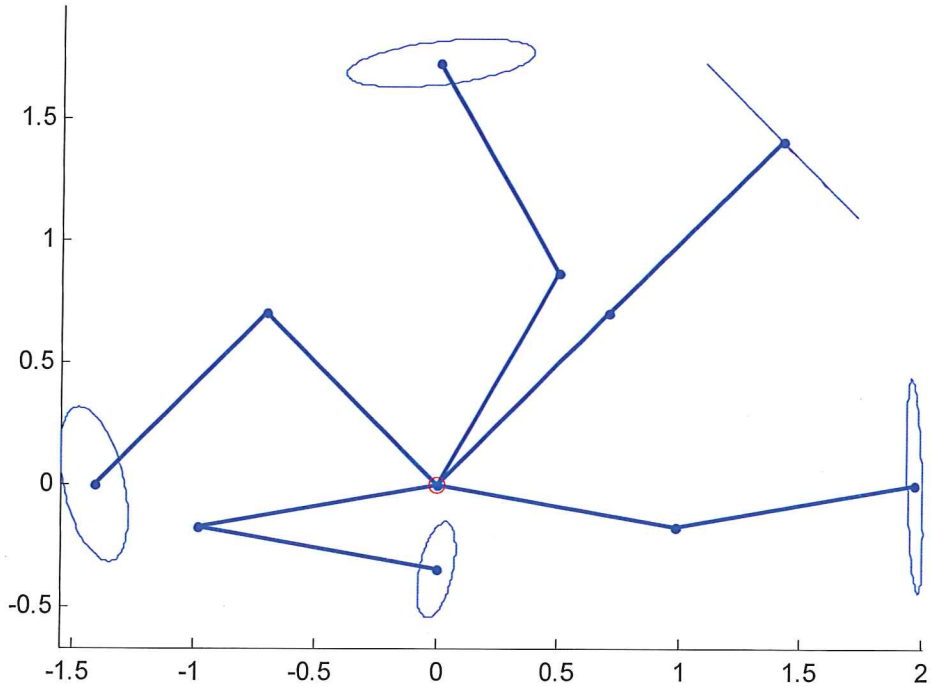


$V_x = -1/\sqrt{2}$

$V_y = \sqrt{2} + 1/\sqrt{2} = \boxed{\begin{matrix} -1/\sqrt{2} \\ 3/\sqrt{2} \end{matrix}}$

5

13


1.5
1
0.5
0
-0.5
-1.5
-1
-0.5
0
0.5
1
1.5
2

In[10]:=
```
(*
ME 449 Homework 2
 Zack Woodruff
 4/24/2014
*)
```

(*Problem I*)

```
Print["Problem 1"];
```

$$\text{SkewSym}[\{a1_, a2_, a3_\}] := \begin{pmatrix} 0 & -a3 & a2 \\ a3 & 0 & -a1 \\ -a2 & a1 & 0 \end{pmatrix};$$

$$\text{Bracket}[\{a1_, a2_, a3_, a4_, a5_, a6_\}] := \begin{pmatrix} 0 & -a3 & a2 & a4 \\ a3 & 0 & -a1 & a5 \\ -a2 & a1 & 0 & a6 \\ 0 & 0 & 0 & 0 \end{pmatrix};$$

```
expS[S_, θ_] := MatrixExp[Bracket[Flatten[S]] * θ]
```

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & L1 + L2 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$S1s = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}; \; S2s = \begin{pmatrix} 0 \\ 0 \\ 1 \\ L1 \\ 0 \\ 0 \end{pmatrix}; \; S3s = \begin{pmatrix} 0 \\ 0 \\ 1 \\ L1 + L2 \\ 0 \\ 0 \end{pmatrix}; \; S4s = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{pmatrix};$$

