To calculate the final term in this equation, we express $T_{i,i-1}$ and $\mathcal{A}_i$ as

$$T_{i,i-1} = \begin{bmatrix} R_{i,i-1} & p \\ 0 & 1 \end{bmatrix} \qquad \text{and} \qquad \mathcal{A}_i = \begin{bmatrix} \omega \\ v \end{bmatrix}.$$

From the fact $\dot{T}_{i,i-1}T_{i,i-1}^{-1} = -[\mathcal{A}_i\dot{\theta}_i]$, we have

$$\dot{R}_{i,i-1} = -[\omega\dot{\theta}_i]R_{i,i-1}, \quad \dot{p} = -[\omega\dot{\theta}_i]p - v\dot{\theta}_i.$$

Then

$$\begin{aligned}
&\frac{d}{dt}([\mathrm{Ad}_{T_{i,i-1}}])\mathcal{V}_{i-1} \\
&\quad = \frac{d}{dt}\begin{bmatrix} R_{i,i-1} & 0 \\ [p]R_{i,i-1} & R_{i,i-1} \end{bmatrix}\mathcal{V}_{i-1} \\
&\quad = \begin{bmatrix} -[\omega\dot{\theta}_i]R_{i,i-1} & 0 \\ [-[\omega\dot{\theta}_i]p - v\dot{\theta}_i]R_{i,i-1} - [p][\omega\dot{\theta}_i]R_{i,i-1} & -[\omega\dot{\theta}_i]R_{i,i-1} \end{bmatrix}\mathcal{V}_{i-1} \\
&\quad = \underbrace{\begin{bmatrix} -[\omega\dot{\theta}_i] & 0 \\ -[v\dot{\theta}_i] & -[\omega\dot{\theta}_i] \end{bmatrix}}_{-[\mathrm{ad}_{\mathcal{A}_i\dot{\theta}_i}]}\underbrace{\begin{bmatrix} R_{i,i-1} & 0 \\ [p]R_{i,i-1} & R_{i,i-1} \end{bmatrix}}_{[\mathrm{Ad}_{T_{i,i-1}}]}\mathcal{V}_{i-1} \\
&\quad = -[\mathrm{ad}_{\mathcal{A}_i\dot{\theta}_i}]\mathcal{V}_i \\
&\quad = [\mathrm{ad}_{\mathcal{V}_i}]\mathcal{A}_i\dot{\theta}_i,
\end{aligned}$$

where the transition from the second equality to the third follows from the Jacobi identity $a \times (b \times c) + b \times (c \times a) + c \times (a \times b) = 0$ for all $a, b, c \in \mathbb{R}^3$, and the transition from the fourth equality to the fifth follows from the identity $[\mathrm{ad}_{\mathcal{V}_1}]\mathcal{V}_2 = -[\mathrm{ad}_{\mathcal{V}_2}]\mathcal{V}_1$. Substituting this result into Equation (8.46), we get ...