

ME449 HW 4

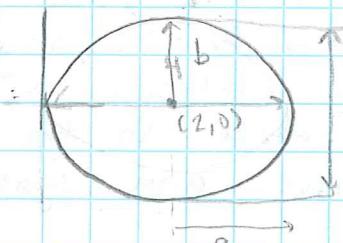
Problems: 9.1, 9.2, 9.21, 9.22, 9.25

54
56

Problem 1 : 9.1

1. Elliptical Path

Start: $(0, 0)$, cw: $(2, 1), (4, 0), (2, -1), (0, 0)$
 $s \in [0, 1]$



$$a = \frac{4}{2} = 2$$

center = $(2, 0)$

$$b = \frac{2}{2} = 1$$

6

$$\begin{cases} x = 2 - 2\cos 2\pi s \\ y = \sin 2\pi s \end{cases}$$

Problem 2 : 9.2

$$\mathbf{x} = (x, y, z) = \begin{bmatrix} \cos 2\pi s \\ \sin 2\pi s \\ 2s \end{bmatrix}$$

$$s(t) = 3t^3 + t^2 - 2t + 10, t \in [0, 2]$$

Write $\dot{\mathbf{x}}, \ddot{\mathbf{x}}$

$$\dot{\mathbf{x}} = \frac{d\mathbf{x}}{dt} = \frac{dx}{ds} \cdot \frac{ds}{dt} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -2\pi s \cdot \sin(2\pi s) \\ 2\pi s \cdot \cos(2\pi s) \\ 2 \end{bmatrix}$$

- See attached
Mathematica
for expansion

$$\ddot{\mathbf{x}} = \frac{d}{dt} \left(\frac{d\mathbf{x}}{dt} \right) = \frac{d\dot{\mathbf{x}}}{ds} \cdot \frac{ds}{dt} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} -4\pi^2 s^2 \cos(2\pi s) - 2\pi s \sin(2\pi s) \\ -4\pi^2 s^2 \sin(2\pi s) + 2\pi s \cos(2\pi s) \\ 2 \end{bmatrix}$$

8

- See attached Mathematica
code for expansion

14

Problem 3 : 9.21

The motion curve A is not possible as s cannot decrease (move backward) in this case, since \dot{s} is always positive, so it can only move forwards. Motion curve B is also not possible because at $\dot{s}=0$, s cannot change, so the curve at that point can only go straight up or down (normal to $\dot{s}=0$) without moving forward, change in s . C is possible because s is always increasing and is normal to $\dot{s}=0$.

6 For a similar reason, motion cones b, and c are not valid because at $\dot{s}=0$, the only valid motions are straight up or down (change in s), since it must accelerate before s can change. So, the only possible motion cones are "positive" and negative accelerations.

Problem 4 : 9.22

- 4 The curve integrated forward from $(s_{lim}, \dot{s}_{start})$ is the maximum deceleration curve. Since the curve F is also the maximum deceleration curve, and so they can't intersect because they are both decelerating, so it will either intersect with the velocity limit curve or $\dot{s}=0$.
- 4 The velocity limit curve represents the limit of inadmissible states, at which point the actuator commands cannot keep the robot on the path. So, the final-time scaling only allows for valid actuator commands, so it can only be tangential to the velocity curve, for it to be controllable.
- 3 In Bang-Bang control, the robot is either at max acceleration or deceleration. The velocity limit represents where min acceleration = max deceleration, so the robot should switch at times when the time scaling touches the velocity limit curve for the path to be time-optimal.

BUT WHY MAX NOT MIN?