$$S1b = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -(L1 + L2) \\ 0 \\ 0 \end{pmatrix}; \; S2b = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -L2 \\ 0 \\ 0 \end{pmatrix}; \; S3b = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}; \; S4b = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{pmatrix};$$

```
(*L1=1;L2=1; d=1;θ1=0;θ2=Pi/2;θ3=0;θ4=0;*)

Print["T04s:"];
(Ts = MatrixExp[Bracket[Flatten[S1s]] * θ1].
     MatrixExp[Bracket[Flatten[S2s]] * θ2].MatrixExp[Bracket[Flatten[S3s]] * θ3].
     MatrixExp[Bracket[Flatten[S4s]] * θ4].M // FullSimplify) // MatrixForm

Print["T04b:"];
(Tb = M.MatrixExp[Bracket[Flatten[S1b]] * θ1].
     MatrixExp[Bracket[Flatten[S2b]] * θ2].MatrixExp[Bracket[Flatten[S3b]] * θ3].
     MatrixExp[Bracket[Flatten[S4b]] * θ4] // FullSimplify) // MatrixForm
```

Problem 1

T04s:

Out[18]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[\theta1+\theta2+\theta3] & -\text{Sin}[\theta1+\theta2+\theta3] & 0 & -\text{L1 Sin}[\theta1]-\text{L2 Sin}[\theta1+\theta2] \\ \text{Sin}[\theta1+\theta2+\theta3] & \text{Cos}[\theta1+\theta2+\theta3] & 0 & \text{L1 Cos}[\theta1]+\text{L2 Cos}[\theta1+\theta2] \\ 0 & 0 & 1 & d-\theta4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

T04b:

Out[20]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[\theta1+\theta2+\theta3] & -\text{Sin}[\theta1+\theta2+\theta3] & 0 & -\text{L1 Sin}[\theta1]-\text{L2 Sin}[\theta1+\theta2] \\ \text{Sin}[\theta1+\theta2+\theta3] & \text{Cos}[\theta1+\theta2+\theta3] & 0 & \text{L1 Cos}[\theta1]+\text{L2 Cos}[\theta1+\theta2] \\ 0 & 0 & 1 & d-\theta4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In[21]:=

```
(*Problem 2*)
Print["Problem 2"];
```

$$M = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 3*L \\ 0 & 0 & -1 & -2*L \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$S1s = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}; S2s = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ -2*L \\ 0 \end{pmatrix}; S3s = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}; S4s = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}; S5s = \begin{pmatrix} 0 \\ 1 \\ 0 \\ L \\ 0 \\ 0 \end{pmatrix}; S6s = \begin{pmatrix} 0 \\ 0 \\ -1 \\ -3*L \\ 0 \\ 0 \end{pmatrix};$$

$$S1b = \begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \\ -3*L \\ 0 \end{pmatrix}; S2b = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ -3*L \end{pmatrix}; S3b = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}; S4b = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{pmatrix}; S5b = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ -L \\ 0 \end{pmatrix}; S6b = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix};$$

```
Print["T04s:"];
(Ts = MatrixExp[Bracket[Flatten[S1s]] * θ1].
      MatrixExp[Bracket[Flatten[S2s]] * θ2].MatrixExp[Bracket[Flatten[S3s]] * θ3].
      MatrixExp[Bracket[Flatten[S4s]] * θ4].MatrixExp[Bracket[Flatten[S5s]] * θ5].
      MatrixExp[Bracket[Flatten[S6s]] * θ6].M // FullSimplify) // MatrixForm

Print["T04b:"];
(Tb = M.MatrixExp[Bracket[Flatten[S1b]] * θ1].
      MatrixExp[Bracket[Flatten[S2b]] * θ2].MatrixExp[Bracket[Flatten[S3b]] * θ3].
      MatrixExp[Bracket[Flatten[S4b]] * θ4].MatrixExp[Bracket[Flatten[S5b]] * θ5].
      MatrixExp[Bracket[Flatten[S6b]] * θ6] // FullSimplify) // MatrixForm

Ts - Tb
```

Problem 2

T04s:

Out[26]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[\theta 1]\ \text{Cos}[\theta 5]\ \text{Sin}[\theta 6] - \text{Sin}[\theta 1]\ (\text{Cos}[\theta 2]\ \text{Cos}[\theta 6] + \text{Sin}[\theta 2]\ \text{Sin}[\theta 5]\ \text{Sin}[\theta 6]) & \text{Cos}[\theta 1]\ \text{Cc} \\ \text{Cos}[\theta 5]\ \text{Sin}[\theta 1]\ \text{Sin}[\theta 6] + \text{Cos}[\theta 1]\ (\text{Cos}[\theta 2]\ \text{Cos}[\theta 6] + \text{Sin}[\theta 2]\ \text{Sin}[\theta 5]\ \text{Sin}[\theta 6]) & \text{Cos}[\theta 5]\ \text{C} \\ \text{Cos}[\theta 6]\ \text{Sin}[\theta 2] - \text{Cos}[\theta 2]\ \text{Sin}[\theta 5]\ \text{Sin}[\theta 6] & \\ 0 & \end{pmatrix}$$

