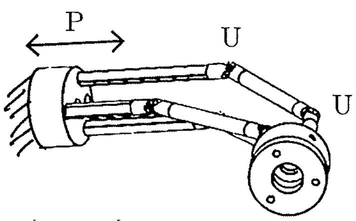
ME 449 Quiz 1
October 19, 2018

Name: Solutions

Always show your work or reasoning so your thought process is clear!

1. The experimental surgical manipulator shown below, developed at the National University of Singapore, is a parallel mechanism with three identical legs, each with a prismatic joint and two universal joints (the joints are marked for one of the legs). Use Grübler's formula to calculate the number of degrees of freedom of this mechanism.



each leg has 2 links & 3 joints
8 links total, including ground and moving platform
9 joints total (3 P, 6 U), with $3(1) + 6(2) = 15$ total freedoms

Grübler: $6(8-1-9) + 15 = 3$ dof

2. (a) Three rigid bodies move in space independently. How many degrees of freedom does this system of three bodies have?

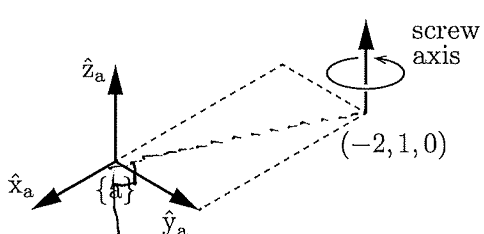
$3(6) = 18$ dof

(b) Now you constrain them so that each body must make contact with at least one of the other two bodies. (The bodies are allowed to slide and roll relative to each other, but they must remain in contact.) How many degrees of freedom does this system of three bodies have?

2 equality constraints, e.g., dist$(A, B) = 0$ and dist$(B, C) = 0$.

$18 - 2 = 16$ dof

3. The zero-pitch screw axis shown below, aligned with $\hat{z}_a$, passes through the point $(-2, 1, 0)$ in the $\{a\}$ frame. What is the twist $\mathcal{V}_a$ if we move about the screw axis at a speed $\dot{\theta} = 5$ rad/s?



$$\omega_a = \dot{\theta}(0,0,1) = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$$

$$\mathcal{V}_a = \begin{bmatrix} \omega_a \\ v_a \end{bmatrix}$$

$$v_a = \dot{\theta}\underbrace{(1,2,0)}_{-\omega_a \times \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}} = \begin{bmatrix} 5 \\ 10 \\ 0 \end{bmatrix}$$

$$\mathcal{V}_a = \begin{bmatrix} 0 \\ 0 \\ 5 \\ 5 \\ 10 \\ 0 \end{bmatrix}$$

4. A wrench F is represented in the {c} frame as $\mathcal{F}_c$. If $T_1 = T_{ab}$ is the configuration of the {b} frame relative to the {a} frame, and $T_2 = T_{ac}$ is the configuration of the {c} frame relative to the {a} frame, express $\mathcal{F}_b$ in terms of T_1, T_2, $\mathcal{F}_c$, and any math operations you need.

$$\hat{\mathcal{F}}_b = [Ad_{T_{cb}}]^T \hat{\mathcal{F}}_c \qquad T_{cb} = T_{ac}^{-1} T_{ab} = T_2^{-1} T_1$$

$$\boxed{\hat{\mathcal{F}}_b = [Ad_{T_2^{-1}T_1}]^T \hat{\mathcal{F}}_c}$$

or

$$\hat{\mathcal{F}}_a = [Ad_{T_{ac}^{-1}}]^T \hat{\mathcal{F}}_c$$

$$\hat{\mathcal{F}}_b = [Ad_{T_{ab}}]^T \hat{\mathcal{F}}_a$$

$$\boxed{\hat{\mathcal{F}}_b = [Ad_{T_1}]^T [Ad_{T_2^{-1}}]^T \hat{\mathcal{F}}_c}$$

