ME 449 Robotic Manipulation
Homework 2
Due at the beginning of class Thursday May 10, 2012

Reading: emailed handout up to page 12; class notes
Office hours: Tues 11-12 (Vose, LIMS lab), Tues 5-6 (Lynch, B221), Wed 2-3 (Ryu, LIMS lab)

1. Parametric representations of surfaces.

   (a) In our derivation of the motion of a moving point on a moving surface, the expression

   $$\dot{s}^T \frac{\partial^2 r}{\partial s^2} \dot{s}$$

   arises, where $s = [u, v]^T$. This is a two-vector (row vector) multiplied by a three-dimensional tensor (representing the curvature of the body) multiplied by a two-vector (column vector), yielding a final column three-vector. Give the equation for each of the three entries of this final column vector in terms of $\dot{u}, \dot{v}, r_{uu}, r_{uv}, r_{vv}$.

   (b) Give a parametric representation $r(s)$ of the surface of an ellipsoid. The body's coordinate frame is centered at the center of the ellipsoid, and the coordinate axes are aligned with the principal axes of the ellipsoid. The half-lengths along the principal axes are a, b, and c in the body's x, y, and z axes, respectively (i.e., the end-to-end length of the ellipsoid in the x direction is $2a$). Make sure that the vector $r_u \times r_v$ is outward pointing.

   (c) For part (b), give the expressions for r_u, r_v, r_{uu}, r_{uv}, and r_{vv}.

2. Show that the first-order roll-slide constraint (24) in the reading can be expressed as ${}^sF^T\ ({}^sV_2 - {}^sV_1) = 0$, where ${}^sV_i$ is the spatial velocity of body i (the linear and angular velocity in a fixed space frame), and ${}^sF$ is the spatial *wrench*, expressed in the same frame, corresponding to a unit force along the contact normal of body 1. (The wrench is a 6-vector consisting of the force and torque.) You should completely transform constraint (24) into ${}^sF^T\ ({}^sV_2 - {}^sV_1) = 0$, or vice-versa, showing all the steps. A couple of relations you may find useful: $(AB)^T = B^T A^T$ and $[a]b = [b]^T a$.

   If body 1 is stationary (e.g., a stationary fixture), we usually just write the first-order roll-slide constraint as ${}^sF^T\ {}^sV = 0$, where ${}^sV$ is the velocity of body 2. The impenetrability constraint is ${}^sF^T\ {}^sV \geq 0$. (In lecture, we also used the notation $w^T t \geq 0$.)

3. Planar velocities, expressed in a space frame, are

   $${}^sV = (v_x, v_y, \cancelto{0}{v_z}, \cancelto{0}{\omega_x}, \cancelto{0}{\omega_y}, \omega_z) = (v_x, v_y, \omega_z).$$

   The following questions refer to planar velocities.

   (a) Express the one-dimensional set of velocities that correspond to a CCW (+) instantaneous rotation center at (x_c, y_c).

   (b) By hand, draw the three-dimensional cone of velocities in the (v_x, v_y, ω_z) space that corresponds to the positive linear span of + rotation centers at $(-1, 1)$, $(-1, -1)$, and $(x, 0)$, where x is a large negative number (essentially at $-\infty$).

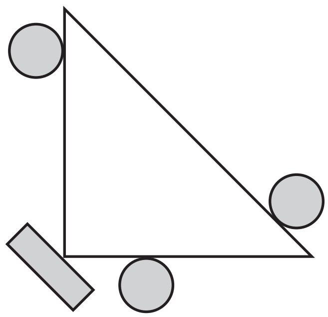
   (c) Draw the positive linear combination of the three rotation centers in the previous problem as a region of rotation centers in the plane.

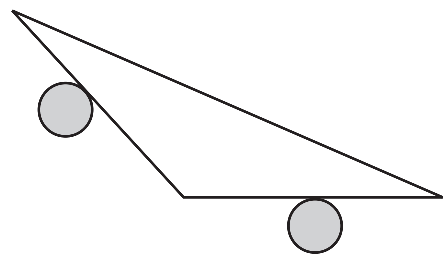
   (d) Draw the rotation centers corresponding to the positive linear combination of $-$ rotation centers at $(1, 0), (-1, 1), (-1, -1)$.

   (e) Draw the rotation centers corresponding to the positive linear combination of the three rotation centers in part (d) as well as a + rotation center at $(0, 0)$.

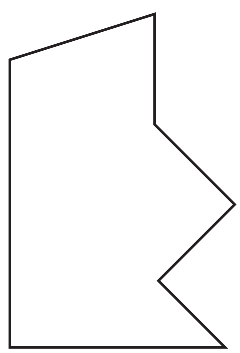
(f) Draw the rotation centers corresponding to the positive linear combination of $+$ rotation centers at $(1, 0)$ and $(-1, 0)$ and a $-$ rotation center at $(0, 0)$.

4. For the two figures below, the shaded bodies are stationary fixtures and the white bodies are movable objects. For each, use Reuleaux's method and indicate the feasible motions of the polygons as $+$ or $-$ rotation centers. For the figure on the right, provide also the contact labels for all the feasible motions (using `B`, `R`, `Sl`, and `Sr`). The label for the contact on the left should be given first.





5. Consider the object below, with concave vertices. A point contact at a concave vertex counts as two contacts, with normals perpendicular to the two edges incident at the vertex. Can the object below be first-order form-closure grasped by two point fingers? Three point fingers? Four point fingers? For each, if the answer is yes, draw a grasp that yields first-order form closure.



6. Two point fingers contact a planar object at $(0, 1)$ and $(0, -1)$. The boundaries of the object are given by segments of the lines $x = 1$ and $x = -1$, as well as smooth (infinitely differentiable) curves $y = f(x) > 0$ and $y = -f(x)$ for $x \in [-1, 1]$. $f(0) = 1$, where the object makes contact with a point finger, and $f'(0) = 0$.

   In this problem we will consider the motion constraints provided by different order analyses. If a first-order analysis indicates penetration or breaking, then this will not change by higher-order analysis. On the other hand, if the first-order analysis indicates roll-slide, a second-order analysis could indicate breaking or penetration. If it also indicates roll-slide, then a third-order analysis could indicate breaking or penetration. And so on. For all problems below, we require $f(0) = 1$ and $f'(0) = 0$.

   (a) By a first-order analysis, which velocities (expressed as rotation centers) are possible for the object? Call this set $\mathcal{V}$.

   (b) Give an example of $f(x)$ for which all the velocities $\mathcal{V}$ are actually possible.

   (c) Give an example of $f(x)$ for which some of $\mathcal{V}$ are possible, but not all.

   (d) Give an example of $f(x)$ for which no motion is actually possible by a second-order analysis (i.e., considering $f''(x)$). This is second-order form closure.

(e) Give an example of $f(x)$ which does not yield second-order form closure but does yield higher-order form closure.

7. Write commented Matlab code to check if a set of contacts yields first-order form closure. The input is a set of x and y coordinates of the contact points as well as the two components of the contact normals. Use the rank condition and the linear programming approach discussed in class. Verify your code is correct on samples you construct. We will also run your code on some sample problems. Turn in your code and the results on one example that fails the test and one that passes the test.

   Your function should be called `fctest` and should take a list of contacts and normals, as in the following command:

   ```
   fctest([[x1,y1,nx1,ny1];[x2,y2,nx2,ny2]; ...  ])
   ```

   Your function should use exactly this format for full credit.