CAN NOW SAFELY MAX ACCEL

(17)

```

(* Ahalya Prabhakar *)
(* ME449 HW4 Problem 2 9.2 *)
X[s_] := {{Cos[2 * Pi * s]}, {Sin[2 * Pi * s]}, {2 * s}};
s[t] = 3 * t^3 + t^2 - 2 * t + 6;
Print["Xdot:"]
(Xdot = FullSimplify[D[X[s[t]], t]]) // MatrixForm
Print["Xddot:"]
(Xddot = FullSimplify[D[Xdot, t]]) // MatrixForm
Xdot:

$$\begin{pmatrix} -2\pi(-2+t(2+9t))\sin[2\pi t(-2+t+3t^2)] \\ 2\pi(-2+t(2+9t))\cos[2\pi t(-2+t+3t^2)] \\ -4+2t(2+9t) \end{pmatrix}$$

Xddot:

$$\begin{pmatrix} -4\pi^2(-2+t(2+9t))^2\cos[2\pi t(-2+t+3t^2)]-4\pi(1+9t)\sin[2\pi t(-2+t+3t^2)] \\ 4\pi((1+9t)\cos[2\pi t(-2+t+3t^2)]-\pi(-2+t(2+9t))^2\sin[2\pi t(-2+t+3t^2)]) \\ 4+36t \end{pmatrix}$$


```

Problem 5 : 9.23

If the velocities at $s=0$ and $s=1$, then the \dot{s}_{start} and \dot{s}_{end} conditions will be different. Depending on these non-zero \dot{s}_{start} and \dot{s}_{end} conditions, a feasible trajectory is not necessarily guaranteed, as there may not exist a trajectory that does not pass through the velocity limit curve. \rightarrow UNDER WHAT SITUATIONS?

3

PROVIDE AN EXAMPLE,

START

END

MIN ACCEL
CURVE

Problem 6 : 9.25

$$\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} \pi/2 s \\ \pi/2 s \end{bmatrix} \quad s \in [0, 1]$$

$$\dot{\theta} = \frac{d\theta}{ds} \cdot \dot{s} = \begin{bmatrix} \pi/2 \dot{s} \\ \pi/2 \dot{s} \end{bmatrix} \quad \ddot{\theta} = \frac{d^2\theta}{ds^2} \dot{s}^2 + \frac{d\theta}{ds} \ddot{s} = \begin{bmatrix} \pi/2 \dot{s} \\ \pi/2 \dot{s} \end{bmatrix}$$

$$M(s) = M(\theta(s)) \frac{d\theta}{ds}$$

$$M(\theta) = \begin{bmatrix} (m_1 + m_2) L_1^2 + m_2(2L_1 L_2 \cos \theta_2 + L_2^2) & m_2(L_1 L_2 \cos \theta_2) + L_2^2 \\ m_2(L_1 L_2 \cos \theta_2) + L_2^2 & m_2 L_2^2 \end{bmatrix}$$

See attached Mathematica code for calculations.

$T_1 :$

$$M_1(s) = \frac{\pi}{2} (2L_2^2 m_2 + L_1^2 (m_1 + m_2) + 3L_1 L_2 m_2 \cos(\frac{\pi}{2}s))$$

$$C_1(s) = -\frac{3\pi^2}{4} L_1 L_2 m_2 \sin(\frac{\pi}{2}s)$$

$$g(s) = 0$$

$T_2 :$

$$M_2(s) = -\frac{\pi}{2} L_2 m_2 (2L_2 + L_1 \cos(\frac{\pi}{2}s))$$

$$C_2(s) = \frac{\pi^2}{4} L_1 L_2 m_2 \sin(\frac{\pi}{2}s)$$

$$g(s) = 0$$

(3)