5. Let the orientation of {b} relative to {a} be

$$R_{ab} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

and a point p be represented in {a} as $p_a = (1, 2, 3)$. What is p_b? (Give a numeric 3-vector.)

$$p_b = R_{ba} p_a = R_{ab}^T p_a = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$$

6. Consider three frames, {a}, {b}, and {c}. You know the representations of these frames in terms of the others, e.g., T_{ab} and T_{bc} (and therefore you can derive T_{ac} and the inverses of these matrices). Give a mathematical expression for the twist $\mathcal{V}_a$ you would need to follow for t seconds to move the {b} frame to be coincident with the {c} frame. Your answer should be symbolic (no numbers), and it should use t, any of the transformation matrices you need, and any math operations you need.

$$t[\mathcal{V}_b] = \log T_{bc}$$

$$[\mathcal{V}_b] = \frac{1}{t} \log T_{bc}$$

$$\mathcal{V}_a = [Ad_{T_{ab}}] \mathcal{V}_b$$

or

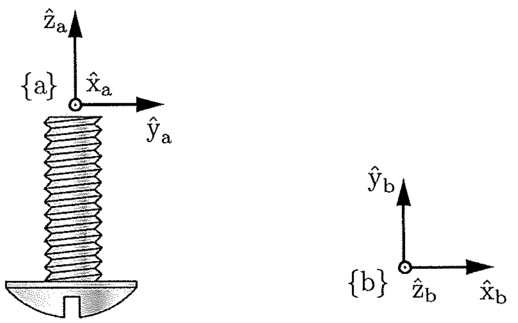
$$T_{ac} = e^{[\mathcal{V}_a]t} T_{ab}$$

$$T_{ac} T_{ab}^{-1} = e^{[\mathcal{V}_a]t}$$

$$\log(T_{ac} T_{ab}^{-1}) = [\mathcal{V}_a]t$$

$$\frac{1}{t} \log(T_{ac} T_{ab}^{-1}) = [\mathcal{V}_a]$$

7. The figure below shows a machine screw. As it advances into a tapped hole, it moves 5 mm linearly for every radian of rotation. A frame {a} has its $\hat{z}_a$-axis along the axis of the screw and its $\hat{x}_a$-axis out of the page. The frame {b} has its origin at $p_a = (0, 3, -2)$ and its orientation is shown in the figure ($\hat{z}_b$ is out of the page). Use mm as your linear units and radians as your angular units.



(a) What is the screw axis $\mathcal{S}_a$ corresponding to advancing into a tapped hole? Give a numerical 6-vector.

$$\mathcal{S}_a = (\underbrace{0, 0, 1}_{\mathcal{S}\omega_a}, \underbrace{0, 0, 5}_{\mathcal{S}v_a})$$

(b) What is the screw axis $\mathcal{S}_b$?

$$\mathcal{S}\omega_b = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \mathcal{S}v_b = \begin{bmatrix} 0 \\ 5 \\ -3 \end{bmatrix} \quad \mathcal{S}_b = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 5 \\ -3 \end{bmatrix}$$

(c) From the initial configuration T_{ab} shown in the figure, the {b} frame follows the screw an angle θ, ending at the final configuration $T_{ab'}$. If we write $T_{ab'} = TT_{ab}$, what is T? Express this symbolically (don't write numbers), using any of $\mathcal{S}_a$, $\mathcal{S}_b$, θ, and any math operations you need.

$$T_{ab'} = \underbrace{e^{[\mathcal{S}_a \theta]}}_{T} T_{ab}$$

(d) Referring to the previous question, if we instead write $T_{ab'} = T_{ab}T$, what is T? Again, express this symbolically (don't write numbers), using any of $\mathcal{S}_a$, $\mathcal{S}_b$, θ, and any math operations you need.

$$T_{ab'} = T_{ab} \underbrace{e^{[\mathcal{S}_b \theta]}}_{T}$$