T04b:

Out[28]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[\theta 1]\ \text{Cos}[\theta 5]\ \text{Sin}[\theta 6] - \text{Sin}[\theta 1]\ (\text{Cos}[\theta 2]\ \text{Cos}[\theta 6] + \text{Sin}[\theta 2]\ \text{Sin}[\theta 5]\ \text{Sin}[\theta 6]) & \text{Cos}[\theta 1]\ \text{Cc} \\ \text{Cos}[\theta 5]\ \text{Sin}[\theta 1]\ \text{Sin}[\theta 6] + \text{Cos}[\theta 1]\ (\text{Cos}[\theta 2]\ \text{Cos}[\theta 6] + \text{Sin}[\theta 2]\ \text{Sin}[\theta 5]\ \text{Sin}[\theta 6]) & \text{Cos}[\theta 5]\ \text{C} \\ \text{Cos}[\theta 6]\ \text{Sin}[\theta 2] - \text{Cos}[\theta 2]\ \text{Sin}[\theta 5]\ \text{Sin}[\theta 6] & \\ 0 & \end{pmatrix}$$

T04s - T04b

Out[29]= {{0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}, {0, 0, 0, 0}}

In[1]:=

(*Problem 3*)

```
Print["Problem 3"];
```

$$\text{SkewSym}[\{a1_, a2_, a3_\}] := \begin{pmatrix} 0 & -a3 & a2 \\ a3 & 0 & -a1 \\ -a2 & a1 & 0 \end{pmatrix}$$

$$\text{Bracket}[\{a1_, a2_, a3_, a4_, a5_, a6_\}] := \begin{pmatrix} 0 & -a3 & a2 & a4 \\ a3 & 0 & -a1 & a5 \\ -a2 & a1 & 0 & a6 \\ 0 & 0 & 0 & 0 \end{pmatrix};$$

```
Adj[b_] :=
  Join[Join[b[[1 ;; 3, 1 ;; 3]], SkewSym[b[[1 ;; 3, 4]]].b[[1 ;; 3, 1 ;; 3]]],
   Join[Table[0, {3}, {3}], b[[1 ;; 3, 1 ;; 3]]], 2];

expS[S_, θ_] := MatrixExp[Bracket[Flatten[S]] * θ]


(* Force for in x dir*)
```

$$\text{Fsx} = \begin{pmatrix} 0 \\ 0 \\ -5 / \text{Sqrt}[2] \\ 5 \\ 0 \\ 0 \end{pmatrix}; \text{Fbx} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 5 \\ 0 \\ 0 \end{pmatrix};$$

```
(* Force for in y dir*)
```

$$Fsy = \begin{pmatrix} 0 \\ 0 \\ 5 * (2 + 1 / Sqrt[2]) \\ 0 \\ 5 \\ 0 \end{pmatrix}; \; Fby = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 5 \\ 0 \end{pmatrix};$$

$$S1s = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}; \; S2s = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}; \; S3s = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 / Sqrt[2] \\ -(1 + 1 / Sqrt[2]) \\ 0 \end{pmatrix};$$

