1. Below is a partially filled-in data sheet for a DC motor. Using what you know about DC motors, fill in the rest of the data sheet.

Nominal voltage (volts) 20
Terminal resistance (ohms) 4
Terminal inductance (henries) 0.01
Max (no load) speed (radians/s) 10,000
Max power coils can continuously dissipate by ohmic heating before overheating (watts) 40
Friction torque (Nm) .... 0 <-- note: this is an ideal, unrealistic case!

Electrical constant k_e (Vs/rad) $\underline{0.002}$ $20V / 10000 \text{ rad/s} = 0.002 \text{ Vs}$

Torque constant k_t (Nm/A) $\underline{0.002}$ $k_e = k_t$ in SI units

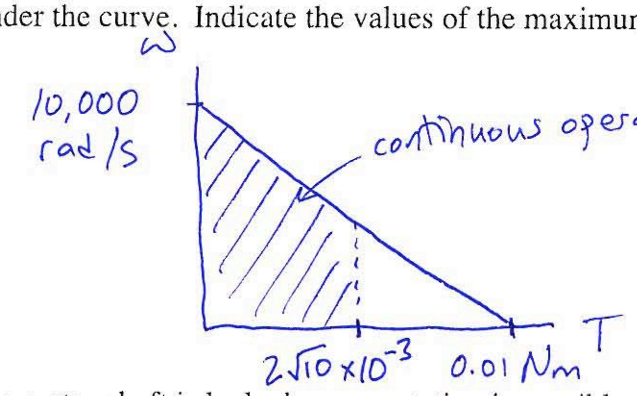
Stall current (A) $\underline{5}$ $20V / 4\Omega = 5A$

Stall torque (Nm) $\underline{0.01}$ $(0.002 \text{ Nm/A})(5A) = 0.01 \text{ Nm}$

Max continuous torque (Nm) $\underline{0.002\sqrt{10}}$ $I^2 R = 40W$, $I^2 = 40/4 = 10$, $I = \sqrt{10}\,A$ so $k_t \sqrt{10}$

Max mechanical power out (watts) $\underline{25}$ $P_{max} = \frac{1}{4} T_{max}\, \omega_{max} = \frac{1}{4}(0.01)(10{,}000) = 25 W$

Draw the speed-torque curve (speed as a function of torque) below and clearly indicate the continuous operating region under the curve. Indicate the values of the maximum speed and torque.



Now assume that the motor shaft is locked, so no rotation is possible. Initially the motor has zero current through it and zero voltage across it. You then power it with 10 V. At the instant after you power it, how much current is flowing? What is the rate of change of the current? Circle your two answers and give units.

Inductor does not allow discontinuous current, so $\boxed{\emptyset A}$ flowing.

$$V = L\frac{dI}{dT} = 0.01\frac{dI}{dt} = 10V$$

$$\boxed{\frac{dI}{dt} = 1000 \text{ A/s}}$$

2. The electrical power you put into a motor is converted to mechanical power plus heat. The less the coils are heated, the more efficient your motor is. Recall "efficiency" is mechanical power out divided by electrical power put in.

a. If your motor is (constantly) producing torque T at an angular velocity ω, give the efficiency of your motor in terms **only** of T, ω, the motor's resistance R, and the motor's torque constant k_t.

$$\eta = \frac{\text{Power out}}{\text{Power in}} = \frac{\overset{\leftarrow \text{mech power}}{T\omega}}{\underbrace{T\omega}_{\text{mech pover}} + \underbrace{I^2 R}_{\text{heat}}} = \frac{T\omega}{T\omega + \left(\frac{T}{k_t}\right)^2 R}$$

b. You have two motors: a low resistance motor with resistance R and torque constant k_t and a high resistance motor with resistance 2R and torque constant $2k_t$. Which motor is more efficient at the operating condition (T, ω)? There are three possible answers: "the low resistance motor," "the high resistance motor," or "it depends on the operating condition (T, ω)." Justify your answer in equations. No credit will be given for an answer without justification.

$$\eta_{low} = \frac{T\omega}{T\omega + \left(\frac{T}{k_t}\right)^2 R} \qquad \eta_{high} = \frac{T\omega}{T\omega + \left(\frac{T}{2k_T}\right)^2 (2R)}$$

$$= \frac{T\omega}{T\omega + \frac{T^2 R}{2k_t^2}}$$

We know $\frac{T^2}{k_t^2} R > \frac{T^2 R}{2k_t^2}$, so

the high resistance motor is more efficient at all (T, ω).

c. (Follow-on question from part a.) Realistically, some of the torque the motor produces is lost to friction, so $T = T_f + T_o$, where T is the torque produced by the motor, T_f is the torque to overcome friction, and T_o is the "output" torque available to actually drive the load. Give the "real" efficiency of your motor in terms **only** of T_f, T_o, ω, the motor's resistance R, and the motor's torque constant k_t.

$$\eta = \frac{T_o \omega}{(T_o + T_f)\omega + \left(\frac{T_o + T_f}{k_t}\right)^2 R}$$