Where we are:

Chap 2  Configuration Space
Chap 3  Rigid-Body Motions
Chap 4  Forward Kinematics
Chap 5  Velocity Kinematics and Statics
Chap 6  Inverse Kinematics
Chap 8  Dynamics of Open Chains
        8.1 Lagrangian Formulation
Chap 9  Trajectory Generation
Chap 11 Robot Control
Chap 13 Wheeled Mobile Robots
Important concepts, symbols, and equations

\[ \tau = M(\theta)\ddot{\theta} + c(\theta, \dot{\theta}) + g(\theta) \]

kinetic energy of a robot:

\[ \mathcal{K}(\theta, \dot{\theta}) = \frac{1}{2} \dot{\theta}^T M(\theta) \dot{\theta} \]

When \( \dot{\theta} = 0 \) and \( g = 0 \),

\( M(\theta) \) maps \( \ddot{\theta} \) to \( \tau \) and

\( M^{-1}(\theta) \) maps \( \tau \) to \( \ddot{\theta} \)
Important concepts, symbols, and equations (cont.)

If $V = J(\theta) \dot{\theta}$ is the e-e velocity and $J$ is invertible (there exists a unique joint velocity for each e-e velocity):

$$\frac{1}{2} V^T \Lambda(\theta) V = \frac{1}{2} \dot{\theta}^T M(\theta) \dot{\theta}$$

$$\dot{\theta}^T J^T(\theta) \Lambda(\theta) J(\theta) \dot{\theta} = \dot{\theta}^T M(\theta) \dot{\theta}$$

$$\Lambda(\theta) = J^{-T}(\theta) M(\theta) J^{-1}(\theta)$$

What if $J$ is tall? wide?

end-effector mass matrix
Important concepts, symbols, and equations (cont.)

When $\dot{\theta} = 0$ and $g = 0$, $\Lambda(\theta)$ maps $\dot{V}$ to $F$ and $\Lambda^{-1}(\theta)$ maps $F$ to $\dot{V}$

Force and acceleration are only parallel along principal axes.
\[ \tau = M(\theta)\ddot{\theta} + c(\theta, \dot{\theta}) + g(\theta) \]

\[ M(\theta) = \begin{bmatrix} I_1 + I_2 + m_1 L_1^2 + m_2 \theta_2^2 & 0 \\ 0 & m_2 \end{bmatrix} \]

What are the e-vals and e-vecs of \( M \)?

Draw the ellipse of \( \tau \) corresponding to a unit circle of \( \ddot{\theta} \) as \( \theta_2 \) increases from zero and \( I_1 = I_2 = m_1 = m_2 = L_1 = 1 \).
At $\theta_1 = 0$, the e-e mass matrix is

$$\Lambda(\theta) = \begin{bmatrix} m_2 & 0 \\ 0 & \frac{(I_1 + I_2 + m_1 L_1^2 + m_2 \theta_2^2)}{\theta_2^2} \end{bmatrix}$$

Draw the ellipse of $F$ corresponding to a unit circle of $\dot{V}$ as $\theta_2$ increases from zero and $I_1 = I_2 = m_1 = m_2 = L_1 = 1$. How does it change as $\theta_1$ changes?