$$B1 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 / Sqrt[2] \\ 2 + 1 / Sqrt[2] \\ 0 \end{pmatrix}; \; B2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 / Sqrt[2] \\ 1 + 1 / Sqrt[2] \\ 0 \end{pmatrix}; \; B3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix};$$

```
(* Space frame Jacobian*)
(Vs1 = S1s // FullSimplify) // MatrixForm;
(Vs2 = Adj[expS[S1s, θ1]].S2s // FullSimplify) // MatrixForm;
(Vs3 = Adj[expS[S1s, θ1].expS[S2s, θ2]].S3s // FullSimplify) // MatrixForm;
Print["Js:"];
(Js = Join[Vs1, Vs2, Vs3, 2]) // FullSimplify // MatrixForm


(* Body frame Jacobian*)
(Vb1 = Adj[(expS[-B3, θ3]).(expS[-B2, θ2])].B1 // FullSimplify) // MatrixForm;
(Vb2 = Adj[(expS[-B3, θ3])].B2 // FullSimplify) // MatrixForm;
(Vb3 = B3 // FullSimplify) // MatrixForm;
Print["Jb:"];
(Jb = Join[Vb1, Vb2, Vb3, 2]) // FullSimplify // MatrixForm

(* Setting angles to zero confic*)
θ1 = 0; θ2 = 0; θ3 = 0;

(* X dir torque calc*)
Print["τsx:"];
τsx = Js^T.Fsx // FullSimplify // MatrixForm
τbx = Jb^T.Fbx // FullSimplify // MatrixForm;

(* Y dir torque calc*)
Print["τsy:"];
τsy = Js^T.Fsy // FullSimplify // MatrixForm
τby = Jb^T.Fby // FullSimplify // MatrixForm;
```

Problem 3

Js:

Out[14]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & \text{Sin}[\theta 1] & \frac{\text{Cos}[\theta 1+\theta 2]}{\sqrt{2}} + \text{Sin}[\theta 1] + \frac{\text{Sin}[\theta 1+\theta 2]}{\sqrt{2}} \\ 0 & -\text{Cos}[\theta 1] & -\text{Cos}[\theta 1] + \frac{-\text{Cos}[\theta 1+\theta 2]+\text{Sin}[\theta 1+\theta 2]}{\sqrt{2}} \\ 0 & 0 & 0 \end{pmatrix}$$

Jb:

Out[19]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \\ -\frac{\text{Cos}[\theta 3]}{\sqrt{2}} + \frac{\text{Sin}[\theta 3]}{\sqrt{2}} + \text{Sin}[\theta 2+\theta 3] & \frac{-\text{Cos}[\theta 3]+\text{Sin}[\theta 3]}{\sqrt{2}} & 0 \\ 1 + \frac{\text{Cos}[\theta 3]}{\sqrt{2}} + \text{Cos}[\theta 2+\theta 3] + \frac{\text{Sin}[\theta 3]}{\sqrt{2}} & 1 + \frac{\text{Cos}[\theta 3]}{\sqrt{2}} + \frac{\text{Sin}[\theta 3]}{\sqrt{2}} & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

τsx:

Out[22]//MatrixForm=

$$\begin{pmatrix} -\frac{5}{\sqrt{2}} \\ -\frac{5}{\sqrt{2}} \\ 0 \end{pmatrix}$$

τsy:

Out[25]//MatrixForm=

$$\begin{pmatrix} 5\left(2 + \frac{1}{\sqrt{2}}\right) \\ 5 + \frac{5}{\sqrt{2}} \\ 5 \end{pmatrix}$$

In[1]:= **(*Problem 4*)**

```
Print["Problem 4"];
```

$$\text{SkewSym}[\{a1_, a2_, a3_\}] := \begin{pmatrix} 0 & -a3 & a2 \\ a3 & 0 & -a1 \\ -a2 & a1 & 0 \end{pmatrix}$$

$$\text{Bracket}[\{a1_, a2_, a3_, a4_, a5_, a6_\}] := \begin{pmatrix} 0 & -a3 & a2 & a4 \\ a3 & 0 & -a1 & a5 \\ -a2 & a1 & 0 & a6 \\ 0 & 0 & 0 & 0 \end{pmatrix};$$

```
Adj[b_] :=
  Join[Join[b[[1 ;; 3, 1 ;; 3]], SkewSym[b[[1 ;; 3, 4]]].b[[1 ;; 3, 1 ;; 3]]],
```

```
    Join[Table[0, {3}, {3}], b[[1 ;; 3, 1 ;; 3]]], 2];

expS[S_, θ_] := MatrixExp[Bracket[Flatten[S]] * θ]
```