L and u were determined from eqn. 9.35

```

Quit

(*Ahalya Prabhakar *)
(* ME449 HW5 Problem 6 (9.25) *)

q = {{θ1[t]}, {θ2[t]}};
dq = D[q, t];
ddq = D[dq, t];

p1 = {L1 * Cos[θ1[t]], L1 * Sin[θ1[t]]};
p2 =
{L1 * Cos[θ1[t]] + L2 * Cos[θ1[t] + θ2[t]], L1 * Sin[θ1[t]] + L2 * Sin[θ1[t] + θ2[t]]};
dp1 = D[p1, t];
dp2 = D[p2, t];

KE = 1/2 * m1 * dp1.dp1 + 1/2 * m2 * dp2.dp2;
PE = 0;
Print["The Lagrangian L is:"];
L = FullSimplify[KE - PE]

(Eq = FullSimplify[(Thread[D[D[L, dq^], t] - D[L, q^] == τ])]) // MatrixForm;

τ1 = Solve[Eq[[1]], τ][[1, 1, 2]] // FullSimplify;
τ2 = Solve[Eq[[2]], τ][[1, 1, 2]] // FullSimplify;
Print["τ1:"]
(FullSimplify[τ1 /. {θ1[t] → θ1, θ2[t] → θ2, θ1'[t] → θ1',
θ2'[t] → θ2', θ1''[t] → θ1'', θ2''[t] → θ2''}]) // MatrixForm
Print["τ2:"]
(FullSimplify[τ2 /. {θ1[t] → θ1, θ2[t] → θ2, θ1'[t] → θ1',
θ2'[t] → θ2', θ1''[t] → θ1'', θ2''[t] → θ2''}]) // MatrixForm

θ1[t] = Pi / 2 * s[t];
θ2[t] = Pi / 2 * s[t];

θ1'[t] = Pi / 2 * s'[t];
θ2'[t] = Pi / 2 * s'[t];

θ1''[t] = Pi / 2 * s''[t];
θ2''[t] = Pi / 2 * s''[t];

Print["τ1[s]:"]
FullSimplify[τ1]

Print["τ2[s]:"]
FullSimplify[τ2]

The Lagrangian L is:


$$\frac{1}{2} \left( (L2^2 m2 + L1^2 (m1 + m2) + 2 L1 L2 m2 \cos[\theta2[t]]) \theta1'[t]^2 + 2 L2 m2 (L2 + L1 \cos[\theta2[t]]) \theta1'[t] \theta2'[t] + L2^2 m2 \theta2'[t]^2 \right)$$


τ1:

```

$$-2 L1 L2 m2 \sin[\theta2] \theta1' \theta2' - L1 L2 m2 \sin[\theta2] (\theta2')^2 + \\ (L2^2 m2 + L1^2 (m1 + m2) + 2 L1 L2 m2 \cos[\theta2]) \theta1'' + L2 m2 (L2 + L1 \cos[\theta2]) \theta2''$$

 $\tau_2 :$

$$L2 m2 (L1 \sin[\theta2] (\theta1')^2 + (L2 + L1 \cos[\theta2]) \theta1'' + L2 \theta2'')$$

4 $\tau_1[s] :$

$$\frac{1}{4} \pi \left(-3 L1 L2 m2 \pi \sin\left[\frac{1}{2} \pi s[t]\right] s'[t]^2 + 2 \left(2 L2^2 m2 + L1^2 (m1 + m2) + 3 L1 L2 m2 \cos\left[\frac{1}{2} \pi s[t]\right] \right) s''[t] \right)$$

4 $\tau_2[s] :$

$$\frac{1}{4} L2 m2 \pi \left(L1 \pi \sin\left[\frac{1}{2} \pi s[t]\right] s'[t]^2 + 2 \left(2 L2 + L1 \cos\left[\frac{1}{2} \pi s[t]\right] \right) s''[t] \right)$$

(8)

```

% Ahalya Prabhakar
% ME449 HW4 Problem 6 (9.25)

clear all
close all
clc

% Set the grid points
S = linspace(0,1,6);

% Set the torque limits on the joints
maxt1 = 2;
maxt2 = 1;
mint1 = -2;
mint2 = -1;

% Define the constants
L1 = 1;
L2 = .5;
m1 = 1;
m2 = .5;

% Iterate through s = 0:.2:1 and sdot = 0:.2:1 and calculate and plot the
% motion cones for each (s, sdot point)
for i = 1:6
    for j = 1:6
        % Set s and sdot
        s = S(i);
        sdot = S(j);

        %Calculate M1(s) and C1(s)
        M1 = (1/4)*pi*2*(2*L2^2*m2 + L1^2*(m1 + m2) +...
            3*L1*L2*m2*cos(1/2*pi*s)) ;
        C1 = 1/4*pi*(-3*L1*L2*m2*pi*sin(1/2*pi*s));

        %Calculate the upper and lower bounds U and L based on the sign of
        % M1(s) based on eqns 9.35
        if M1 > 0
            LL1 = (mint1-C1*sdot^2)/M1;
            U1 = (maxt1-C1*sdot^2)/M1;

        elseif M1 < 0
            LL1 = (maxt1-C1*sdot^2)/M1;
            U1 = (mint1-C1*sdot^2)/M1;
        end

        %Calculate M2(s) and C2(s)
        M2 = (1/4)*L2*m2*pi*2*(2*L2 + L1*cos(1/2*pi*s));
        C2 = 1/4*L2*m2*pi*(L1*pi*sin(1/2*pi*s));

        %Calculate the upper and lower bounds U and L based on the sign of
        % M2(s) based on eqns 9.35
    end
end