$$M = \begin{pmatrix} 1 & 0 & 0 & L1+L2+L3+L4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$S1s = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}; \; S2s = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ -L1 \\ 0 \end{pmatrix}; \; S3s = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ -(L1+L2) \\ 0 \end{pmatrix}; \; S4s = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ -(L1+L2+L3) \\ 0 \end{pmatrix};$$

$$B1 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ (L1+L2+L3+L4) \\ 0 \end{pmatrix}; \; B2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ (L2+L3+L4) \\ 0 \end{pmatrix}; \; B3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ L3+L4 \\ 0 \end{pmatrix}; \; B4 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ L4 \\ 0 \end{pmatrix};$$

```
Print["T04s:"];
(Ts = expS[S1s, θ1].expS[S2s, θ2].expS[S3s, θ3].expS[S4s, θ4].M // FullSimplify) //
 MatrixForm

(*Part b, Body Jacobian*)
(* Body frame Jacobian*)
(Vb1 = Adj[(expS[-B4, θ4]).(expS[-B3, θ3]).(expS[-B2, θ2])].B1 // FullSimplify) //
  MatrixForm;
(Vb2 = Adj[(expS[-B4, θ4]).(expS[-B3, θ3])].B2 // FullSimplify) // MatrixForm;
(Vb3 = Adj[(expS[-B4, θ4])].B3 // FullSimplify) // MatrixForm;
(Vb4 = B4 // FullSimplify) // MatrixForm;
Print["Jb:"];
(Jb = Join[Vb1, Vb2, Vb3, Vb4, 2]) // FullSimplify // MatrixForm

(* Torque Calculation part c*)
θ1 = 0; θ2 = 0; θ3 = Pi / 2; θ4 = -Pi / 2;
```

$$Fb = \begin{pmatrix} 0 \\ 0 \\ 10 \\ 10 \\ 10 \\ 0 \end{pmatrix};$$

```
Print["τc:"];
(τc = Jb^T.Fb // FullSimplify) // MatrixForm

(* Torque Calculation part d*)
```

```
θ1 = 0; θ2 = 0; θ3 = Pi / 2; θ4 = -Pi / 2;
```

$$\mathbf{Fb} = \begin{pmatrix} 0 \\ 0 \\ -10 \\ -10 \\ 10 \\ 0 \end{pmatrix};$$

```
Print["τd:"];
(τd = Jb^T.Fb // FullSimplify) // MatrixForm
```

Problem 4

T04s:

Out[10]//MatrixForm=

$$\begin{pmatrix} \mathrm{Cos}[\theta1+\theta2+\theta3+\theta4] & -\mathrm{Sin}[\theta1+\theta2+\theta3+\theta4] & 0 & \mathrm{L1\,Cos}[\theta1]+\mathrm{L2\,Cos}[\theta1+\theta2]+\mathrm{L3\,Cos}[\theta1+\theta2+ \\ \mathrm{Sin}[\theta1+\theta2+\theta3+\theta4] & \mathrm{Cos}[\theta1+\theta2+\theta3+\theta4] & 0 & \mathrm{L1\,Sin}[\theta1]+\mathrm{L2\,Sin}[\theta1+\theta2]+\mathrm{L3\,Sin}[\theta1+\theta2+ \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Jb:

Out[16]//MatrixForm=

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ \mathrm{L3\,Sin}[\theta4]+\mathrm{L2\,Sin}[\theta3+\theta4]+\mathrm{L1\,Sin}[\theta2+\theta3+\theta4] & \mathrm{L3\,Sin}[\theta4]+\mathrm{L2\,Sin}[\theta3+\theta4] & \mathrm{L3} \\ \mathrm{L4}+\mathrm{L3\,Cos}[\theta4]+\mathrm{L2\,Cos}[\theta3+\theta4]+\mathrm{L1\,Cos}[\theta2+\theta3+\theta4] & \mathrm{L4}+\mathrm{L3\,Cos}[\theta4]+\mathrm{L2\,Cos}[\theta3+\theta4] & \mathrm{L4+I} \\ 0 & 0 \end{pmatrix}$$

τc:

Out[20]//MatrixForm=

$$\begin{pmatrix} 10\,(1+\mathrm{L1}+\mathrm{L2}-\mathrm{L3}+\mathrm{L4}) \\ 10\,(1+\mathrm{L2}-\mathrm{L3}+\mathrm{L4}) \\ 10\,(1-\mathrm{L3}+\mathrm{L4}) \\ 10\,(1+\mathrm{L4}) \end{pmatrix}$$

τd:

Out[24]//MatrixForm=

$$\begin{pmatrix} 10\,(-1+\mathrm{L1}+\mathrm{L2}+\mathrm{L3}+\mathrm{L4}) \\ 10\,(-1+\mathrm{L2}+\mathrm{L3}+\mathrm{L4}) \\ 10\,(-1+\mathrm{L3}+\mathrm{L4}) \\ 10\,(-1+\mathrm{L4}) \end{pmatrix}$$

In[1]:= **(*Problem 5*)**

```
Print["Problem 5"];
```

$$\text{SkewSym}[\{a1_, a2_, a3_\}] := \begin{pmatrix} 0 & -a3 & a2 \\ a3 & 0 & -a1 \\ -a2 & a1 & 0 \end{pmatrix};$$

$$\text{Bracket}[\{a1_, a2_, a3_, a4_, a5_, a6_\}] := \begin{pmatrix} 0 & -a3 & a2 & a4 \\ a3 & 0 & -a1 & a5 \\ -a2 & a1 & 0 & a6 \\ 0 & 0 & 0 & 0 \end{pmatrix};$$

```
Adj[b_] :=
  Join[Join[b[[1 ;; 3, 1 ;; 3]], SkewSym[b[[1 ;; 3, 4]]].b[[1 ;; 3, 1 ;; 3]]],
   Join[Table[0, {3}, {3}], b[[1 ;; 3, 1 ;; 3]]], 2];
```

$$M = \begin{pmatrix} 1 & 0 & 0 & L1 + L2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};$$

$$S1s = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}; \; S2s = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ -L1 \\ 0 \end{pmatrix};$$

```
Print["T02s:"];
(Ts = MatrixExp[Bracket[Flatten[S1s]] * θ1[t]].
     MatrixExp[Bracket[Flatten[S2s]] * θ2[t]].M // FullSimplify) // MatrixForm
expS[S_, θ_] := MatrixExp[Bracket[Flatten[S]] * θ]

(* Space frame Jacobian*)
(Vs1 = S1s // FullSimplify) // MatrixForm;
(Vs2 = Adj[expS[S1s, θ1]].S2s // FullSimplify) // MatrixForm;

(Js = Join[Vs1, Vs2, 2]) // FullSimplify // MatrixForm;

Print["J:"];
D[Ts[[1 ;; 2, 4]], {{θ1[t], θ2[t]}}] /. θ1[t] → θ1 /. θ2[t] → θ2 // MatrixForm

(*D[Ts,t]//FullSimplify //MatrixForm
    {{θ1'[t]},{θ2'[t]},{0},{1}}//FullSimplify //MatrixForm

   D[Ts,t].{{θ1'[t]},{θ2'[t]},{0},{1}}//FullSimplify//MatrixForm*)

(*D[Ts,t].Inverse[Ts]//FullSimplify//MatrixForm

   Inverse[Ts].D[Ts,t]//FullSimplify//MatrixForm*)
```

Problem 5

T02s:

Out[8]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[\theta 1[t]+\theta 2[t]] & -\text{Sin}[\theta 1[t]+\theta 2[t]] & 0 & \text{L1 Cos}[\theta 1[t]]+\text{L2 Cos}[\theta 1[t]+\theta 2[t]] \\ \text{Sin}[\theta 1[t]+\theta 2[t]] & \text{Cos}[\theta 1[t]+\theta 2[t]] & 0 & \text{L1 Sin}[\theta 1[t]]+\text{L2 Sin}[\theta 1[t]+\theta 2[t]] \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

J:

Out[14]//MatrixForm=

$$\begin{pmatrix} -\text{L1 Sin}[\theta 1]-\text{L2 Sin}[\theta 1+\theta 2] & -\text{L2 Sin}[\theta 1+\theta 2] \\ \text{L1 Cos}[\theta 1]+\text{L2 Cos}[\theta 1+\theta 2] & \text{L2 Cos}[\theta 1+\theta 2] \end{pmatrix}$$