```

```

if M2 > 0
    LL2 = (mint2-C2*sdot^2)/M2;
    U2 = (maxt2-C2*sdot^2)/M2;
elseif M2 < 0
    LL2 = (maxt2-C2*sdot^2)/M2;
    U2 = (mint2-C2*sdot^2)/M2;
end

% Set the output L and U
L = max([LL1 LL2]);
U = min([U1 U2]);
Lmat(i,j) = L;
Umat(i,j) = U;

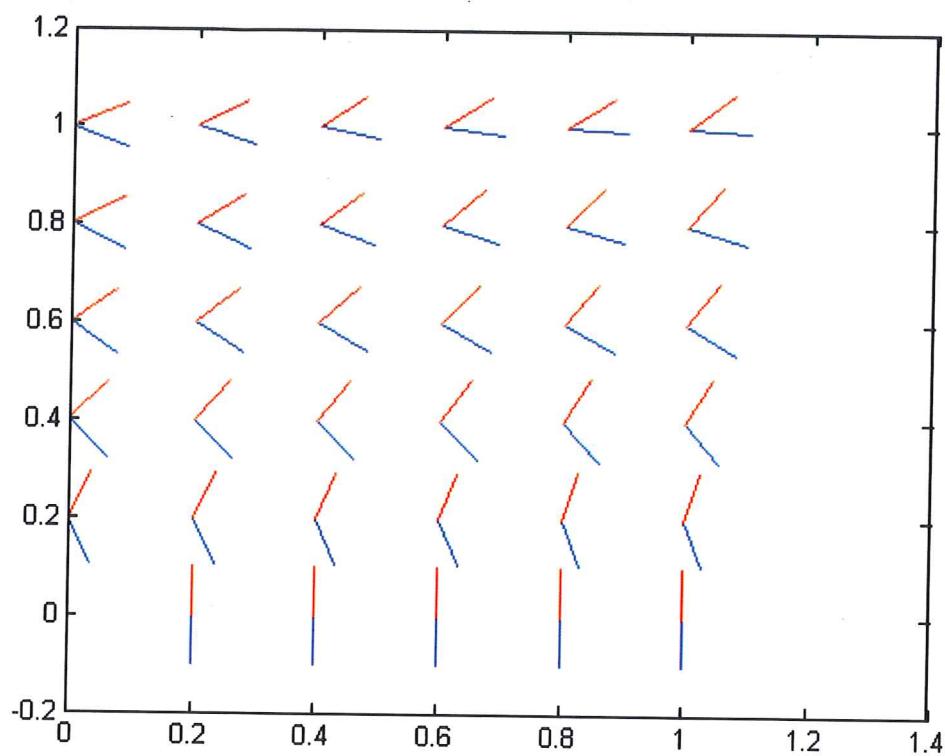
figure(1)

% Create the L and U vectors in order to normalize the plotted
% vectors
Lvec = [sdot,L];
Uvec = [sdot,U];

% Plot the resulting motion cones with a scaling of 1/10 and
% normalized vectors
p1 = [s, sdot];
scale = 10;
Lpt = p1 + Lvec/norm(Lvec)/scale;
Upt = p1 + Uvec/norm(Uvec)/scale;
plot([p1(1);Lpt(1)],[p1(2);Lpt(2)],'b')
hold on
plot([p1(1);Upt(1)],[p1(2);Upt(2)],'r')
hold on

end
end

```



Q

Published with MATLAB® R2013b

(12)